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Edge-iterated graph parameters: theory and applications to wireless sensor networks

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ABSTRACT

In this paper, we study three graph parameters–the iterated chromatic number $\chi^n(G)$, the iterated domination number $\gamma^n(G)$, and the iterated covering number $\beta^n(G)$—through the line-graph transformations applied successively on a connected graph G. These edge-iterated parameters track how coloring, domination, and covering structures evolve as the graph undergoes successive line-graph transformations. For standard graph families such as paths, cycles, and grid graphs, we derive exact formulas and establish upper and lower bounds, revealing both periodic and divergent behaviours depending on graph structure. Since exact computation of these parameters becomes intractable for large networks, we propose a BFS-based greedy algorithm for estimating $\chi^n(G)$, and benchmark it against the Welsh–Powell and DSATUR algorithms. The simulation results validate the theoretical bounds and show that the proposed method is computationally efficient without significant loss in coloring quality. We further show that these parameters have natural interpretations in wireless sensor networks (WSNs): $\chi^n(G)$ informs frequency and time-slot assignment under multi-hop interference, $\gamma^n(G)$ identifies minimal supervisory structures for fault tolerance, and $\beta^n(G)$ guides energy-aware link monitoring. The framework thus connects iterated graph theory to concrete design problems in sensor network optimization.

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1. INTRODUCTION

Iterated graph parameters under line graph transformations reveal deep structural insights and have notable implications for real-world network systems. Let G be a connected graph. The chromatic number $\chi(G)$, the domination number $\gamma(G)$, and the covering number $\beta(G)$ are three classical invariants with rich theoretical foundations and numerous applications in scheduling, coverage, control, and fault-tolerant design [1]–[3].

A fundamental graph transformation is the line graph $L(G)$, whose vertices correspond to the edges of G, with adjacency determined by edge incidence in G. The operation of taking successive line graphs gives

rise to iterated line graphs $L^n(G)$, where $L^1(G) = L(G)$ and $L^n(G) = L(L^{n-1}(G))$ for $n \geq 2$. This concept was explored in detail by Beineke and Bagga [4]. The graph invariants of this iterated structure is a hot area of research [5]–[10]. For each such iterated structure, one may define corresponding edge-iterated parameters as follows:
- The edge-iterated chromatic number is defined by $\chi^n(G) = \chi(L^n(G))$,
- The edge-iterated domination number is defined by $\gamma^n(G) = \gamma(L^n(G))$,
- The edge-iterated covering number is defined by $\beta^n(G) = \beta(L^n(G))$.

These parameters show how the complexity and control potential of a graph change as it goes through a series of line-graph transformations. A lot of research has been done on chromatic and domination parameters in classical graph theory [1]–[3], [11]–[15], but not much has been done on how they behave when graphs are iterated under the line-graph transformations. In this paper, we look at $\chi^n(G), \gamma^n(G), \beta^n(G)$ for different types of graphs in a unified way. We find the exact values and limits for these parameters, figure out how they repeat, and look at how they behave as they get bigger. To deal with the problems that come up when calculating edge iterated chromatic numbers, we suggest a greedy algorithm for estimating $\chi^n(G)$. This algorithm works well on large, sparse graphs. The study's practical purpose comes from its use in wireless sensor networks (WSNs), where graph-theoretic models are used to improve communication efficiency, coverage, and reliability [16]–[19]. The edge-iterated graph parameters, like the edge-iterated chromatic number, the edge-iterated domination number, and the edge-iterated covering number, are helpful for assigning frequencies and minimising interference. They also help finding fault-tolerant supervisory structures and support energy-efficient network coverage. This makes them a mathematically sound way to design, operate, fault-tolerant, and resilient sensor node networks.

In graph theory, the chromatic number $\chi(G)$, the domination number $\gamma(G)$, and the covering number $\beta(G)$ are some of the most important optimization parameters. Harary [2] gives an easy-to-understand introduction to vertex and edge coloring and the chromatic number. Cockayne and Hedetniemi [1] do the same for domination sets and the domination number. Monographs by Haynes *et al.* [3], Haynes and Slater [15], and Sampathkumar and Walikar [12] give great detail about domination variants. Covering numbers have also been used to solve problems with reliability and resource allocation [16], [20]. The line graph transformation, where vertices stand for the edges of the original graph, gives us a natural framework to look at link-centric properties. Beineke and Bagga [4] studied the iterated line graphs $L^n(G)$, but not much research has been done on how the classical invariants behave when they are iterated. Sreelatha *et al.* [21] investigated the edge-iterated independence number and the edge-iterated clique number and talked about how they could be used to improve WSNs.

For large graphs, it is impossible to calculate these three graph invariants exactly using an algorithm. This is why heuristics are used. Greedy coloring is often used for sparse networks, but DSATUR and Welsh–Powell offer stronger baselines with different trade-offs between quality and runtime. Structure-aware methods, like Telle's decomposition strategies for partial k-trees [22], show how important it is to use graph properties. Our edge-aware greedy algorithm builds on this work by taking advantage of the structure of iterated line graphs, which makes it more scalable and accurate in the iterated setting. Graph-theoretic models are very important for communication, scheduling, and reliability in WSNs [23]–[28]. Graph coloring is important for assigning frequencies and time slots, domination is important for clustering and fault tolerance, and covering helps with energy-efficient monitoring [20]. Graph-based monitoring is also used in IoT-driven deployments [29], and chromatic partitioning and sensitivity analysis are talked about in [18], [19], [13]. Most WSN studies, on the other hand, use vertex-centric formulations, like power graphs for k-hop neighbourhoods. Link-centric approaches, on the other hand, have not been studied as much.

Table 1 shows a comparison of previous research in this area and show the research gap. The table shows that current research either looks at single invariants or uses classical models on WSNs without taking iterations into account. None of the current methods use or combine the edge-iterated chromatic number, the edge-iterated domination number, and the edge-iterated covering number in a single framework for WSN. This gap is what led us to create our unified framework, which uses these edge-iterated parameters to solve problems with scalability, reliability, and fault tolerance in WSNs.

The principal contributions of this paper are summarized as follows:
- We present a unified analytical framework to study three edge-iterated graph parameters together: the chromatic number $\chi^n(G)$, the domination number $\gamma^n(G)$, and the covering number $\beta^n(G)$. These parameters are defined through a series of line-graph transformations of a connected graph G. Unlike earlier work that

studies each parameter separately or avoids iteration, our approach treats them in a combined setting.
- For basic graph classes such as paths, cycles, and grid graphs, we obtain exact values and bounds for the edge-iterated chromatic, domination, and covering numbers. These findings help explain how graph structure changes when line-graph operations are applied repeatedly.
- We introduce a BFS-based greedy algorithm to compute the edge-iterated chromatic number $\chi^n(G)$. The method makes use of the structural features of iterated line graphs and performs well on large, sparse graphs.
- We compare the proposed algorithm with standard graph coloring methods, including Welsh–Powell and DSATUR. The results show that our algorithm produces similar chromatic numbers while requiring considerably less computation time.
- We also demonstrate the practical usefulness of the framework in WSNs. In this setting, edge-iterated graph parameters assist in frequency allocation, interference reduction, fault tolerance, and the design of energy-efficient communication backbones.

Table 1. Summary of prior work on iterated graph parameters and WSN applications

Author(s)	Parameter(s)	Graph class	WSN focus	Contribution	Gap addressed in this paper
Harary [2]	$\chi(G)$	General graphs	None	Introduced chromatic number	No iteration, no WSN
Cockayne and Hedetniemi [1]	$\gamma(G)$	General graphs	None	Early domination theory	No iteration, no WSN
Beineke and Bagga [4]	Line graphs	General graphs	None	Extended to $L^n(G)$	Did not link to WSN invariants
Haynes *et al.* [3], Haynes and Slater [15]	Domination variants	General graphs	None	Comprehensive survey	No iterated framework
Fattah and Leung [17]	$\chi(G)$	Wireless networks	Indirect	Scheduling via coloring	Vertex-based, not iterated
Albert *et al.* [20]	Robustness metrics	Complex networks	Indirect	Redundancy-aware design	No edge-iterated modeling
Sreelatha *et al.* [21]	Independence, clique	Iterated line graphs	None	Edge-iterated graph parameters	No WSN applications
Proposed work	$\chi^n(G), \gamma^n(G), \beta^n(G)$	Iterated line graphs	Direct	Unified framework, WSN case studies	**First integration with WSN design**

2. RELATED WORK

Graph invariants such as the chromatic number $\chi(G)$, domination number $\gamma(G)$, and covering number $\beta(G)$ have long been central topics in classical graph theory. They play key roles in optimization and decision-making problems on graphs. Harary [2] introduced chromatic concepts in the context of vertex and edge coloring. Cockayne and Hedetniemi [1] established early foundations in domination theory. Detailed treatments of domination and its variants, including connected and paired domination, are given in the monographs by Haynes *et al.* [3], Haynes and Slater [15], and Sampathkumar and Walikar [12]. Covering numbers, closely related to domination, have also been studied in connection with network reliability and resource allocation.

The line graph transformation, where vertices correspond to edges of the original graph, provides an important structural tool for studying edge-based properties. Beineke and Bagga [4] formalized this transformation and examined iterated line graphs $L^n(G)$, focusing on structural behavior and convergence. Although chromatic properties of line graphs have been widely studied, much less is known about how classical invariants behave under repeated line-graph iterations. Sreelatha *et al.* [21] recently contributed to this direction by introducing edge-iterated independence and clique numbers.

Computing graph invariants exactly becomes difficult for large networks. This is especially true under successive line-graph transformations. As a result, heuristic and structure-based methods are often used. In graph coloring, greedy algorithms are popular because they are simple and perform well on sparse graphs. Our proposed method for estimating $\chi^n(G)$ follows this idea. While exact computation is impractical in iterated settings, the greedy approach provides a scalable approximation for large networks. Structural decomposition techniques, such as Telle's work on partial k-trees [22], offer additional strategies that may improve performance for special graph classes.

Graph-theoretic parameters are widely applied in WSNs. They help model communication structures, schedule transmissions, maintain robustness, and reduce energy consumption. Fattah and Leung [17] discussed scheduling methods based on graph coloring for frequency and time-slot assignment. Chromatic partitioning and sensitivity analysis have also been explored in [18], [19]. The impact of faults and attacks in complex networks was studied by Albert *et al.* [20], who emphasized the importance of redundancy-aware design. This connects naturally with domination-based approaches for supervision and fault tolerance in WSNs.

Related ideas appear in other scientific fields as well. For example, structural models in quantum physics, such as Feynman graphs [30], use layered graph constructions that resemble iterated models. These similarities suggest that edge-iterated parameters may have applications beyond communication networks. Overall, these studies highlight the importance of examining edge-iterated chromatic, domination, and covering numbers within a unified framework. Such an approach strengthens theoretical understanding of iterated graph transformations and supports the design of efficient and fault-tolerant sensor node networks.

3. PRELIMINARIES

Throughout this paper, we consider finite, simple, connected graphs. For a graph G, let $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively. The degree of a vertex $v \in V(G)$ is denoted by $\deg_G(v)$, or simply $\deg(v)$ when the graph is clear from context.

3.1. Classical graph parameters

The chromatic number of a graph G, denoted $\chi(G)$, is the minimum number of colors required to color the vertices of G such that adjacent vertices receive different colors.

A subset $S \subseteq V(G)$ is called a dominating set if every vertex $v \in V(G) \setminus S$ is adjacent to some vertex in S. The *domination number* of G, denoted $\gamma(G)$, is the minimum cardinality of a dominating set in G.

A subset $S \subseteq V(G)$ is called a covering (or vertex cover) if every edge of G has at least one endpoint in S. The covering number of G, denoted $\beta(G)$, is the minimum cardinality of a covering set.

3.2. Line graph and iterated line graph

The line graph of a graph G, denoted $L(G)$, is the graph whose vertices correspond to the edges of G, where two vertices in $L(G)$ are adjacent if and only if their corresponding edges in G share a common vertex.

For $n \geq 1$, the n-fold iterated line graph of G, denoted $L^n(G)$, is defined recursively by:

$$L^1(G) = L(G), \quad L^n(G) = L(L^{n-1}(G)) \text{ for } n \geq 2.$$

3.3. Edge-iterated parameters

Using the iterated line graph, we define the edge-iterated versions of the classical parameters as follows:

- The edge-iterated chromatic number of G is defined as:

$$\chi^n(G) = \chi(L^n(G)).$$

- The edge-iterated domination number of G is defined as:

$$\gamma^1(G) = \gamma(L(G)), \quad \gamma^n(G) = \gamma(L^n(G)) \text{ for } n \geq 2.$$

- The edge-iterated covering number of G is defined as:

$$\beta^1(G) = \beta(L(G)), \quad \beta^n(G) = \beta(L^n(G)) \text{ for } n \geq 2.$$

These edge-iterated parameters describe how the chromatic, domination, and covering characteristics of a graph evolve under successive line graph transformations.

4. MAIN RESULTS

In this section, we present the main theoretical results for the edge-iterated chromatic number $\chi^n(G)$, edge-iterated domination number $\gamma^n(G)$, and edge-iterated covering number $\beta^n(G)$ for different graph classes. The definitions and notations used are consistent with those established in the Preliminaries.

4.1. Edge-iterated chromatic number $\chi^n(G)$

We begin by formally introducing the concept of the edge-iterated chromatic number for a graph G. Throughout this study, we assume that G is a connected graph. This assumption is justified because, for a disconnected graph, the chromatic number can be determined by computing the chromatic number of each connected component individually—the maximum among these values then serves as the chromatic number for the entire graph.

Definition 4.1. *Let G be a connected graph, and let $\chi(G)$ denote the chromatic number of G. Define the first edge-iterated chromatic number of G as $\chi^1(G) = \chi(L(G))$, where $L(G)$ is the line graph of G. Recursively, define:*

$$\chi^2(G) = \chi(L^2(G)) = \chi(L(L(G))),$$

and in general, the k-th edge-iterated chromatic number of G is given by:

$$\chi^k(G) = \chi(L^k(G)),$$

where $L^k(G)$ denotes the k-fold iterated line graph of G, that is,

$$L^k(G) = L(L^{k-1}(G)), \quad \text{with } L^0(G) = G.$$

Theorem 1 (edge-iterated chromatic number of a path)**.** *Let P_n be a path with n vertices. Then the edge-iterated chromatic number of P_n is given by:*

$$\chi^k(P_n) = \begin{cases} 2, & \text{for } 1 \leq k \leq n-2, \\ 1, & \text{for } k = n-1. \end{cases}$$

Proof. The line graph of P_n, denoted $L(P_n)$, is isomorphic to P_{n-1}. P_n has n vertices and $(n-1)$ edges. Since each edge in a path shares a vertex with at most two other edges, $L(P_n)$ is again a path P_{n-1}. So, $L(P_n) \cong P_{n-1}$. Hence,

$$\chi^1(P_n) = \chi(L(P_n)) = \chi(P_{n-1}) = 2.$$

Continuing the iteration, we have:

$$\chi^2(P_n) = \chi(L^2(P_n)) = \chi(P_{n-2}) = 2.$$

In general, for all k such that $n - k \geq 2$, the k-th iterated line graph, $L^k(P_n)$, is a path P_{n-k}, which has chromatic number 2. That is,

$$\chi^k(P_n) = \chi(L^k(P_n)) = \chi(P_{n-k}) = 2 \quad \forall (n-k) \geq 2$$

or,

$$\chi^k(P_n) = 2, \quad \text{for } 1 \leq k \leq n-2.$$

Finally, when $k = n - 1$, the graph $L^{n-1}(P_n)$ consists of a single vertex (i.e., K_1), which has chromatic number 1. Thus,

$$\chi^{n-1}(P_n) = \chi(L^{n-1}(P_n)) = \chi(K_1) = 1.$$

Theorem 2 (edge-iterated chromatic number of a cycle)**.** *Let C_n be a cycle with $n \geq 3$ vertices. Then the edge-iterated chromatic number of C_n is given by:*

$$\chi^k(C_n) = \begin{cases} 2, & \text{if } n \text{ is even}, \\ 3, & \text{if } n \text{ is odd} \end{cases} \quad \text{for all } k \geq 1.$$

Proof. The line graph of a cycle C_n, denoted $L(C_n)$, is again C_n. Therefore,

$$\chi^1(C_n) = \chi(L(C_n)) = \chi(C_n) = \begin{cases} 2, & \text{if } n \text{ is even,} \\ 3, & \text{if } n \text{ is odd.} \end{cases}$$

Similarly,

$$\chi^2(C_n) = \chi(L^2(C_n)) = \chi(C_n),$$

and this pattern continues for all $k \geq 1$, as each iteration yields the same cycle C_n. Hence,

$$\chi^k(C_n) = \chi(C_n) = \begin{cases} 2, & \text{if } n \text{ is even,} \\ 3, & \text{if } n \text{ is odd,} \end{cases} \quad \text{for all } k \geq 1.$$

Lemma 1 (lower bound for edge-iterated chromatic number)**.** *If a connected graph G has an induced subgraph isomorphic to K_3 or $K_{1,3}$, then*

$$\chi^k(G) \geq 3 \quad \textit{for all } k \geq 1.$$

Proof. If G contains an induced subgraph isomorphic to K_3 or $K_{1,3}$, then its k-th iterated line graph $L^k(G)$ contains a triangle for all $k \geq 1$. Since the chromatic number of a graph that contains a triangle is at least 3, it follows that:

$$\chi^k(G) = \chi(L^k(G)) \geq 3.$$

Corollary 1 (lower bound for edge-iterated chromatic number of K_n)**.** *Let K_n be the complete graph with n vertices, where $n \geq 3$. Then,*

$$\chi^k(K_n) \geq 3 \quad \textit{for all } k \geq 1.$$

Proof. The result follows directly from Lemma 1, since K_n contains an induced subgraph isomorphic to K_3 for all $n \geq 3$. Therefore,

$$\chi^k(K_n) \geq 3 \quad \text{for all } k \geq 1.$$

Corollary 2 (lower bound for edge-iterated chromatic number of $K_{m,n}$)**.** *Let $K_{m,n}$ be a complete bipartite graph with parts m and n. If $\max\{m, n\} \geq 3$, then*

$$\chi^k(K_{m,n}) \geq 3 \quad \textit{for all } k \geq 1.$$

Proof. When $\max\{m, n\} \geq 3$, the graph $K_{m,n}$ contains an induced subgraph isomorphic to $K_{1,3}$. By Lemma 1, it follows that

$$\chi^k(K_{m,n}) \geq 3 \quad \text{for all } k \geq 1.$$

Theorem 3 (edge-iterated chromatic number for trees with maximum degree $\Delta(G) \geq 3$)**.** *If G is a tree with maximum degree $\Delta(G) \geq 3$, then*

$$\chi^k(G) \geq 3 \quad \textit{for all } k \geq 1.$$

Proof. Since G is a tree with $\Delta(G) \geq 3$, it contains a vertex v with degree at least 3. The vertex v together with three of its neighbors forms an induced subgraph isomorphic to $K_{1,3}$. By Lemma 1, it follows that

$$\chi^k(G) \geq 3 \quad \text{for all } k \geq 1.$$

Theorem 4 (edge-iterated chromatic number for trees with maximum degree $\Delta(G) = 2$)**.** *Let G be a tree with n vertices and maximum degree $\Delta(G) = 2$. Then,*

$$\chi^k(G) = \begin{cases} 2, & 1 \leq k \leq n-2, \\ 1, & k = n-1. \end{cases}$$

Proof. If G is a tree with $\Delta(G) = 2$, then G must be a path on n vertices. By Theorem 1, it follows that

$$\chi^k(G) = \begin{cases} 2, & 1 \leq k \leq n-2, \\ 1, & k = n-1. \end{cases}$$

Theorem 5 (edge-iterated chromatic number for trees with maximum degree $\Delta(G) = 1$)**.** *Let G be a tree with maximum degree $\Delta(G) = 1$. Then,*

$$\chi(G) = 2 \quad \textit{and} \quad \chi^2(G) = 1.$$

Proof. If G is a tree with $\Delta(G) = 1$, then G must be a path on 2 vertices. Therefore, the chromatic number is $\chi(G) = 2$, and since the line graph of G has only one vertex, we get $\chi^2(G) = 1$.

Theorem 6 (limiting behaviour of edge-iterated chromatic number)**.** *Let G be a connected graph that is neither a cycle, nor a path, nor isomorphic to $K_{1,3}$. Then,*

$$\lim_{k\to\infty} \chi^k(G) = \infty.$$

Proof. If G is a connected graph not equal to a cycle, a path, or $K_{1,3}$, then

$$\lim_{k\to\infty} \delta(L^k(G)) = \infty,$$

where δ denotes the minimum degree. Since the edges incident at a vertex in G form a clique in $L(G)$, the iterated line graphs $L^k(G)$ will contain arbitrarily large cliques. As a result, their chromatic numbers will also grow without bound. Hence,

$$\lim_{k\to\infty} \chi^k(G) = \infty.$$

4.2. A greedy algorithm for the edge-iterated chromatic number

In this section, we propose a greedy algorithm, presented in Algorithm 1 to compute the edge-iterated chromatic number of a graph. The algorithm follows a constructive approach that sequentially assigns colors to the edges of a graph while maintaining the constraint that no two adjacent edges share the same color. To do this efficiently, we designed a function called `EDGE-GREEDY-CHROM`. The function traverses the edges of the graph using a breadth-first search. Every time an edge is traversed, it assigns a color to that edge. We pick the smallest available color not used by any of its adjacent edges. This approach guarantees correct coloring without backtracking or advanced decision-making.

This function follows a greedy approach. For every vertex, it uses the smallest available color that does not clash with its incident edges. This is an easy, practical, and intuitive method for approximating the edge-chromatic number. It is especially helpful when calculating the exact chromatic index is computationally expensive. Our method is easy to implement and exhibits encouraging results on a wide range of graph classes, and hence it is a good heuristic for real-world applications.

The EDGE-GREEDY-CHROM(G) estimates the edge-chromatic number of a graph G using a greedy coloring algorithm that integrates breadth-first edge traversal. To illustrate the functioning of our proposed algorithm, we consider a sample graph, as depicted in Figure 1. We begin by performing a breadth-first traversal, starting from edge a. The traversal proceeds in the following order: we first visit edge a, then move to its neighboring edges b and c. From edge b, we proceed to visit its neighboring edge d, followed by edge e, which is adjacent to edge c. This results in the following BFS ordering of the edges in the graph G:

$$a \to b \to c \to d \to e.$$

Starting with edge a, we assign color 1 and set the edge chromatic number (ECN) to 1. Next, edge b is adjacent to a (color 1), so we assign b color 2, updating ECN to 2. Then edge c is adjacent only to a (color 1), so c also receives color 2, and ECN remains 2. Edge d is adjacent to b (color 2), so we assign d color 1, leaving ECN at 2. Finally, edge e is adjacent to c (color 2), and thus e is assigned color 1, with ECN remaining 2 throughout.

Algorithm 1 Edge-iterated chromatic number

Require: Graph G; Positive integer n
Ensure: $\chi^n(G)$: The n-th edge-iterated chromatic number of G
$G_0 \leftarrow G$
$ECN_0 \leftarrow$ EDGEGREEDYCHROM(G_0)
for $k = 1$ to n **do**
    $G_k \leftarrow L(G_{k-1})$
    $ECN_k \leftarrow$ EDGEGREEDYCHROM(G_k)
end for
return ECN_n

Procedure EDGEGREEDYCHROM(Graph G)
Create an empty queue Q and visited edge set
Select an arbitrary edge $e_0 \in E(G)$
Enqueue e_0 into Q and mark it visited
Initialize dictionary for edge colors
$ECN \leftarrow 1$
Assign color 1 to e_0
while Q not empty **do**
    Dequeue edge e
    for each adjacent unvisited edge e' of e **do**
        Enqueue e', mark visited
        $A(e') \leftarrow$ set of edges adjacent to e'
        $C_{\text{adj}}(e') = \{\text{color}(e) \mid e \in A(e'), e \text{ colored}\}$
        Assign smallest color $c \notin C_{\text{adj}}(e')$
        if $c > ECN$ **then**
            $ECN \leftarrow c$
        end if
    end for
end while
return ECN

The example shown in Figure 1 demonstrates the efficiency of the greedy edge-coloring algorithm in minimizing color usage while ensuring that no two adjacent edges share the same color. The final edge-chromatic number for this graph is 2, highlighting the effectiveness of our approach in producing an optimal or near-optimal coloring with minimal computational effort.

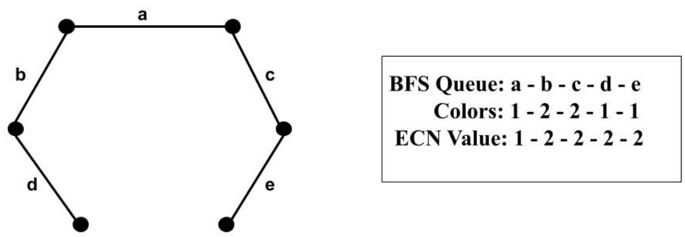

Figure 1. A graph, BFS list of its edges, color assignments and the updated value of ECN for each color assignment

The total time complexity of the proposed algorithm for computing the edge-iterated chromatic number is:

$$O\left(\sum_{k=0}^{n} |E_k| \cdot \Delta_k\right)$$

where $|E_k|$ denotes the number of edges and Δ_k the maximum degree of the graph G_k at the k^{th} iteration. At each step, the algorithm constructs the line graph $G_k = L(G_{k-1})$ and applies the EDGE-GREEDY-CHROM procedure, which performs a breadth-first traversal over the edges and assigns colors greedily to ensure that no two adjacent edges receive the same color. For each edge, determining the valid color involves checking all adjacent edges, which takes $O(\Delta_k)$ time. Therefore, the coloring step incurs $O(|E_k| \cdot \Delta_k)$ time per iteration.

Assuming $|E_k| \leq E_{\text{max}}$ and $\Delta_k \leq \Delta_{\text{max}}$ for all $k \in \{0, 1, \ldots, n\}$, the overall time complexity can be bounded by:

$$O(n \cdot E_{\text{max}} \cdot \Delta_{\text{max}}) \qquad .$$

This reflects how the runtime scales with the number of iterations, the growth of the edge set, and the complexity of edge adjacencies in successive line graphs.

4.3. Comparison with other coloring algorithms

To evaluate the effectiveness of our approach, we compare it against two classical graph coloring strategies, Welsh-Powell and DSATUR-followed by our proposed edge-aware BFS-based greedy method. Each algorithm is described below with its operational principle, pseudocode, and complexity analysis.

4.31. Welsh–Powell's algorithm (degree-ordered greedy coloring)

The Welsh–Powell's method shown in Algorithm 2 generates an ordering of vertices sorted by non-increasing degree (ties are broken by neighbor degree). Greedy coloring is then applied in this order, biasing the coloring process toward central vertices with high impact. This heuristic tends to reduce the number of colors on iterated line graphs while remaining computationally efficient.

Algorithm 2 Degree-ordered greedy graph coloring

Input: Graph $G = (V(G), E(G))$, where $|V(G)| = n$, $|E(G)| = m$
Output: color$[v]$ for all $v \in V(G)$
Order the vertices of $V(G)$ in descending order of $\deg(v)$
for each v in the ordered list **do**
    used-colors $\leftarrow$ $\{$color$[u]$: $u \in N(v)$ and color$[u]$ assigned$\}$
    color$[v]$ $\leftarrow$ smallest positive integer not in used-colors
end for
return color$[v]$

Complexity: sorting requires $O(n \log n)$. Neighbor color checks cost $O(m)$ in total, giving overall complexity $O(n \log n + m)$. A naive adjacency-matrix implementation leads to $O(n^2)$ in dense graphs, whereas adjacency lists and hashing maintain $O(n \log n + m)$. This method always produces a valid coloring but does not guarantee optimality.

4.32. DSATUR algorithm (saturation-degree heuristic algorithm for vertex coloring)

Algorithm 3 illustrates the DSATUR approach, which repeatedly selects the uncolored vertex with maximum saturation degree (number of distinct neighbor colors). Ties are broken by degree. This strategy adapts dynamically to color distribution but involves heavier bookkeeping compared to Welsh–Powell.

Algorithm 3 DSATUR graph coloring algorithm

```
Input: Graph G = (V(G), E(G)), where |V(G)| = n, |E(G)| = m
Output: color[v] for all v ∈ V(G)
for each v ∈ V(G) do
    color[v] ← UNCOLORED
    sat[v] ← 0                                    ▷ number of distinct neighbour colors
    forb[v] ← ∅                                   ▷ set of neighbour colors
end for
v_0 ← arg max_v deg(v)                            ▷ vertex of maximum degree
color[v_0] ← 1
Update sat and forb for neighbors of v_0
while there exists an uncolored vertex do
    Choose v among uncolored vertices maximizing (sat[v], deg(v)) lexicographically
    c ← smallest positive integer not in forb[v]
    color[v] ← c
    for each u ∈ N(v) do
        if c ∉ forb[u] then
            Add c to forb[u]
            sat[u] ← sat[u] + 1
        end if
    end for
end while
return color[v]
```

Complexity: the naive version runs in $O(n^2)$. With efficient data structures, complexity improves to $O(n \log n + m \log n)$. In practice, DSATUR incurs higher overhead than Welsh–Powell due to frequent priority updates.

4.33. Proposed BFS based greedy algorithm for vertex coloring

Our proposed algorithm generates a vertex ordering tailored to iterated line graphs. It begins with a high-degree vertex and constructs a BFS traversal, breaking ties by neighbor degree. The greedy coloring is then applied in this BFS order. This approach prioritizes central vertices and distributes colors more effectively across the network.

Time complexity: the breadth-first traversal has a cost of $O(m + n)$.

- Neighbor sorting: ordering the neighbors of each vertex by degree requires a total cost of $O(n \log \Delta)$, where Δ denotes the maximum degree of the graph.
- Coloring: the greedy coloring phase requires scanning all adjacency lists once, contributing $O(m)$ overall.
- Overall cost: the total complexity is therefore $O(m + n + m \log \Delta)$. In practice, this can be expressed succinctly as $O(m \log m + n)$ when degrees are large and global sorting is applied.
- Comparison with DSATUR: unlike DSATUR, which requires frequent priority updates and set manipulations, the proposed algorithm avoids such overhead. Consequently, it scales more efficiently and is typically much faster on large sparse graphs.

4.4. Edge-iterated domination and covering numbers

In this subsection, we formally introduce the edge-iterated domination number $\gamma^n(G)$ and edge-iterated covering number $\beta^n(G)$, both defined through successive applications of the line graph operator. We also establish exact values for these parameters for fundamental graph classes such as paths and cycles.

Definition 4.2. *A subset S of nodes in a graph G is called a* dominating set *if every node of G that is not in S is adjacent to some node in S. The minimum cardinality of a dominating set in G is called the* domination number *of G, denoted $\gamma(G)$. Let $\gamma^1(G) = \gamma(L(G))$, $\gamma^2(G) = \gamma(L^2(G)) = \gamma(L(L(G)))$, and so on. In general, the nth edge-iterated domination number is defined as:*

$$\gamma^n(G) = \gamma(L^n(G)) \quad .$$

Definition 4.3. *A subset S of nodes in a graph G is called a* (vertex) covering *if every edge of G has at least one end-vertex in S. The minimum cardinality of such a set is called the* covering number *of G, denoted $\beta(G)$. Let $\beta^1(G) = \beta(L(G))$, $\beta^2(G) = \beta(\beta^1(G))$, and so on. In general, the nth edge-iterated covering number is defined as:*

$$\beta^n(G) = \beta(L^n(G)) \quad .$$

If G has no isolated vertices, then every covering set is also a dominating set, and hence $\gamma(G) \leq \beta(G)$.

Theorem 7. $\gamma^k(P_n) = \left\lceil \frac{n-k}{3} \right\rceil$ *where* $1 \leq k < n-1$.

Proof. Since the line graph of P_n is P_{n-1}, we have $\gamma^1(P_n) = \gamma(L(P_n)) = \gamma(P_{n-1}) = \left\lceil \frac{n-1}{3} \right\rceil$. Also, $\gamma^2(P_n) = \gamma(L^2(P_n)) = \gamma(P_{n-2}) = \left\lceil \frac{n-2}{3} \right\rceil$. Thus, $\gamma^k(P_n) = \gamma(L^k(P_n)) = \gamma(P_{n-k}) = \left\lceil \frac{n-k}{3} \right\rceil$ for $n-k \geq 2$.

Theorem 8. *For* $n \geq 3$, $\gamma^k(C_n) = \left\lceil \frac{n}{3} \right\rceil$ *where* $1 \leq k \leq n-3$.

Proof. The result follows since $\gamma(C_n) = \left\lfloor \frac{n}{3} \right\rfloor$ for $n \geq 3$ and $L(C_n) = C_n$ for all integers $n \geq 3$.

Theorem 9. $\beta^k(P_n) = \left\lceil \frac{n-k}{2} \right\rceil$ *where* $1 \leq k < n-1$.

Proof. Since the line graph of P_n is P_{n-1}, we have $\beta^1(P_n) = \beta(L(P_n)) = \beta(P_{n-1}) = \left\lceil \frac{n-1}{2} \right\rceil$. Also, $\beta^2(P_n) = \beta(L^2(P_n)) = \beta(P_{n-2}) = \left\lceil \frac{n-2}{2} \right\rceil$. Thus, $\beta^k(P_n) = \beta(L^k(P_n)) = \beta(P_{n-k}) = \left\lceil \frac{n-k}{2} \right\rceil$ for $n-k \geq 2$.

Theorem 10. *For* $n \geq 3$, $\beta^k(C_n) = \left\lceil \frac{n}{2} \right\rceil$ *for* $k \geq 1$.

Proof. Since a vertex of a cycle can cover exactly two edges, $\beta(C_n) = \left\lceil \frac{n}{2} \right\rceil$ for $n \geq 3$. Also, $L(C_n) = C_n$ for all $n \geq 3$, hence the result follows for all $k \geq 1$.

5. APPLICATIONS OF EDGE-ITERATED GRAPH PARAMETERS IN WSNS

WSNs are naturally modeled using graph theory. In this representation, each sensor is treated as a vertex, and any communication link between two sensors is represented by an edge. Thus, a connected graph $G = (V, E)$ provides a clear mathematical abstraction of a sensor network, where V denotes the set of sensor nodes and E represents the communication links among them. Designing such networks, however, involves several practical challenges. Issues such as optimal resource allocation, interference-free communication scheduling, and resilience against node or link failures must be addressed carefully. Edge-iterated graph parameters—specifically the chromatic number $\chi^n(G)$, domination number $\gamma^n(G)$, and covering number $\beta^n(G)$— offer rigorous mathematical tools to analyze and resolve these concerns.

The edge-iterated chromatic number $\chi^n(G)$ plays a crucial role in managing interference. In practical terms, it corresponds to assigning communication frequencies or time slots in such a way that conflicting transmissions do not overlap. To capture higher-order interactions in the network, we iteratively construct the line graphs $L^n(G)$. In $L^n(G)$, vertices represent communication links from the previous iteration, thereby modeling indirect or multi-hop interactions between sensor nodes. The edge-iterated chromatic number is formally defined as:

$$\chi^n(G) = \chi(L^n(G)) \quad .$$

This quantity gives the minimum number of frequencies (or time slots) required to ensure interference-free communication at the n-th level of interaction. By partitioning communication channels into mutually non-interfering sets, this approach enhances spectrum utilization and improves overall network efficiency.

The edge-iterated domination number $\gamma^n(G)$ addresses reliability and supervisory control. In a sensor network, certain nodes or communication links must act as controllers or monitors to maintain connectivity and enable recovery from failures. When we examine the iterated line graph $L^n(G)$, minimal dominating sets correspond to critical communication structures that oversee the network at progressively deeper interaction levels. Formally,

$$\gamma^n(G) = \gamma(L^n(G)).$$

Each iteration reveals increasingly essential supervisory components. Determining $\gamma^n(G)$ therefore quantifies the minimum number of nodes or links required to maintain effective control and connectivity, particularly in environments where failures are frequent or topology changes dynamically.

Similarly, the edge-iterated covering number $\beta^n(G)$ focuses on monitoring and resource management. It identifies the smallest subset of vertices in $L^n(G)$ that collectively cover all communication links. Formally,

$$\beta_n(G) = \beta(L^n(G)).$$

This parameter highlights the minimal set of nodes or links necessary to supervise the entire network structure. Optimizing $\beta^n(G)$ supports balanced load distribution and energy-efficient operation, both of which are critical in WSNs where nodes typically operate on limited battery power. By guiding the design of energy-aware topologies, this measure helps extend network lifetime while ensuring comprehensive monitoring and connectivity. Together, these edge-iterated parameters provide a structured and scalable framework for analyzing interference control, fault tolerance, and energy efficiency in WSNs, thereby linking theoretical graph invariants with practical network design considerations.

5.1. Iterative optimization methodology

We propose an iterative optimization framework for WSN leveraging the above parameters:

- Model the initial sensor network as a graph G.
- Compute the initial chromatic, domination, and covering numbers $\chi(G)$, $\gamma(G)$, and $\beta(G)$.
- Construct Iterative line graphs $L^n(G)$ and compute respective edge-iterated parameters $\chi^n(G)$, $\gamma^n(G)$, and $\beta^n(G)$.
- Identify a small collection of $\chi^n(G)$, $\gamma^n(G)$, or $\beta^n(G)$ nodes (structurally significant nodes) that can supervise and execute network optimization functions such as maintainability, fault tolerance, minimal interference, and topology design.

Through iterative computation, this methodology provides a comprehensive analysis that identifies network structures critical to maintaining robust, energy-efficient, and interference-free sensor network designs.

5.2. Illustrative visualisation example for the edge-iterated parameters in a sample wireless sensor network

In this section, we illustrate the visualization of these three concepts using a small sensor network G with six nodes, although the approach is applicable to any WSN topology such as star, mesh, or grid. The network topology is shown in Figure 2, where nodes represent sensors and edges indicate that the respective nodes are within their communication range. By iteratively computing $\chi^n(G)$, $\gamma^n(G)$, and $\beta^n(G)$, we identify structurally significant nodes and communication links. These iterative evaluations systematically highlight the nodes and communication links that play a critical role in sustaining reliable network functionality. Consequently, they not only strengthen the operational robustness of the network but also underscore the practical significance and theoretical depth of edge-iterated graph parameters.

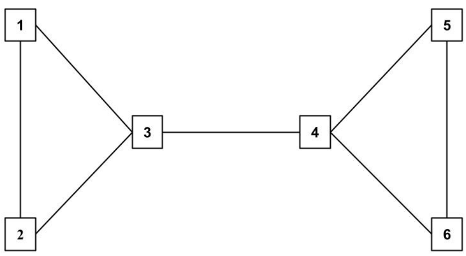

Figure 2. A WSN with six sensor nodes

- Edge-iterated chromatic number analysis

Step 1: the chromatic number of G is $\chi(G) = 3$, with color classes {1,4}, {2,5}, and {3,6}. Each color class consists of mutually non-adjacent sensor nodes, meaning the nodes within a class can transmit simultaneously without causing interference. Thus, these classes represent disjoint subsets of sensors that can operate in parallel while still collectively ensuring complete network coverage.

Step 2: next, we construct the derived sensor channel network, namely the line graph $L(G)$, in which every communication link of G is represented as a vertex. The vertex set of $L(G)$ is:

$$\{12,\ 13,\ 23,\ 34,\ 45,\ 46,\ 56\}.$$

As illustrated in Figure 3, the derived graph also has chromatic number 3. One possible proper coloring yields the color classes:

$$\{12,\ 34,\ 56\},\quad \{13,\ 45\},\quad \{23,\ 46\}.$$

These classes group together non-adjacent communication links, indicating sets of channels that can function simultaneously without mutual interference.
Step 3: the first color class spans the entire sensor network using three mutually non-interfering links, demonstrating that a carefully chosen minimal set of channels can maintain global communication. Among these, the link $\{34\}$ is particularly significant. It connects nodes 3 and 4, which appear repeatedly in structurally important configurations and occupy central positions in the network, as evident from the topology shown in Figure 3. This highlights their structural importance in maintaining robust communication.

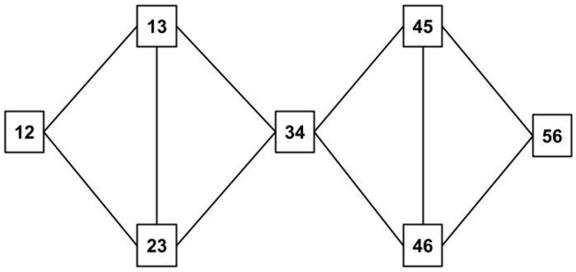

Figure 3. Derived sensor node channel network for the newtork in Figure 2

- Edge-iterated domination number analysis

Step 1: in the original graph G, the domination number satisfies $\gamma(G) = 2$, and there are nine minimal dominating sets of cardinality 2:

$$\{1,4\},\ \{2,4\},\ \{3,4\},\ \{1,5\},\ \{2,5\},\ \{3,5\},\ \{1,6\},\ \{2,6\},\ \{3,6\}.$$

Each of these sets ensures that every node in the network is either included in the set or adjacent to a node in the set. Operationally, such configurations support localized supervision and efficient monitoring of the network.
Step 2: in the derived graph $L(G)$, the domination number is $\gamma(L(G)) = 2$. The four minimal dominating sets of size 2 are:

$$\{13,45\},\ \{13,46\},\ \{23,45\},\ \{23,46\}.$$

These pairs of communication links collectively dominate all other links in the derived network, ensuring that each channel is either selected or directly adjacent to a selected channel.
Step 3: a closer inspection of these dominating sets shows that nodes 3 and 4 repeatedly appear in the corresponding link structures, confirming their central role in network supervision and resilience. In particular, the link $\{34\}$ serves as a structurally critical connector, contributing significantly to fault tolerance and sustained connectivity.

This combined analysis demonstrates that both the edge-iterated chromatic number and the edge-iterated domination number identify the same structurally influential nodes and links. Nodes 3 and 4, together with the communication channel $\{34\}$, consistently emerge as vital components of the network. Their importance in maintaining connectivity, supporting fault recovery, and enabling interference-free communication validates the effectiveness of edge-iterated parameters in guiding the design of resilient, energy-aware, and interference-conscious WSNs.

- Edge-iterated covering number analysis

Since a cycle of length 3 requires at least two vertices to cover all its edges, the graph G must contain at least four vertices in any minimum vertex cover.

Step 1: in the original graph G, the covering number is $\beta(G) = 4$. The minimal vertex covers of cardinality 4 are:

$$\{1, 3, 5, 6\},\ \{1, 3, 4, 6\},\ \{1, 3, 4, 5\},\ \{1, 2, 4, 5\},\ \{2, 3, 4, 6\},\ \{2, 3, 4, 5\}.$$

Each of these sets intersects every edge of G, ensuring that all communication links are monitored by at least one selected node. Operationally, such configurations support link supervision and network maintenance.

Step 2: in the derived graph $L(G)$, the covering number satisfies $\beta(L(G)) = 4$. The unique minimum vertex cover of size 4 is:

$$\{13,\ 23,\ 45,\ 46\}.$$

This set collectively covers all edges of the derived channel network, meaning every communication interaction between links is supervised.

Step 3: a closer examination shows that nodes 3 and 4 repeatedly appear in the structural components corresponding to these covering sets. This consistency reinforces their central role in ensuring network robustness and resilience. From the network diagram, the link $\{34\}$ emerges as structurally critical, serving as a key connector that contributes significantly to fault tolerance and sustained connectivity.

Taken together, the analyses of the edge-iterated chromatic number, domination number, and covering number consistently identify the same structurally influential nodes—namely 3 and 4—along with the link $\{34\}$. These components are fundamental to maintaining connectivity, supporting fault recovery, and enabling interference-aware communication. The experimental observations therefore confirm that edge-iterated graph parameters provide a reliable mathematical framework for detecting critical nodes and links in WSNs.

Depending on the intended application, different parameters may guide network design decisions: the chromatic number is suitable for partitioned or task-oriented scheduling, the domination number is appropriate for node-level monitoring and supervisory control, and the covering number is particularly useful when complete link-level supervision is required. Each parameter thus offers a distinct yet complementary perspective for achieving resilient, energy-efficient, and interference-aware network configurations.

6. RESULTS AND DISCUSSION

To validate the theoretical results and to assess the practical performance of the proposed approach, simulation experiments were conducted on representative graph classes, namely path, cycle, and grid graphs. For each graph $G = (V, E)$, successive line-graph iterations $L^n(G)$ were generated for $n = 0, 1, 2, 3$, and the corresponding edge-iterated chromatic number was computed (Table 2). The value of $\chi^n(G)$ in Table 2 indicates the number of channels required to avoid interference in a WSN with the specified structure. The proposed BFS-based greedy algorithm was evaluated alongside the classical Welsh-Powell and DSATUR algorithms (refer Table 3). The computed edge-iterated parameter values, along with the corresponding computational performance, experimentally validate the theoretical results and demonstrate the applicability of the proposed framework to WSNs.

Table 2. Edge iterated chromatic number $\chi^n(G)$ for various graphs

Graph	n	$\|V(L^n(G))\|$	$\|E(L^n(G))\|$	$\chi^n(G)$
C_6	0	6	6	2
C_6	1	6	6	2
C_6	2	6	6	2
C_6	3	6	6	2
P_6	0	6	5	2
P_6	1	5	4	2
P_6	2	4	3	2
P_6	3	3	2	2
$P_5 \times P_5$ (5×5 grid)	0	25	40	2
$P_5 \times P_5$ (5×5 grid)	1	40	94	4
$P_5 \times P_5$ (5×5 grid)	2	94	372	7
$P_5 \times P_5$ (5×5 grid)	3	372	2730	13

Table 3. Comparison of edge-iterated chromatic numbers across coloring algorithms

Graph	k	$\|V(L^k(G))\|$	$\|E(L^k(G))\|$	χ^k (WP)	χ^k (DSATUR)	χ^k (proposed BFS)
C_6	0	6	6	2	2	2
	1	6	6	2	2	2
	2	6	6	2	2	2
	3	6	6	2	2	2
C_6	0	6	5	2	2	2
	1	5	4	2	2	2
	2	4	3	2	2	2
	3	3	2	2	2	2
$P_5 \times P_5$ (5×5 grid)	0	25	40	2	2	2
	1	40	94	5	4	4
	2	94	372	7	7	7
	3	372	2730	12	11	13

The simulation results confirm the theoretical analysis for path, cycle, and grid graphs. Although all algorithms produce the same parameter values, the proposed BFS-based greedy algorithm computes them more efficiently, making it suitable for large and energy-constrained WSNs. For a better understanding of the experimental results in WSNs, the edge-iterated graph parameters are linked to their network-level interpretations. Table 4 explains how the edge-iterated chromatic, domination, and covering numbers change across iteration levels, with higher iterations representing increased interaction and interference among communication links. The table illustrates how these parameters support practical WSN tasks such as interference-free scheduling, cluster-head selection, fault tolerance, and energy-efficient network design.

Table 4. Interpretation of edge-iterated parameters in WSNs

Parameter	Iteration n	Graph-theoretic meaning	Interpretation in WSN	Practical benefit
$\chi^n(G)$	$n = 0$	Chromatic number of G	Minimum frequencies/time slots for direct communication	Interference-free scheduling
	$n = 1$	Chromatic number of $L(G)$	Channel allocation for links	Link-level interference avoidance
	$n \geq 2$	Chromatic number of $L^n(G)$	Scheduling under higher-order interference	Reliable scheduling in dense WSNs
$\gamma^n(G)$	$n = 0$	Minimum dominating set	Cluster heads/controllers	Efficient supervision
	$n = 1$	Domination in $L(G)$	Control of links	Fault-tolerant monitoring
	$n \geq 2$	Domination in $L^n(G)$	Supervision of link dependencies	Robust failure control
$\beta^n(G)$	$n = 0$	Minimum vertex cover	Active nodes covering links	Energy-efficient sensing
	$n = 1$	Vertex cover of $L(G)$	Active links covering communications	Backbone construction
	$n \geq 2$	Vertex cover of $L^n(G)$	Coverage of higher-order interactions	Longer network lifetime

The analytical and simulation results together show that edge-iterated graph parameters capture higher-order structural features that classical vertex-based measures do not reveal. As line-graph transformations are applied repeatedly, the edge-iterated chromatic, domination, and covering numbers increase in a way that reflects the growing complexity of edge-level interactions. The simulation results support the correctness of the derived bounds and exact values for standard graph families. The agreement between theoretical and simulated values strengthens the validity of the proposed framework. From a computational point of view, the proposed greedy algorithm produces coloring results comparable to DSATUR, while requiring less computation time. This advantage becomes more noticeable for large and sparse graphs, where scalability is important.

In WSNs, the edge-iterated chromatic number naturally models frequency or time-slot allocation under higher-order interference constraints. The edge-iterated domination and covering numbers help identify structures for fault-tolerant supervision and energy-efficient monitoring. Together, these findings suggest that edge-iterated parameters offer a more expressive and practical modeling framework, particularly for dense and interference-sensitive WSN deployments.

7. CONCLUSION

The present work introduces three graph-theoretic parameters defined through edge-iterated line graph transformations of a connected graph. These parameters offer a deeper understanding of graph structure by examining how fundamental concepts such as coloring, domination, and covering evolve under successive line-graph operations. We studied their behaviour for basic graph families, including paths, cycles, and grid graphs, and established explicit bounds and recursive relationships. The iterative framework illustrates how communication constraints, fault tolerance, and supervisory control mechanisms emerge at higher levels of graph abstraction. In addition, we developed a greedy algorithm that efficiently estimates the edge-iterated chromatic number for large and sparse networks.

The practical relevance of the framework is demonstrated through its application to WSN. The edge-iterated chromatic number supports interference-free communication by guiding channel allocation. The edge-iterated domination number identifies the minimum set of nodes or links required for supervision, recovery, and system stability. The edge-iterated covering number ensures complete link monitoring and supports efficient scheduling strategies. An experimental study on a six-node WSN shows how these parameters converge to highlight structurally critical nodes and links. This leads to network designs that maintain reliable communication with minimal interference and reduced energy consumption. Overall, the unified framework strengthens theoretical understanding of iterated graph operations while offering practical tools for designing resilient and efficient network systems.

Some possible directions for future work include: i) identifying the classes of graphs for which the sequences $\chi^n(G)$, $\gamma^n(G)$, and $\beta^n(G)$ converge, ii) designing faster parallel or hybrid algorithms to compute edge-iterated parameters in large and dynamic networks, iii) studying the relationship among chromatic, domination, and covering numbers, and understanding their combined impact on network reconfiguration and fault prediction, iv) analyzing how these parameters respond to changes in graph density, node degree, and topology, along with the trade-off between optimization quality and computational cost, v) performing simulation studies to compare traditional WSN designs with edge-iterated parameter-based models, including integration with protocols such as LEACH and PEGASIS, and vi) exploring multi-objective optimization strategies that jointly consider interference control, supervision, and energy efficiency.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal Analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject Administration
Fu : **Fu**nding Acquisition

CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflict of interest.

DATA AVAILABILITY

In this study, no external datasets were used. The data and simulation code supporting the findings of this study are available from the corresponding author upon reasonable request.

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