Fault-tolerant trajectory tracking of mobile robots via model-free intelligent PID and fault observer

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ABSTRACT

This paper addresses the problem of unexpected errors, such as model uncertainties and actuator faults, that degrade the performance of wheeled mobile robots (WMR). To overcome these challenges, a fault-tolerant control (FTC) approach is developed in which model-free control (MFC) is merged with an intelligent-proportional integral derivative (i-PID) controller and complemented by a fault observer (FO). Unlike existing approaches, the proposed controller does not rely on accurate system modeling; instead, MFC ensures robustness to time-varying parameters, and while i-PID enhances trajectory tracking through adaptive gain adjustment. The FO estimates actuator faults in real time and compensates for their effects, ensuring reliable operation even under severe conditions. The closed-loop stability is rigorously analyzed via Lyapunov theory. MATLAB/Simulink results show reduced tracking errors, improved stability, and strong robustness under both model uncertainties and actuator faults, including time-varying mass and inertia, validating the effectiveness and practical potential of the proposed FTC.

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1. INTRODUCTION

Fault-tolerant control (FTC) is becoming a critical requirement for wheeled mobile robots (WMR) used in real-world applications such as automated logistics, warehouse robotics, and service robots, where safety and reliability are essential. A key control objective is trajectory tracking, which ensures the robot follows a desired path accurately under varying conditions [1]–[5]. However, real-world environments often introduce uncertainties such as actuator degradation, parameter variations, and external disturbances. These lead to faults, defined as significant deviations from normal operation [6], [7], which can compromise system stability and performance. Traditional controllers are typically insufficient to handle such conditions.

To enhance robustness, pioneering studies by researcher [6], [7] laid the foundations for fault diagnosis and FTC [8]–[11], which have since been extended to address complex robotic systems. Modern FTC approaches incorporate techniques such as sliding mode control (SMC), fuzzy logic, neural networks (NN), backstepping, and MPC, which offer improved adaptability and resilience, against dynamic faults and disturbances.

SMC is widely recognized for its robustness and efficiency in handling uncertainties in WMR [12], [13]. An adaptive SMC-based FTC was proposed in [14] to compensate for actuator faults without requiring online fault information. However, it did not consider time-varying parameters during motion and remains limited by chattering effects under turbulent conditions.

Adaptive fault-tolerant backstepping controllers have been introduced in [15], [16] to estimate parameter variations and component faults. However, they do not account for continuously time-varying dynamics during robot operation, limiting their adaptability. Meanwhile, MPC offers a flexible alternative, as demonstrated in [17], [18]. The integration of MPC with disturbance observers (DOB) in [17] addresses trajectory tracking under input constraints and disturbances, while the approach in [18] enables rapid fault compensation by optimizing control actions—though it assumes fixed model parameters.

With the advancement of hardware and computational power, data-driven approaches such as NN have become popular for modeling and control of nonlinear robotic systems due to their strong approximation capabilities. Recent studies applied NN to tasks like adaptive backstepping hierarchical SMC [19], trajectory tracking [20], and adaptive controllers for wheeled robots [21]. However, their performance is highly sensitive to input selection, network architecture, and hyperparameter tuning [22]. Radial basis function (RBF) networks require extensive training data, while poor initialization can lead to suboptimal performance and overfitting, especially under time-varying conditions [23]. These challenges highlight the need for improved robustness and generalization, as also noted in broader applications [24]–[27].

Fuzzy logic controllers, while flexible, rely heavily on expert-defined rule sets, making performance dependent on designer experience. Adaptive fuzzy control has been combined with backstepping and dynamic surface control to handle faults [28], while other fuzzy-based FTC strategies for mobile robots have been validated in MATLAB/Simulink [29], [30]. However, these methods often focus only on kinematic models and neglect time-varying dynamic parameters like mass and inertia, which are crucial in practical scenarios.

MFC [31] has recently gained attention as a data-driven approach that designs controllers based on input–output behavior without relying on accurate system models. When combined with proportional integral derivative (PID) control, MFC leverages the strengths of both components: the PID ensures accurate feedback regulation, while MFC enhances robustness to model uncertainties and disturbances. This hybrid strategy offers fast adaptation to dynamic conditions, simplifies implementation, and reduces the need for complex modeling or expert knowledge, making it an attractive solution for practical control applications.

Unlike existing FTC approaches that either rely on precise models or require heavy offline training, our proposed method combines MFC, i-PID, and FO to achieve robust performance under time-varying dynamics and actuator faults. The main contributions of this paper are as: i) the combined MFC and i-PID-based FTC is proposed, and the effectiveness is demonstrated via comparison with some other controllers such as PID and i-PID-MFC without FO, ii) the proposed controller still ensures high performance in tracking the robot's trajectory, under the effect of faults, the robot's mass and moment variable over time, and iii) the system stability is guaranteed through Lyapunov theory, and the effectiveness of the proposed strategy is demonstrated by simulation results.

The remainder of this work is organized as follows. Section 2 presents the mathematical modeling of the nonholonomic mobile robot, developed following the method described in [32]–[35] and provides a model of the actuator faults on WMR. Sections 3 and 4 present the calculations and design of the FTC and FO. Section 5 presents several simulation scenarios and provides comparative results to demonstrate the effectiveness and superiority of the proposed control strategy. Finally, the main conclusions are summarized in section 6.

2. DESCRIPTION SYSTEM

2.1. Kinematic-dynamic model of mobile robot

The mobile robot model present in Figure 1 includes 2 independent drive active wheels and 2 passive front and rear rotating wheels. $P_i(x_i,y_i)$ denotes the position of the robot's center, and θ represents its yaw angle. In this model, m is the robot's mass and I is its total moment of inertia about the central axis. The symbol r denotes the radius of the steering wheel, whereas $2R$ corresponds to the spacing between the two rear rudders.

The position of the WMR in the inertial Cartesian coordinate frame {Oxy} is completely concretized by the vector $q = [x_i \quad y_i \quad \theta_i]^T$. The relationship between the velocity between two fixed and moving coordinates [29], [30]:

$$\dot{q} = S(q)V(t) \tag{1}$$

where, $S(q)=\begin{bmatrix} cos\,\theta & 0\\ sin\,\theta & 0\\ 0 & 1\end{bmatrix}, V(t)=\begin{bmatrix} v\\ \omega\end{bmatrix}$, v and ω represent the translational and angular velocities of the robot, respectively. The kinematic model of the robot is then expressed as (2):

$$\begin{bmatrix}\dot{x}_P\\ \dot{y}_P\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix} cos\,\theta & 0\\ sin\,\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} v\\ \omega\end{bmatrix} \quad (2)$$

A broad class of non-holonomic muscle systems is described by the following dynamical form based on the Euler-Lagrange formula [32]-[35]:

$$M_0(q)\ddot{q}+C_0(q,\dot{q})\dot{q}+G_0(q)+\tau_d=B_0(q)\tau+J^T(q)\lambda \quad (3)$$

Non-holonomic constraints:

$$J(q)\dot{q}=0 \quad (4)$$

Here, $q\in R^n$ denotes the generalized coordinate vector. The inertia matrix $M_0\in R^{nxn}$ is symmetric and strictly positive definite, ensuring well-posed dynamics. The term $C_0\dot{q}=0$ accounts for Coriolis and centrifugal effects, while $G_0\in R^n$ represents the gravitational contributions. The disturbance vector $\tau_d\in R^n$ accounts for modeling uncertainties and external perturbations. The matrix $B_0\in R^{nx\alpha},(\alpha<n)$ describes the input transformation, τ is the α -dimensional input vector; λ is the Lagrange multiplier associated with the non-holonomic constraints. With the mobile robot considered here, we have:

$$M_0=\begin{bmatrix} m & 0 & 0\\ 0 & m & 0\\ 0 & 0 & I\end{bmatrix}, C_0(q,\dot{q})=\begin{bmatrix}0\\0\\0\end{bmatrix}, G_0=0, B_0=\frac{1}{r}\begin{bmatrix} cos\,\theta & cos\,\theta\\ sin\,\theta & sin\,\theta\\ R & -R\end{bmatrix}, \tau=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}, J^T=\begin{bmatrix}-sin\,\theta\\ cos\,\theta\\ 0\end{bmatrix}$$

The dynamics model of the robot be presented as (5):

$$M_0(q)\ddot{q}=B_0(q)\tau+J^T(q)\lambda \quad (5)$$

where, τ_1 and τ_2 represent the torques generated by the left and right motors, respectively.
Non-holonomic constraints can be rewrite:

$$\dot{x}\,sin\,\theta-\dot{y}\cos\theta=0 \quad (6)$$

According the dynamics (5), we have as (7):

$$\begin{cases}\ddot{x}=\frac{(\tau_1+\tau_2)}{rm}\cos\theta+\frac{\lambda}{m}sin\,\theta\\ \ddot{y}=\frac{(\tau_1+\tau_2)}{rm}sin\,\theta-\frac{\lambda}{m}\cos\theta\\ \ddot{\theta}=\frac{R(\tau_1-\tau_2)}{rI}\end{cases} \quad (7)$$

Differential (1):

$$\ddot{q}=\dot{S}(q)\begin{bmatrix} v\\ \omega\end{bmatrix}+S(q)\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix}\Leftrightarrow\begin{cases}\ddot{x}=-v\dot{\theta}\,sin\,\theta+\dot{v}\cos\theta\\ \ddot{y}=v\dot{\theta}\cos\theta+\dot{v}\,sin\,\theta\\ \ddot{\theta}=\dot{\omega}\end{cases} \quad (8)$$

Combine (7) and (8), we obtained the dynamic model of robot:

$$\begin{bmatrix}\dot{v}\\ \dot{\omega}\end{bmatrix}=\frac{1}{r}\begin{bmatrix}\frac{1}{m} & \frac{1}{m}\\ \frac{R}{I} & \frac{-R}{I}\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \quad (9)$$

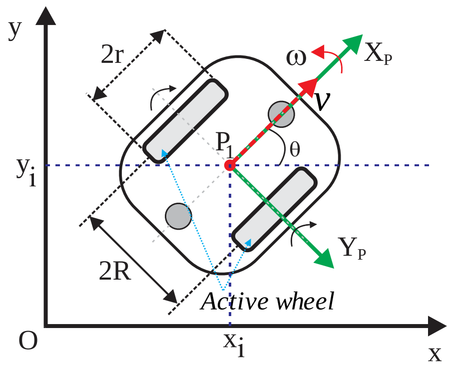

Figure 1. The perspective of WMR

2.2. Fault model

In this work, the actuator fault be considered reduces the efficiency of the actuator on the 2 motors of robot. The dynamic model of the robot in this case can be written:

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \frac{1}{r}\begin{bmatrix} \frac{1}{m+\Delta m} & \frac{1}{m+\Delta m} \\ \frac{R}{I+\Delta I} & \frac{-R}{I+\Delta I} \end{bmatrix}\begin{bmatrix} \tau_1 - \Delta\tau_1 \\ \tau_2 - \Delta\tau_2 \end{bmatrix} \tag{10}$$

The unknown vector input $\Delta\tau = [\Delta\tau_1 \; \Delta\tau_2]^T$ presented the actuator fault affecting the mobile robot, and m, I aren't constance. The values of v and ω will change by an unknown quantity, $\Delta v(t)$ and $\Delta\omega(t)$ respectively:

$$\begin{bmatrix} v_{real} \\ \omega_{real} \end{bmatrix} = \begin{bmatrix} v - \Delta v \\ \omega - \Delta\omega \end{bmatrix} \tag{11}$$

Figure 2 illustrates the proposed structure of the nonlinear fault observer (NFO), which estimates the actuator faults in real time and compensates for their effects.

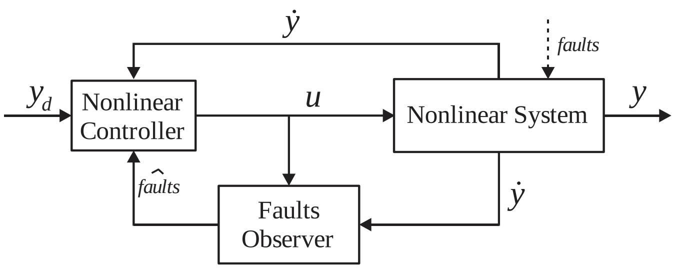

Figure 2. Structure of the NFO

3. CONTROLLER DESIGN

The FTC for the robot is designed with a structure of two control loops. The outer loop is the control law of tracking the set trajectory. It is built based on the kinematics problem of the robot to calculate and determine the value of the desired velocity and angular velocity. The inner loop control law ensures that the deviation of the velocity and angular velocity of the model from the desired value converges to zero. The problem posed here is that the moving robot is affected by actuator faults, and in the case of the parameters *m, I* are time-varying.

3.1. Tracking controller

The diagram shows the robot's coordinate axes (Figure 3). It is moving and tracking the trajectory. $q = [x \; y \; \theta]^T$ is position of the robot in the origin coordinate; $q_r = [x_r \; y_r \; \theta_r]^T$ is position of the desired robot in the origin coordinate system.

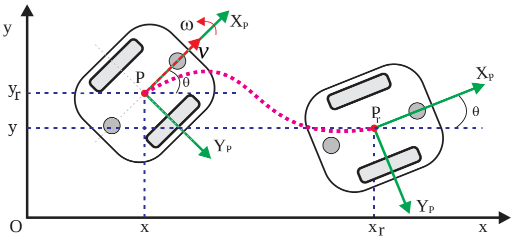

Figure 3. The coordinate axes of the robot

Consider the robot's motion trajectory as a set of n small enough segments $\Delta l > 0$. With $P(x, y)$ being the coordinates of the robot, we can completely deduce the position of point $P_1(x_1, y_1)$, which is the coordinates of the robot after moving with that segment Δl. Thus:

$$\begin{cases} x_1 = x + \Delta l \cos\theta \\ y_1 = y + \Delta l \sin\theta \end{cases} \tag{12}$$

Tracking the robot's motion trajectory can be considered and replaced by controlling $P_1(x_1, y_1)$ to follow the set trajectory. Differential in (12) and combine with (1), we have as (13):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\Delta l \sin\theta \\ \sin\theta & \Delta l \cos\theta \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{13}$$

Thus:

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\frac{\sin\theta}{\Delta l} & \frac{\cos\theta}{\Delta l} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; \ (\dot{x}_1 = u_1; \dot{y}_1 = u_2) \tag{14}$$

The final tracking controller is:

$$\begin{cases} u_1 = x_{1d} + k_1(x_{1d} - x_1) \\ u_2 = y_{1d} + k_2(y_{1d} - y_1) \\ e_1 = [x_{1d} - x_1 \ y_{1d} - y_1]^T \end{cases} => \begin{cases} v = k_1 e_{1x} \cos\theta + k_2 e_{1y} \sin\theta + \dot{x}_1 \cos\theta + \dot{y}_1 \sin\theta \\ \omega = \frac{1}{\Delta l}\left(-k_1 e_{1x} \sin\theta + k_2 e_{1y} \cos\theta + \dot{x}_1 \sin\theta + \dot{y}_1 \cos\theta\right) \end{cases} \tag{15}$$

3.2. Dynamic controller base on intelligent-proportional integral derivative-model-free control

MFC, as presented in [31], [36], replaces the unknown complex dynamics with an ultra-local model, allowing the nonlinear system to be described by (16):

$$y^{\vartheta} = F + Au \tag{16}$$

where, $y^{\vartheta}$is the derivative of order $\vartheta \geq 1$ of $y \in R^n$; u and $A \in R^{n\,x\,n}$ are signal vectors of controller and matrix of non-physical constant parameter, respectively; F is dimensionless vectors, play the role of faults, regularly updated in progress. We consider the dynamic system in (10) and rewrite with form (16) we have:

$$\dot{y} = F + Au \tag{17}$$

where, $\dot{y} = \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix}$; $u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$; $\Delta u = \begin{bmatrix} \Delta\tau_1 \\ \Delta\tau_2 \end{bmatrix}$; $A = \begin{bmatrix} \frac{1}{rm} & \frac{1}{rm} \\ \frac{R}{rI} & \frac{-R}{rI} \end{bmatrix}$; $F = f(m, I, \Delta m, \Delta I, u, \Delta u)$

According to Precup *et al*. [31] with reversible matrix A, we have signal vectors via the i-PID controller:

$$u_{ctrl} = A^{-1}\left(\dot{y}_d - \hat{F} + \kappa_p e_y + \kappa_i \int e_y dt + \kappa_d \dot{e}_y\right) \tag{18}$$

where, $\kappa_p, \kappa_i, \kappa_d$ are parameter's i-PID, $\hat{F}$ are estimates from observer of F; y_d is the reference trajectory, $e_y = y_d - y$ is the tracking error. The inner loop controller proposed is shown on Figure 4.

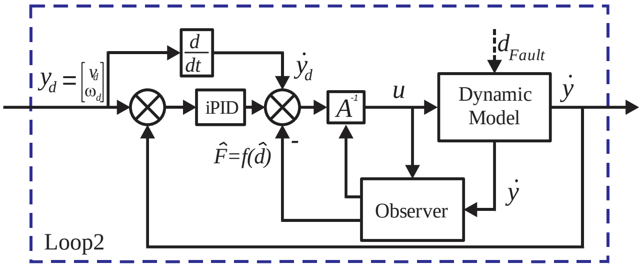

Figure 4. The inner loop control law

4. FAULTS OBSERVER MODEL

The relationship between the reference coordinate system and the local coordinate system when moving the robot is shown by (19):

$$q_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{19}$$

With non-holonomic constraints, we yield as (20):

$$\dot{q}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_r \cos\theta - v + y_e\omega \\ v_r \sin\theta_e - x_e\omega \\ \omega_r - \omega \end{bmatrix} \tag{20}$$

From (11), in (20) can be rewritten as (21):

$$\dot{q}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_r \cos\theta - (v + \Delta v) + y_e(\omega + \Delta\omega) \\ v_r \sin\theta_e - x_e(\omega + \Delta\omega) \\ \omega_r - (\omega + \Delta\omega) \end{bmatrix} \tag{21}$$

From (7) and (9), we can see that the fault affecting the actuator can be considered as $\Delta v(t), \Delta\omega(t)$. According Yano and Terashima [37], for a nonlinear system described by (22):

$$\begin{cases} \dot{x}(t) = f(x(t)) + g_1(x(t))u + g_2(x(t))F_a(t) \\ y(t) = h(x(t)) \end{cases} \tag{22}$$

with: $x \in R^n$ (state vector), $u \in R$ (input), $F_a \in R$ (faults). We can build a corresponding faults observer, respectively. In (21) can be rewritten as (23):

$$\dot{q}_e = \begin{bmatrix} y_e\omega_r \\ v_r \sin\theta_e - x_e\omega_r \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -y_e \\ 0 & x_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \cos\theta_e - v \\ \omega_r - \omega \end{bmatrix} + \begin{bmatrix} 1 & -y_e \\ 0 & x_e \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta\omega \end{bmatrix} \tag{23}$$

where: $f(q_e(t)) = \begin{bmatrix} y_e\omega_r \\ v_r \sin\theta_e - x_e\omega_r \\ 0 \end{bmatrix}; u = \begin{bmatrix} v_r \cos\theta_e - v \\ \omega_r - \omega \end{bmatrix}; g_1(q_e(t)) = g_2(q_e(t)) = \begin{bmatrix} 1 & -y_e \\ 0 & x_e \\ 0 & 1 \end{bmatrix}; F_a(t) = \begin{bmatrix} \Delta v \\ \Delta\omega \end{bmatrix}$

Choose parameter for observer:

$$p(q_e(t)) = \begin{bmatrix} x_e \\ \theta_e \end{bmatrix}; L(q_e(t)) = k_{FO} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; k_{FO} = 200$$

The FO model is designed as (24):

$$\begin{cases} \dot{z} = -L(q_e(t))(g_2(q_e(t))z + g_2(q_e(t))p(q_e(t) + f(q_e(t)) + g_1(q_e(t))u) \\ \hat{F}_a(t) = z + p(q_e(t)) \end{cases} \tag{24}$$

With $\hat{F}_a$ is value estimated of F, accoding to [38], it has also been shown that $e_F = \hat{F}_a - F$ stabilizes to zero. So, we can estimate for $\hat{F}$ in the inner loop controller proposed (Figure 4). The controller design proposed have structured in Figure 5.

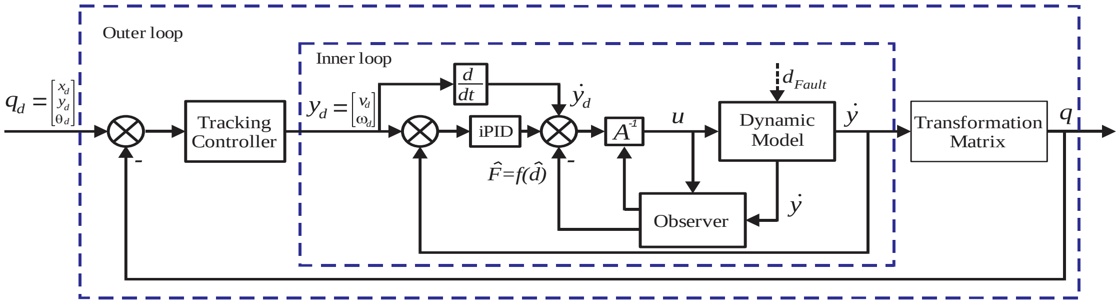

Figure 5. Structure of controller proposed

Theorem 1: the dynamic system described by (17), together with the fault estimates ($\hat{F}$) provided by the observer in (24), is asymptotically stable, and the tracking error converges to zero under the MFC-iPID control law in (18).
Proof 1: the Lyapunov function candidate be selected:

$$L_V(t) = \frac{1}{2}\left(e_y^T\left(e_y + \kappa_d e_y\right) + \rho^T \kappa_i \rho\right), \text{with } \rho = \int e_y dt \tag{25}$$

The time derivative of $V(t)$ yield:

$$\dot{L}_V(t) = e_y^T \dot{e}_y + e_y^T \kappa_d \dot{e}_y + e_y^T \kappa_i \rho = e_y^T(\dot{y}_d - F - Au) + e_y^T \kappa_d \dot{e}_y + e_y^T \kappa_i \int e_y dt \tag{26}$$

With controller MFC-iPID, replace u_{ctrl} into (26) we get:

$$\begin{aligned}\dot{L}_V(t) &= e_y^T\left[\dot{y}_d - F - AA^{-1}\left(\dot{y}_d - \hat{F} + \kappa_p e_y + \kappa_i \int e_y dt + \kappa_d \dot{e}_y\right)\right] + e_y^T \kappa_d \dot{e}_y + e_y^T \kappa_i \int e_y dt \\ &= -e_y^T\left(\kappa_p e_y + e_F\right)\end{aligned} \tag{27}$$

e_F is stabilizes to zero, so $\dot{L}_V(t) < 0$. Based on the Lyapunov stability criterion [39], because $L_V(t) > 0$ and $\dot{L}_V(t) < 0$, the system in (17) is asymptotically stable, and $e_y \to 0$.
Remark 1: the controller necessitates the computation of the second derivative of the desired output vector. In practice, this can be achieved provided that the term $\hat{F}$ is observed online and continuously updated.

5. PERFORMANCE EVALUATION
5.1. Simulation setup

In this subsection, the simulation results obtained using the proposed FO model are presented and discussed. The simulation parameters are provided in Table 1.

Table 1. The parameter values for robot

Variable	Detail	Value	Unit
m	The mass of robot	15	kg
r	The wheel radius	0.1	m
I	The overall moment of inertia of the robot	2.5	kg.m^2
R	Half of the distance between the 2 rear rudders	0.5	m

Tracking with:

$$q_d = [x_d \quad y_d \quad \theta_d]^T = [5\,cos(0.3t) \quad 5\,sin(0.3t) \quad 0.3t + 0.5\pi]^T \tag{28}$$

Initial state: $x_0 = 0; y_0 = 0; \theta_0 = 0$

To demonstrate the feasibility and effectiveness of the proposed controller, PID and i-PID without FO are given for comparison in the simulation. The control parameters of the relevant controllers are listed in Table 2.

Table 2. The control parameters of three controllers

Controller	Control gains
PID	$K_{Pv} = 150, K_{Iv} = 0.01, K_{Dv} = 0.001; K_{P\omega} = 50, K_{I\omega} = 0.1, K_{D\omega} = 0.001; k_1 = 5, k_2 = 5$
i-PID without FO	$K_{Pv} = 150, K_{Iv} = 0.01, K_{Dv} = 0.001; K_{P\omega} = 50, K_{I\omega} = 0.1, K_{D\omega} = 0.001; k_1 = 5, k_2 = 5$
Proposed controller	$\kappa_P = 12, \kappa_I = 2, \kappa_D = 0.01; k_1 = 5, k_2 = 5; k_{FO} = 200$

The control parameters in Table 2 were determined through iterative tuning in MATLAB/Simulink to ensure stability and minimize trajectory tracking error. Sensitivity analysis shows that small variations (±10%) in these parameters result in minor changes in tracking error, while larger deviations may lead to slower convergence or oscillations. The observer gain k_{FO} was tuned to balance fast fault estimation and noise rejection.

To have the most objective results, the controllers used will simulate both the no-fault and fault cases in which the robot's parameters are changed: i) only model error and without actuator faults and ii) both model error and actuator faults.

5.2. Simulation results

Case study 1: first, three controllers are evaluated for trajectory tracking under fault-free conditions. The control responses are shown in Figure 6, where Figures 6(a) and (b) present the translational and angular speed responses, respectively. The proposed controller exhibits faster convergence and smoother behavior than PID and i-PID without FO. The tracking errors are illustrated in Figure 7, with Figures 7(a)–(c) corresponding to the X-axis, Y-axis, and heading angle errors, respectively, all of which are smaller for the proposed controller. The overall trajectory tracking performance is depicted in Figure 8, confirming the superior accuracy of the proposed method.

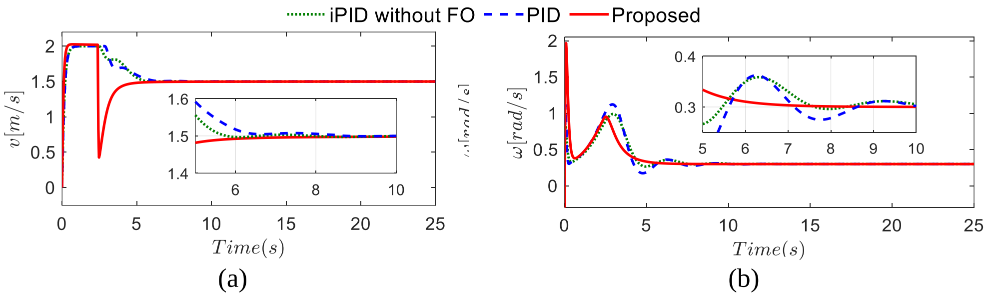

Figure 6. Signal controls of three controllers in case 1; (a) translational speed and (b) angular speed

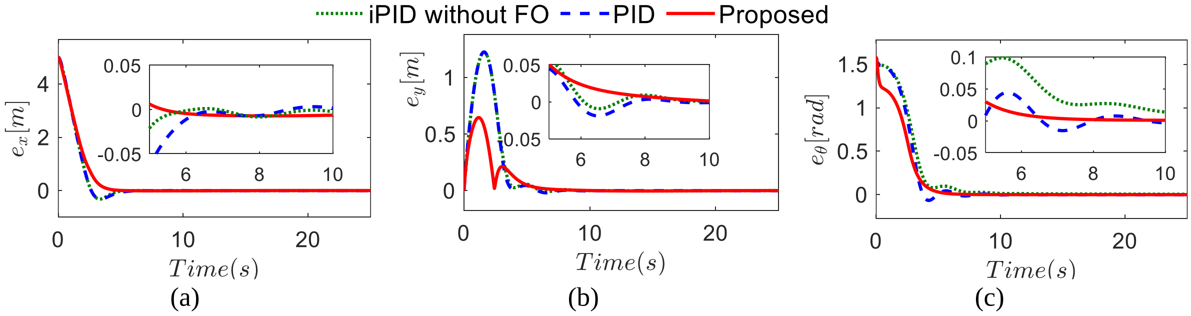

Figure 7. Tracking errors of three controllers in case 1; (a) X-axis, (b) Y-axis, and (c) heading theta

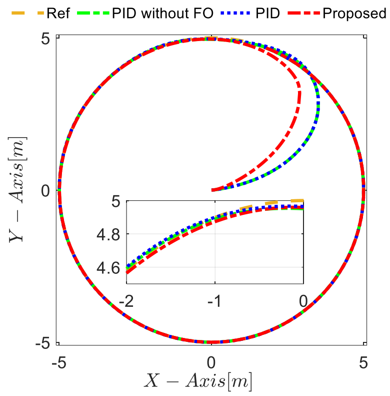

Figure 8. The trajectory tracking of relevant controllers in case study 1

Case study 2: in the second scenario, actuator faults are introduced at $t = 8s$ with $\Delta v = 0.25v\, sin(t)$ and $\Delta\omega = 0.3\omega$ while the robot mass m and inertia I are also varied. Figures 9 to 12 present the results. The tracking errors shown in Figure 9, including Figures 9(a)-(c) for the X-axis, Y-axis, and heading angle, respectively, indicate that the proposed controller maintains smaller deviations after the fault, whereas the PID and i-PID without FO exhibit larger residual errors and slower recovery. The control inputs are shown in Figure 10, where Figures 10(a) and (b) represent the translational and angular speed inputs, respectively. The proposed controller yields smoother signals with smaller oscillations, while PID and i-PID without FO exhibit larger fluctuations after the fault.

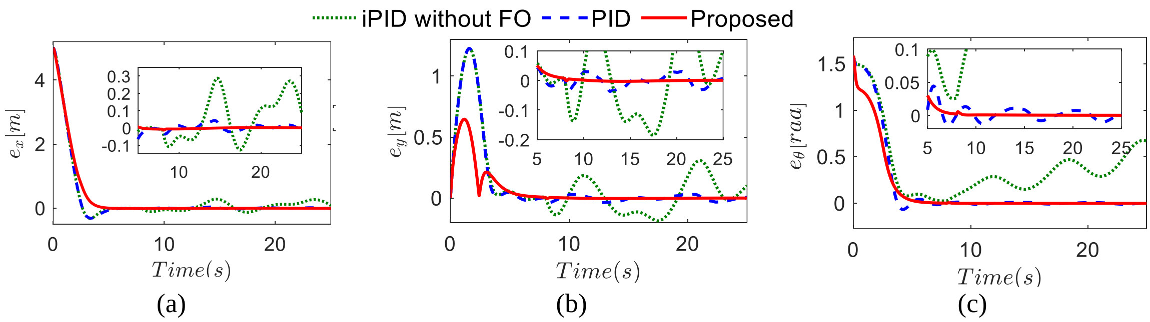

Figure 9. Tracking errors of three controllers in case 2; (a) X-axis, (b) Y-axis, and (c) heading theta

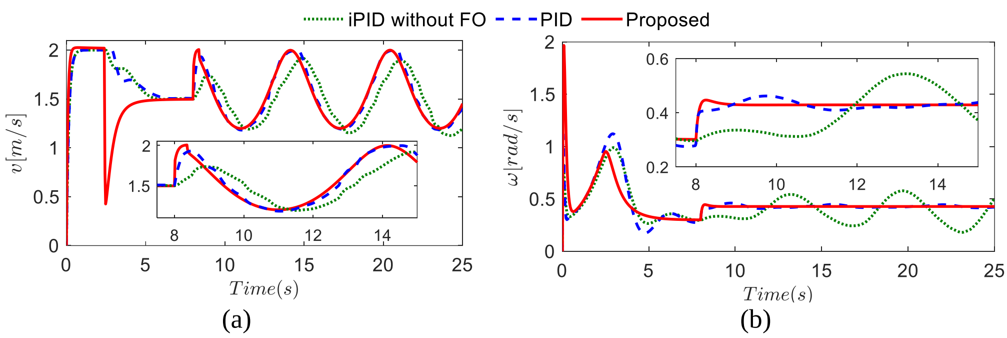

Figure 10. Control inputs of three controllers in case 2; (a) translational speed and (b) angular speed

Figure 11 demonstrates the role of the FO. The estimated Figure 11(a) faults $\Delta\hat{v}$ and Figure 11(b) $\Delta\hat{\omega}$ converge rapidly to the true values, with the detection occurring at approximately 8.007 s after fault injection. This accurate estimation enables effective fault compensation within the inner control loop. Finally, the trajectory tracking in Figure 12 confirms that the proposed controller preserves high tracking accuracy despite actuator degradation and parameter variations, whereas the benchmark methods exhibit significant deviations, especially in curved sections. Overall, the results confirm that combining MFC, i-PID, and FO greatly improves fault tolerance, robustness, and trajectory tracking over conventional PID-based methods. Accordingly, a proposed FTC was built based on iPID-MFC and FO in this article. It will be compensated for the effect of faults and ensure orbital tracking performance for the robot.

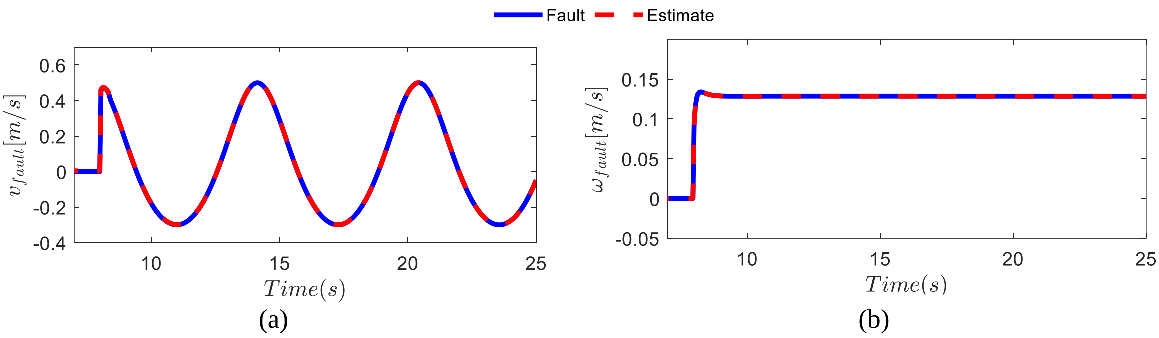

Figure 11. Fault estimation results of the WMR in case study 2; (a) translational speed and (b) angular speed

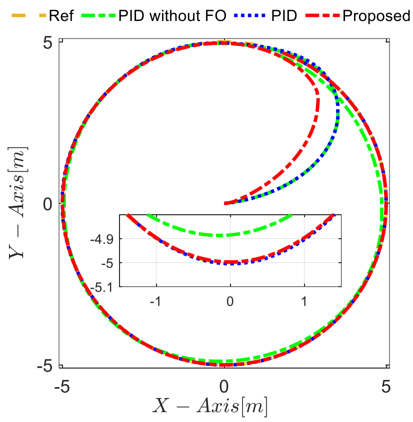

Figure 12. The trajectory tracking of relevant controllers in case 2

6. CONCLUSION

This paper presented the development of a robust FTC for trajectory tracking of WMRs under challenging conditions, including actuator faults and time-varying parameters. The proposed control scheme integrates MFC, i-PID, and FO to ensure reliable performance despite model uncertainties and external disturbances.

The stability of the closed-loop system has been rigorously proven using Lyapunov theory, providing a solid theoretical foundation for controller design. Extensive simulations in MATLAB/Simulink demonstrated the effectiveness of the proposed FTC in both fault-free and faulty scenarios. Results show significant improvements in trajectory tracking accuracy, robustness, and fault compensation compared to conventional PID and i-PID-MFC controllers without fault observation. For future work, we plan to extend the proposed FTC framework to consider sensor faults, enhance its real-time implementation, and conduct experiments on physical robot platforms to validate the method under real-world operating conditions.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

REFERENCES

[1] B. B. Mevo, M. R. Saad, and R. Fareh, "Adaptive Sliding Mode Control of Wheeled Mobile Robot with Nonlinear Model and Uncertainties," *Canadian Conference on Electrical and Computer Engineering*, vol. 2018-May, 2018, doi: 10.1109/CCECE.2018.8447570.

[2] B. Moudoud, H. Aissaoui, and M. Diany, "Fuzzy adaptive sliding mode controller for electrically driven wheeled mobile robot for trajectory tracking task," *Journal of Control and Decision*, vol. 9, no. 1, pp. 71–79, Jan. 2022, doi: 10.1080/23307706.2021.1912665.

[3] J. Bai, J. Du, T. Li, and Y. Chen, "Trajectory Tracking Control for Wheeled Mobile Robots with Kinematic Parameter Uncertainty," *International Journal of Control, Automation and Systems*, vol. 20, no. 5, pp. 1632–1639, May 2022, doi: 10.1007/s12555-021-0212-z.

[4] Y. Zheng, J. Zheng, K. Shao, H. Zhao, Z. Man, Z. Sun, "Adaptive fuzzy sliding mode control of uncertain nonholonomic wheeled mobile robot with external disturbance and actuator saturation," *Information Sciences*, vol. 663, 2024, 120303, doi: 10.1016/j.ins.2024.120303.

[5] S. P. Ho, D. T. Duong, V. C. Le, V. V. Phan, H. C. Ta, and T. T. Nguyen, "Fault-Tolerant Control for Mobile Robots: Integrating Low-Pass Filters for Improved Actuator Fault Compensation," *2025 3rd International Conference on Smart Systems for applications in Electrical Sciences (ICSSES)*, Tumakuru, India, 2025, pp. 1-6, doi: 10.1109/ICSSES64899.2025.11009298.

[6] R. Isermann, *Fault Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance*, Springer, New York, NY, USA, 2006.

[7] M. Witczak, *Fault Diagnosis and Fault-Tolerant Control Strategies for Non-Linear Systems*, Springer, vol. 266, pp. 1–229, 2014.

[8] P. Yazdjerdi and N. Meskin, "Design and real-time implementation of actuator fault-tolerant control for differential-drive mobile robots based on multiple-model approach," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 232, no. 6, pp. 652–661, Jul. 2018, doi: 10.1177/0959651818779849.

[9] V. D. Phan, H. V. A. Truong, and K. K. Ahn, "Actuator failure compensation-based command filtered control of electro-hydraulic system with position constraint," *ISA Transactions*, vol. 134, pp. 561–572, 2023, doi: 10.1016/j.isatra.2022.08.023.

[10] V. D. Phan, C. P. Vo, H. V. Dao, and K. K. Ahn, "Robust Fault-Tolerant Control of an Electro-Hydraulic Actuator with a Novel Nonlinear Unknown Input Observer," *IEEE Access*, vol. 9, pp. 30750–30760, 2021, doi: 10.1109/ACCESS.2021.3059947.

[11] H. V. Dao, D. T. Tran, and K. K. Ahn, "Active Fault Tolerant Control System Design for Hydraulic Manipulator with Internal Leakage Faults Based on Disturbance Observer and Online Adaptive Identification," *IEEE Access*, vol. 9, pp. 23850–23862, 2021, doi: 10.1109/ACCESS.2021.3053596.

[12] A. E. S. B. Ibrahim, "Wheeled mobile robot trajectory tracking using sliding mode control," *Journal of Computer Science*, vol. 12, no. 1, pp. 48–55, 2016, doi: 10.3844/jcssp.2016.48.55.

[13] A. Bessas, A. Benalia, and F. Boudjema, "Integral Sliding Mode Control for Trajectory Tracking of Wheeled Mobile Robot in Presence of Uncertainties," *Journal of Control Science and Engineering*, vol. 2016, no. 1, 2016, doi: 10.1155/2016/7915375.

[14] Y. H. Pham, T. L. Nguyen, T. T. Bui, and T. V. Nguyen, "Adaptive Active Fault Tolerant Control for a Wheeled Mobile Robot under Actuator Fault and Dead Zone," *IFAC-PapersOnLine*, vol. 55, no. 37, pp. 314–319, 2022, doi: 10.1016/j.ifacol.2022.11.203.

[15] X. Z. Jin, Y. X. Zhao, H. Wang, Z. Zhao, and X. P. Dong, "Adaptive fault-tolerant control of mobile robots with actuator faults and unknown parameters," *IET Control Theory and Applications*, vol. 13, no. 11, pp. 1665–1672, 2019, doi: 10.1049/iet-cta.2018.5492.

[16] B. Xiao, L. Cao, S. Xu, and L. Liu, "Robust Tracking Control of Robot Manipulators with Actuator Faults and Joint Velocity Measurement Uncertainty," *IEEE/ASME Transactions on Mechatronics*, vol. 25, no. 3, pp. 1354–1365, 2020, doi: 10.1109/TMECH.2020.2975117.

[17] Z. Sun, Y. Xia, L. Dai, K. Liu, and D. Ma, "Disturbance rejection MPC for tracking of wheeled mobile robot," *IEEE/ASME Transactions on Mechatronics*, vol. 22, no. 6, pp. 2576–2587, 2017, doi: 10.1109/TMECH.2017.2758603.

[18] G. C. Karras and G. K. Fourlas, "Model Predictive Fault Tolerant Control for Omni-directional Mobile Robots," *Journal of Intelligent & Robotic Systems*, vol. 97, no. 3–4, pp. 635–655, Mar. 2020, doi: 10.1007/s10846-019-01029-7.

[19] S. T. Dang, X. M. Dinh, T. D. Kim, H. L. Xuan, and M. H. Ha, "Adaptive Backstepping Hierarchical Sliding Mode Control for 3-Wheeled Mobile Robots Based on RBF Neural Networks," *Electronics (Switzerland)*, vol. 12, no. 11, p. 2345, May 2023, doi: 10.3390/electronics12112345.

[20] D. Trujillo, L. A. Morales, D. Chávez, and D. F. Pozo, "Trajectory Tracking Control of a Mobile Robot using Neural Networks," *Emerging Science Journal*, vol. 7, no. 6, pp. 1843–1862, Dec. 2023, doi: 10.28991/ESJ-2023-07-06-01.

[21] N. Hassan and A. Saleem, "Neural Network-Based Adaptive Controller for Trajectory Tracking of Wheeled Mobile Robots," *IEEE Access*, vol. 10, pp. 13582–13597, 2022, doi: 10.1109/ACCESS.2022.3146970.

[22] S. Li *et al.*, "Adaptive NN-based finite-time tracking control for wheeled mobile robots with time-varying full state constraints," *Neurocomputing*, vol. 403, pp. 421–430, Aug. 2020, doi: 10.1016/j.neucom.2020.04.104.

[23] S. Rhili, H. Trabelsi, and J. Hmad, "PI and PR Current Controllers of Single Phase Grid Connected PV system : Analysis, Comparaison and Testing," in *16th International Multi-Conference on Systems, Signals and Devices, SSD 2019*, pp. 700–705, Mar. 2019, doi: 10.1109/SSD.2019.8893232.

[24] L. A. Tuan, Y. H. Joo, L. Q. Tien, and P. X. Duong, "Adaptive neural network second-order sliding mode control of dual arm robots," *International Journal of Control, Automation and Systems*, vol. 15, no. 6, pp. 2883–2891, 2017, doi: 10.1007/s12555-017-0026-1.

[25] M. Boukens, A. Boukabou, and M. Chadli, "Robust adaptive neural network-based trajectory tracking control approach for nonholonomic electrically driven mobile robots," *Robotics and Autonomous Systems*, vol. 92, pp. 30–40, 2017, doi: 10.1016/j.robot.2017.03.001.

[26] X. Lu, X. Zhang, G. Zhang, and S. Jia, "Design of Adaptive Sliding Mode Controller for Four-Mecanum Wheel Mobile Robot," *Chinese Control Conference (CCC)*, 2018, vol. 2018, pp. 3983–3987, doi: 10.23919/ChiCC.2018.8483388.

[27] Q. Wen, X. Yang, C. Huang, J. Zeng, Z. Yuan, and P. X. Liu, "Disturbance Observer-based Neural Network Integral Sliding Mode Control for a Constrained Flexible Joint Robotic Manipulator," *International Journal of Control, Automation and Systems*, vol. 21, no. 4, pp. 1243–1257, 2023, doi: 10.1007/s12555-021-0972-5.

[28] Y. Lu and W. Liu, "Disturbance Observer-based Adaptive Fuzzy Fault-tolerant Tracking Control for Uncertain Nonlinear Systems with Input Delay and Actuator Faults," *International Journal of Control, Automation and Systems*, vol. 21, no. 3, pp. 820–828, 2023, doi: 10.1007/s12555-021-1117-6.

[29] A. M. Alshorman, O. Alshorman, M. Irfan, A. Glowacz, F. Muhammad, and W. Caesarendra, "Fuzzy-based fault-tolerant control for omnidirectional mobile robot," *Machines*, vol. 8, no. 3, p. 55, Sep. 2020, doi: 10.3390/MACHINES8030055.

[30] M. Qin, B. Guo, J. Liu, Q. Xiao, R. Guo, and S. Y. Dian, "Fault-tolerant control of trajectory tracking for mobile robot," *Proceedings - 2021 6th International Conference on Automation, Control and Robotics Engineering (CACRE 2021)*, 2021, pp. 269–273, doi: 10.1109/CACRE52464.2021.9501392.

[31] R. E. Precup, R. C. Roman, and A. Safaei, "Data-Driven Model-Free Controllers," *Data-Driven Model-Free Controllers*, pp. 1–403, 2021, doi: 10.1201/9781003143444.

[32] B. Moudoud, H. Aissaoui, and M. Diany, "Robust Adaptive Trajectory Tracking Control Based on Sliding Mode of Electrical Wheeled Mobile Robot," *International Journal of Mechanical Engineering and Robotics Research*, vol. 10, no. 9, pp. 505–509, 2021, doi: 10.18178/ijmerr.10.9.505-509.

[33] D. L. Tran, N. T. D. Cao, V. D. Phan, D. T. Duong, and S. P. Ho, "Advanced trajectory tracking control for wheeled mobile robots under actuator faults and slippage," *Bulletin of Electrical Engineering and Informatics*, vol. 14, no. 3, pp. 1746–1757, 2025, doi: 10.11591/eei.v14i3.9047.

[34] H. S. Phuong, N. M. Tien, N. D. Tan, M. T. Anh, and D. D. Tu, "Proposal of a Fault-tolerant controller for wheeled mobile robots with faulty actuators," *Proceedings - 12th IEEE International Conference on Control, Automation and Information Sciences (ICCAIS 2023)*, 2023, pp. 507–512, doi: 10.1109/ICCAIS59597.2023.10382286.

[35] V. T. Ha, T. T. Thuong, and L. N. Truc, "Trajectory tracking control based on genetic algorithm and proportional integral derivative controller for two-wheel mobile robot," *Bulletin of Electrical Engineering and Informatics*, vol. 13, no. 4, pp. 2348–2357, 2024, doi: 10.11591/eei.v13i4.7847.

[36] M. Fliess and C. Join, "Model-free control," *International Journal of Control*, vol. 86, no. 12, pp. 2228–2252, Dec. 2013, doi: 10.1080/00207179.2013.810345.

[37] K. Yano and K. Terashima, "Sloshing suppression control of liquid transfer systems considering a 3-D transfer path," *IEEE/ASME Transactions on Mechatronics*, vol. 10, no. 1, pp. 8–16, Feb. 2005, doi: 10.1109/TMECH.2004.839033.

[38] A. Kaldmäe and Ü. Kotta, "A brief tutorial overview of disturbance observers for nonlinear systems: Application to flatness-based control," *Proceedings of the Estonian Academy of Sciences*, vol. 69, no. 1, pp. 57–73, 2020, doi: 10.3176/proc.2020.1.07.

[39] H. Khalil and W. G. Jessy, *Nonlinear Systems*. NJ: Prentice hall: Nonlinear Systems, 3rd ed. Upper Saddle River, 2002.

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