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Evaluating random–Nyquist sampling ratios in combined compressed sensing magnetic resonance imaging

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ABSTRACT

Compressed sensing (CS) has been widely applied in magnetic resonance imaging (MRI) to accelerate the image acquisition without significantly reducing its image quality. In Cartesian MRI, acquisition time can be reduced by skipping phase-encoding steps for faster data acquisition. However, the balance between random under-sampling and Nyquist sampling at the k-space center strongly determines image quality. In this study, we systematically evaluate the impact of different random-to-Nyquist sampling ratios for both single-coil (CS-MRI) and multi-coil (CS-pMRI) reconstructions. Simulation results reveal that dense Nyquist sampling around the k-space center is essential for maintaining image fidelity, whereas reconstruction quality deteriorates sharply when random sampling exceeds approximately 60% of the total under-sampled data. Moreover, CS-pMRI consistently outperforms CS-MRI under equivalent under-sampling factors, benefiting from additional coil sensitivity information that improves resilience against aliasing and noise. These findings provide practical guidelines for hybrid under-sampling design, emphasizing that sufficient Nyquist sampling coverage of central k-space is crucial for achieving high-quality reconstructions while enabling high acceleration in CS-MRI.

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1. INTRODUCTION

Magnetic resonance imaging (MRI) is one of the most powerful non-invasive imaging methods in advanced medical diagnosis, providing high-resolution anatomical and functional information without using ionizing radiation. However, a major limitation of conventional MRI is its time consuming acquisition time, which can lead to motion artifacts, patient discomfort, and limited throughput in clinical practice. Reducing the acquisition time without considerably sacrificing image quality is therefore a central research goal in MRI technology. This challenge is often addressed by under-sampling k-space data and reconstructing the image using proper algorithms that exploit data redundancy and prior knowledge.

Over the last two decades, numerous strategies have been developed to accelerate MRI acquisition. The parallel MRI (pMRI) is one of these methods, which allows much faster image acquisition than the normal MRI due to its simultaneous signals receiving with an array of RF coils (multi-coils). Some of the common pMRI methods are used for reducing image scan time, such as the simultaneous acquisition of spatial harmonics (SMASH) [1] and sensitivity encoding (SENSE) [2], and generalized autocalibrating partially parallel acquisitions (GRAPPA) [3].

Many of other techniques primarily focus on k-space, a concept was first proposed by Donald B. Twieg in 1983 for describing nuclear magnetic resonance (NMR) image reconstruction by 2D Fourier encoding [4]. When taking an MRI scan, the encoded NMR signals are collected and sampled by the Nyquist sampling criteria as k-space for image reconstruction. The Nyquist criteria ensure the reconstructed image is free of aliasing artifacts. However, it has been found that a NMR image can be sparsely represented in a known transform domain [5]. Therefore it can be recovered from randomly under-sampled k-space data using nonlinear reconstruction [6], [7]. Compressed sensing MRI (CS-MRI), introduced by Lustig *et al.* [8], leverages sparsity in the transform domain and incoherent under-sampling to accurately recover images from far fewer k-space samples than Nyquist theory requires [9]. Several hybrid approaches that combine CS and pMRI have been proposed [10]-[13], showing improved reconstruction quality over either method alone. Sampling pattern design has also been extensively studied, random sampling with variable density, chaotic under-sampling, and a combination of random and Nyquist sampling being common choices to achieve incoherence while maintaining coverage of low-frequency components [14]-[16]. Although these studies clearly demonstrated the need for central k-space coverage and incoherent sampling, most studies heuristically propose fixed under-sampling masks and stop short of quantifying how the balance between random and Nyquist samples for ensuring image fidelity. Recently, data-driven and deep learning (DL)-based methods [17]-[19] have emerged, enabling joint optimization of sampling and reconstruction while yielding state-of-the-art performance at acceleration factors previously unattainable with conventional techniques. More recently, CS has been combined with deep adaptive perceptual generative adversarial network (DAPGAN) [20], [21] and self-supervised contrastive learning [22] for improving the accuracy of under-sampled image reconstruction. However, these methods typically require large, high quality training datasets to generalize well across anatomies, pathologies, and scanner vendors for image reconstruction. Moreover, training and deploying state-of-the-art DL models demand high performance hardware, particularly GPUs with large memory which may not be available in all clinical or research environments.

Despite these advances, there remain open problems in optimization of under-sampling pattern for Cartesian MRI acquisitions. Specifically, while random sampling satisfies the incoherence condition required by CS theory, excessive randomness can degrade the coverage of low-frequency k-space, which is essential for preserving image contrast and overall structure [8], [23], [24]. Conversely, sampling strategies that heavily favor the k-space center may under-sample high-frequency details, leading to blur image reconstruction [25]. Our previous works have acknowledged this trade-off but have not systematically quantified the effect of the relative proportion of low-frequency (Nyquist sampling) and random sampling on reconstruction quality, especially in the context of combined CS and pMRI reconstruction [11], [15]. However, these studies have either adopted fixed sampling patterns or relied on heuristic density functions, without establishing empirical guidelines on the optimal balance between random and deterministic sampling regions.

In this study, we address this issue by systematically evaluating hybrid sampling patterns that combine a fully-sampled low-frequency region (near k-space center) with a randomly under-sampled high-frequency region (periphery of k-space). By using retrospective simulations on both single-coil CS (CS-MRI) and multi-coil (CS-pMRI) reconstructions, we vary the proportion of random sampling and analyze its impact on quality metrics of image reconstruction. We propose a threshold in the random sampling ratio beyond which reconstruction quality degrades sharply, and also demonstrate that the CS-pMRI approach consistently outperforms CS-MRI alone under equivalent under-sampling factors. These findings provide guidance for practical MRI protocol design for high acceleration of CS.

2. METHOD

2.1. The basics of compressed sensing in magnetic resonance imaging

CS is known as a method that reduces the number of measurements required for reconstructing the signal or image without significantly decreasing its quality. In traditional MRI, the NMR signals are sampled with finite frequency information. In a MRI scan, data measurements are required to obtain a complete k-space dataset for proper image reconstruction [26]. Unlike frequency encoding, phase encoding takes a much longer time to be completed. Therefore, if the phase encoding lines are reduced, the scan time will be greatly reduced. Moreover, the highest magnitude data is distributed around the center of the k-space (low frequency region) and rapidly decays toward its border. The most important information that seems to be required to reconstruct an NMR image is concentrated at k-space center. Thus, taking the under-sampling data higher around the k-space origin will be more realistic and practical. The required time for acquiring k-space is determined by the number of phase encoding (k_y) measurements (the red lines are 2D Cartesian samples), as shown in Figure 1. In this figure, all k-space lines (indicated by the red arrows pointing toward the left) are acquired, followed by the frequency-encoding and phase-encoding steps.

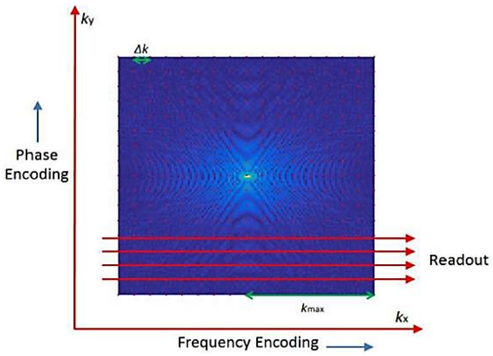
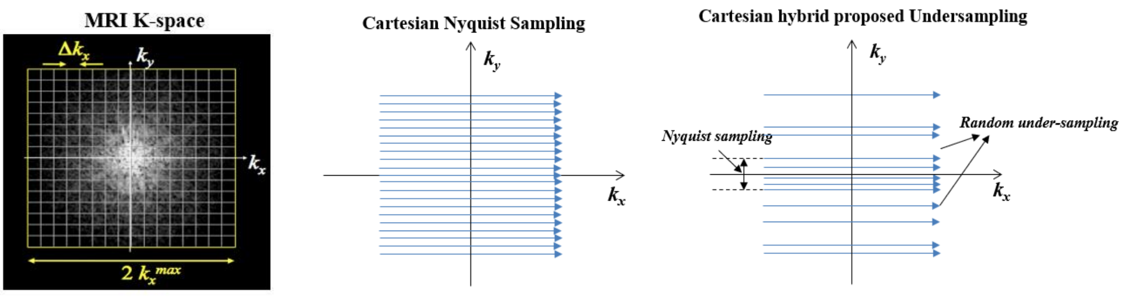


Figure 1. K-space data acquisition in traditional Cartesian MRI [26]

2.2. The proposal of new hybrid under-sampling with varying parameters in CS-MRI

Previously, we proposed a combination of random and Nyquist sampling (a hybrid under-sampling method) applied for MRI k-space (Figure 2(a)), which divided the taken number of phase encoding lines into two separate parts (the random under-sampling distributed at high frequency domain and the Nyquist sampling at the low frequency domain of the k-space [11], [15]). This method has been proven that it is capable of eliminating the disadvantages of the traditional random CS. The quality of the reconstructed image still remains with a small value of the under-sampling ratio (R). Different from the Cartesian Nyquist sampling (Figure 2(b)), the k-space matrix (N×N) is under-sampled by a majority of measurements defined by both a random sampling ratio R1 and a Nyquist sampling R2 ratio and (Figure 2(c)). Here, R1 represents the amount of random sampling part in the whole k-space, while R2 represents the Nyquist sampling around the k-space center. The total number of measurement samples is defined by the under-sampling ratio R (R=R1+R2). The advantage of this method is to ensure that certain amount of sampling data around the k-space origin is always taken by the Nyquist samplings, which had been confirmed the advantages of this method as compared to the traditional random and regular under-sampling methods. In these previous studies, we proposed a hybrid undersampling method by using R1=70% for random under-sampling and R2=30% for the Nyquist sampling. However, this ratio was heuristic and based on the need to guarantee minimal low-frequency coverage at the center of k-space while retaining incoherence with random sampling. Although this hybrid under-sampling has obviously outweighed traditional random methods, the influence of random under-sampling/Nyquist sampling (R1/R2) ratio on the quality of image reconstruction is still required to be thoroughly and systematically evaluated.



(a) (b) (c)

Figure 2. The model of CS using the hybrid under-sampling method; (a) MRI K-space, (b) Cartesian Nyquist sampling, and (c) Cartesian hybrid proposed undersampling [15]

Therefore, in this study, we investigate the effect of varying the proportion between low-frequency fully-sampled (Nyquist) and high-frequency randomly-sampled regions in the hybrid Cartesian under-sampling for both CS-MRI and CS-pMRI reconstruction. The methodology was proposed to be fully applicable to a broad range of Cartesian MRI acquisitions. Figure 3 illustrates the experimental workflow, which consists of: i) MRI image data; ii) generation of the hybrid under-sampling pattern with varying parameters; iii) image reconstruction; and iv) statistically quantitative evaluations. In this workflow, MRI image data (single-coil or multi-coils) is loaded to obtain fully sampled k-space. This fully sampled k-space is then under-sampled using a hybrid under-sampling mask with varying parameters. A proper reconstruction algorithm is selected to recover the image from the under-sampled data. Finally, image quality metrics are calculated to evaluate efficiency of the under-sampling method.

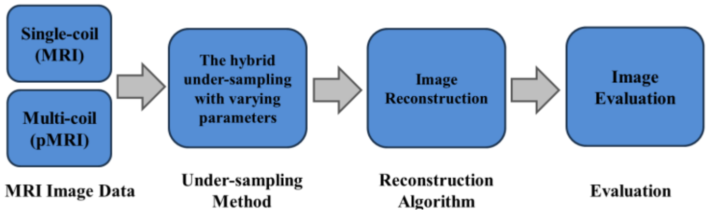


Figure 3. The workflow of this proposed under-sampling method and evaluation

The experimental workflow of hybrid under-sampling evaluations with varying parameters for CS-MRI and CS-pMRI acceleration:

- Step 1: load MRI data (single coil and multi-coil image) for transforming into fully-sampled k-space.
- Step 2: generate the under-sampled data (k_x, k_y) defined by the hybrid under-sampling pattern. The number of sampled data is based on the pre-defined under-sampling ratio R and the varying proportion between random and Nyquist sampling R1/R2 (assign R1 for random sampling and R2 for Nyquist sampling, and R=R1+R2). Determine coordinates (k_x, k_y) of the k-space for each case and store them as a template. Then, obtain the digital data of the under-sampled k-space based on the template and store them in a vector for each case.
- Step 3: perform the single coil and multi-coil reconstruction using the NCG algorithm.
- Step 4: calculate average values of image quality metrics and perform statistical evaluations.

2.3. Datasets and simulations

In this work, two datasets were used: single coil images (for CS-MRI experiments) is 2D images selected from the MRI dataset from MATLAB that contains 27 slices of a human head phantom (128×128 matrix, complex-valued) arranged into a 4-D array of size 128-by-128-by-1-by-27 and multi-coil images (for CS-pMRI experiments) is a 3-D MPRAGE dataset acquired with an eight-channel head coil (TE/TR=3.45/2530 ms, TI=1100 ms, FOV=25.6 cm, matrix=256×256, and slice thickness=1.33 mm), which was obtained from this work [27]. For evaluation of different anatomical context, cardiac MRI images (from [28]) and fastMRI knee images (from [29]) were also used in this work with dataset user agreement.

To study the effect of different ratios between random and Nyquist sampling to the reconstructed image quality, the value of R1 will be varied from 0.1 to 0.9 for different under-sampling ratio R in a range between 0.05 and 0.5. For each trial: the complex-valued k-space data was computed via 2D FFT for each slice. The under-sampling mask generated by the hybrid under-sampling method was applied retrospectively in the phase-encoding direction only. For each under-sampling ratio, 100 under-sampling masks are randomly repeated per trial for computational feasibility. These statistical evaluations based on numerical simulations will be performed for different under-sampling ratios with varying random R1 applying for both CS-MRI and CS-pMRI. The simulations were performed with a personal computer (Core i5-12100, 16 GB RAM, and SSD drive), the consuming time for each trial is about 10s (CS-MRI) and 2s (CS-pMRI).

2.4. Image reconstruction and evaluations

In the CS-MRI, the single coil MRI image reconstruction was carried out by the optimized nonlinear conjugate gradient (NCG) as described in our previous studies [11], [15]. This algorithm adapted from Lustig *et al.* [8], which was proposed to reconstruct an MRI image from under-sampled k-space data using CS principles. It combines data consistency with sparsity priors in both the wavelet domain and the total variation (TV) domain, and solves the optimization via NCG for minimizing the following objective as shown in (1):

$$\frac{min}{x}\frac{1}{2}||F_u x - y||_2^2 + \lambda_{TV}||TV(x)||_1 + \lambda_w||W(x)\,||_1 \tag{1}$$

This NCG reconstruction algorithm used in this work is described as following parameters: the maximum number of iterations: 30; the sparsity transform, ψ: daubechies D4 wavelet basis; L_1-wavelet norm smoothing: $\lambda_w = 0.005$; weight for TV penalty: $\lambda_{TV} = 0.002$. These values were selected and optimized based on our previous studies [11], [15], which were empirically tuned to balance noise suppression with preservation of fine anatomical details, ensuring reproducibility of the reported results.

In the CS-pMRI, the multi-coil MRI image reconstruction was performed by 2D SENSE reconstruction using the NCG algorithm. This NCG configuration was chosen for its robustness and relatively low computational cost compared to more complex iterative algorithms, while still achieving high quality reconstruction in both CS-MRI and CS-pMRI settings.

For quantitative evaluations, different image quality indexes such as mean squared error (E) for an average squared pixel error of the reconstructed MRI image compared to the ground-truth fully sampled image, the universal image quality index (Q) [30] for a perceptual metric sensitive to luminance and contrast; and the structural similarity index measure (SSIM) [31] for perceptual similarity between reconstructed and reference images, which were used to assess the simulation results. Each experiment was repeated 100 times with independent random masks to ensure statistical significance as aforementioned.

3. RESULTS AND DISCUSSION

3.1. Image reconstruction from hybrid under-sampled data with varying random proportions

To illustrate the impact of sampling ratios between random and fixed sampling amounts, the numerical simulations were performed for both the CS-MRI and CS-pMRI at the same under-sampling ratio of 0.15 but with different random R1 proportions. Figures 4 and 5 show the simulation results with the random sampling amounts (R1) of 0.1, 0.5, and 0.9 for CS-MRI and CS pMRI, respectively. The upper images show the distributions of taken sampling data (the under-sampling pattern), while the lower images are their respectively reconstructed images. When the random sampling amount (R1) increases, it can be noticed that the general quality of the reconstructed image reduces in both cases, especially the aliasing noise appearance at the high amount of random sampling (R1=0.9) as indicated by direct observation and image quality indexes (E, Q, SSIM). This means that by same under-sampled data of k-space, the reconstructed image quality will be improved when more sampling data is distributed around the center of k-space. Moreover, by the same under-sampling ratio R and random sampling/Nyquist sampling ratio, the CS pMRI proves better image quality than the CS-MRI as indicated by all image quality indexes. This finding highlights that while random sampling satisfies the incoherence condition required by CS theory (introduced by Lustig *et al.* [8]), fully sampling (Nyquist sampling) data compromises the coverage of low-frequency information, which is essential for preserving global image contrast and anatomy.

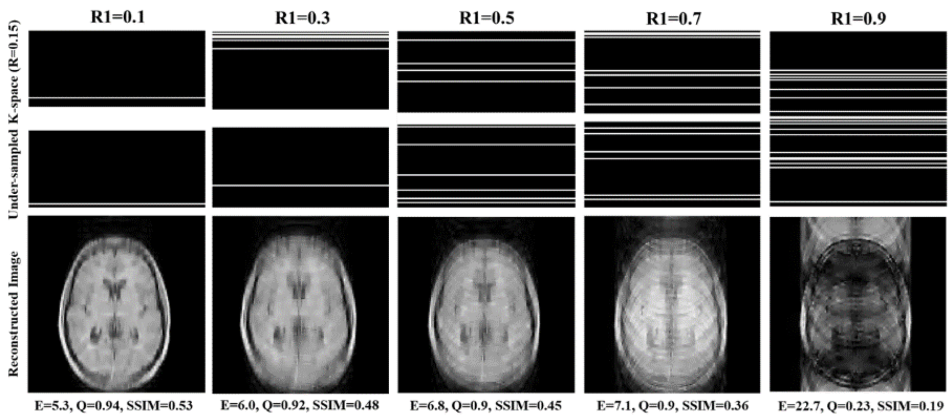


Figure 4. CS-MRI simulations at R=0.15 with different random sampling proportions R1: under-sampled k-space patterns and reconstructed images

In this work, MRI images of different anatomical structures have been also used for more thoroughly evaluating the proposed hybrid under-sampling method with varying the random proportion. Figure 6 shows the simulation results of CS-MRI on brain, cardiac, and knee MRI images for under-sampling ratio R of 0.15 and different random proportions R1 of 0.1, 0.5, and 0.9. As can be seen from upper photos in the figure, the quality of reconstructed images is crucially affected by the random sampling proportion, especially at high values of R1 (0.9). The quantitative evaluation of image quality indexes (normalized average error E, Q-index, and SSIM) shown on the lower-row graphs of Figure 6 was averaged by 30 simulations, and the scale bar in each column is standard deviation (SD), which has confirmed these observations. These findings suggest that under highly under-sampled ratios (R=0.15), maintaining a smaller proportion of random sampling yields the best trade-off between error, visual quality, and structural fidelity of reconstructed images from the under-sampled rata. Excessive random sampling in the CS-MRI reduces the effectiveness of k-space coverage, resulting in severe loss of diagnostic content, which is critically detrimental to the anatomical detail of all brain, cardiac, and knee image reconstruction.

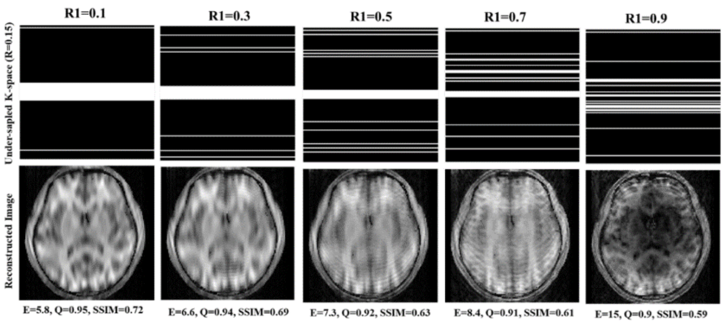


Figure 5. CS-pMRI simulations at R=0.15 with different random sampling proportions R1: under-sampled k-space patterns and reconstructed images

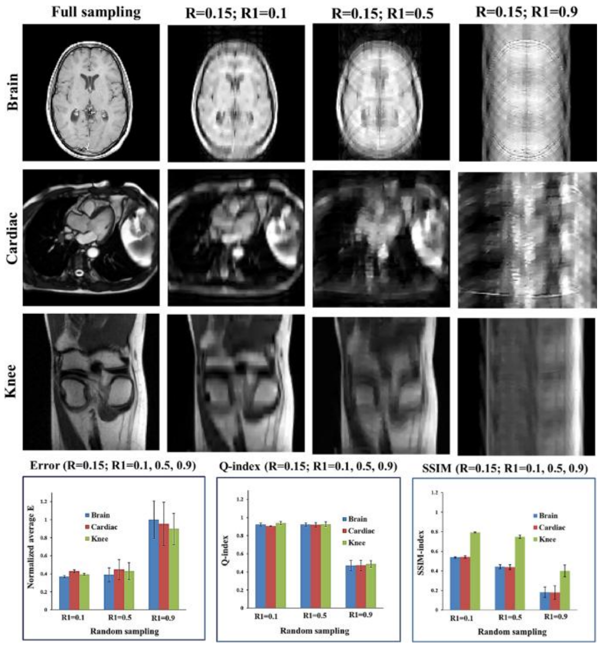
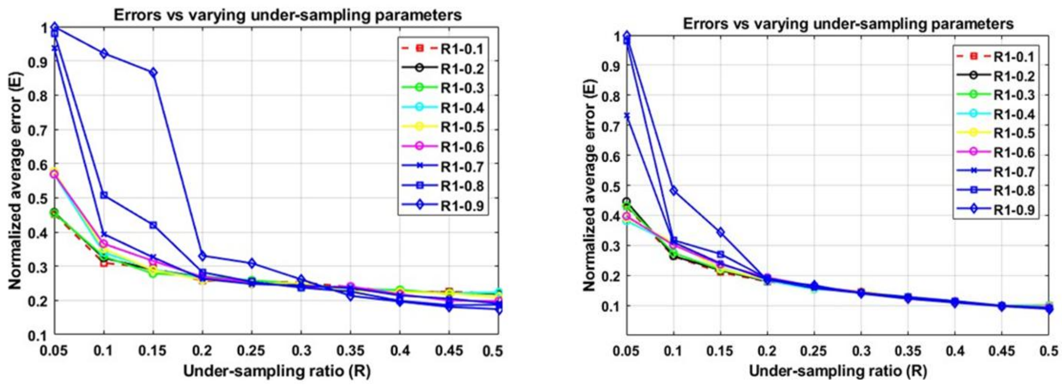


Figure 6. Evaluation of random sampling proportions (R1) in CS-MRI at an under-sampling ratio (R=0.15) for brain, cardiac, and knee MRI images

3.2. Statistical evaluation of image reconstruction with different under-sampling parameters

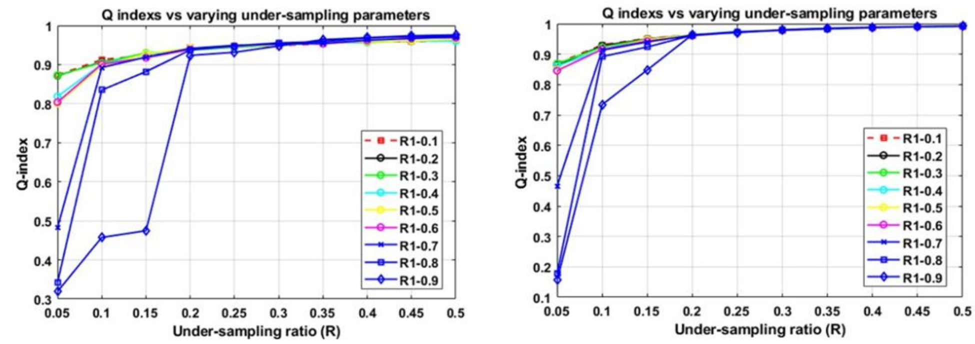
To provide a more rigorous assessment, 100 independent simulations were performed for each combination of under-sampling ratio (R) and random sampling proportion (R1). The results clearly indicate that image quality deteriorates significantly when the proportion of random under-sampling exceeds approximately 60% of the total acquired k-space data. Here, the repeated simulations were performed for each under-sampling ratio R, and the sampling amount of random under-sampling R1 varies from 0.1 to 0.9 (inversely, the Nyquist sampling amount R1 will reduce from 0.9 down to 0.1). Figure 7 shows the statistical simulated estimation of the average of normalized mean squared error (E) for each sampling portion in the CS-MRI (Figure 7(a)) and the CS-pMRI (Figure 7(b)). It can be seen that the error value E is lowest for the cases of random under-sampling amounts lower than 60% (from R1-0.1 to R1-0.6) in both CS-MRI and CS-pMRI. From this observation, across both CS reconstructions, dense Nyquist sampling of the central k-space can consistently preserve contrast, reduced structural distortion, and yielded higher image quality metric values. Moreover, it can be seen that the CS-pMRI outperforms CS-MRI alone in all tested configurations, demonstrating the advantage of leveraging coil sensitivity encoding alongside sparsity constraints.



(a) (b)

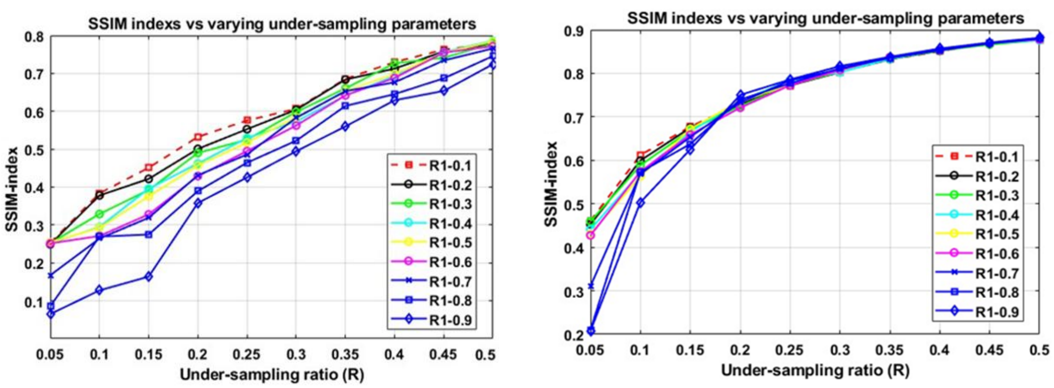
Figure 7. The dependence of the normalized average error (E) on varying under-sampling ratios (R) and random sampling proportions (R1) for; (a) CS-MRI and (b) parallel CS-pMRI

We also calculate average values (by 100 repeated simulations) for the universal image index Q and the structural similarity index SSIM for systematically studying the influence of different random sampling portions on the quality of image reconstruction in both CS-MRI and the CS-pMRI. Similar observations as seen on the error E, averaged Q and SSIM indexes calculated with different random under-sampling proportions (0.1 to 0.9) also confirm better image quality metrics when the random sampling amounts are smaller than 60% as shown in Figures 8 and 9. Figures 8(a) and 9(a) respectively show the quantitative evaluations of Q and SSIM indexes for CS-MRI, while Figures 8(b) and 9(b) show the same image quality metrics for CS-pMRI.



(a) (b)

Figure 8. The dependence of the average Q-index on varying under-sampling ratios (R) and random sampling proportions (R1) for; (a) CS-MRI and (b) CS-pMRI



(a) (b)

Figure 9. The dependence of the average SSIM-index on varying under-sampling ratios (R) and random sampling proportions (R1) for; (a) CS-MRI and (b) CS-pMRI

From these observations, in all cases, image quality metrics remain relatively stable and favorable when R1≤0.6, but degrade sharply beyond this threshold. Dense Nyquist sampling at the k-space center consistently preserves contrast, reduces structural distortion, and yields higher perceptual similarity. Importantly, CS-pMRI achieves superior performance compared to CS-MRI across all tested configurations. By leveraging coil sensitivity encoding in addition to sparsity constraints, CS-pMRI demonstrates improved resilience against aliasing artifacts and noise, confirming its synergistic advantage over single-coil CS-MRI.

3.3. Discussion on our evaluated compressed sensing method

In this work, by statistically evaluation of the hybrid under-sampling with varying parameters, our study lies in finding out a quantitative threshold in this hybrid under-sampling method. When random sampling exceeds ~60% of total under-sampled k-space data, reconstruction quality drops markedly. While earlier studies proposed hybrid or variable-density under-sampling strategies, they typically relied on heuristic functions or fixed masks and did not quantify the relationship between random/Nyquist ratios and image quality [11], [14], [15], [32]. Commonly, the center part of k-space contains high magnitude signals determining the general contrast, brightness, and shapes of the image, while its outer part determines the edges, details, and sharp transitions [33]. Thus, it is supposed that a perfect quality of the reconstructed image will be defined by a balance between both these regions of k-space. In contrast, from our statistical evaluations of different simulations, it can be confirmed that the sampled data is crucially required to be densely taken around the original of k-space for better quality of reconstructed images.

Our result is consistent with the finding reported by Kojima *et al.* [14], which showed that optimal under-sampling should include dense sampling near the k-space center. However, in our study, a more comprehensive study has been performed to find out that a higher random sampling proportion of ~60% will detrimentally affect the image quality, especially when high acceleration ratios are applied. Although this threshold might be dependent on datasets and reconstruction algorithm, it provides a useful empirical guideline for designing hybrid under-sampling patterns in Cartesian acquisitions.

Moreover, our work also found that the consistent superiority of CS-pMRI over CS-MRI highlights the synergistic benefit of coil sensitivity encoding in resolving aliasing artifacts and improving noise resilience. This is consistent with earlier studies [10], [12] and more recent CS-pMRI integrations [17], where multi-coil data provides additional spatial encoding that relaxes the incoherence requirements of CS.

Compared to recent advanced DL-based MRI reconstruction studies [34]-[36], our method achieves competitive quality without requiring large training datasets or high-end GPUs. DL methods have demonstrated remarkable performance at higher acceleration factors by learning powerful image priors and optimizing the sampling mask. For instance, Zibetti *et al.* [17], [18] employed a data-driven approach for optimizing pMRI sampling tailored to specific coil geometries and further proposed alternation learned adaptive masks coupled with unrolled networks to outperform heuristic variable-density patterns. More recently, Kim *et al.* [36] has deeply reviewed on using spatiotemporal and multi-contrast redundancies for deep-learning based reconstruction. However, these DL approaches have certain major limitations of data requirement and hardware demand. In fact, large, diverse training datasets are essential for generalization across anatomies, pathologies, and scanner vendors. Modern DL reconstructions are also often required powerful VGA card (GPUs with >16 GB VRAM) for efficient inference and reconstruction. In contrast, our method is model-free, easy to integrate into existing Cartesian MRI protocols, and computationally feasible on standard CPUs for moderate-sized datasets.

From a clinical perspective, our findings may provide practical implications for clinical MRI protocol design. Real MRI scanners impose hardware and software constraints on how under-sampling masks can be implemented. In clinical workflow for informatics integration, the hybrid under-sampling parameters (R and R1) can be embedded in sequence of data acquisition in user interface (UI) software before MRI scanning. The image reconstruction can be performed with NCG reconstruction or proper CS framework. Our results suggest that ensuring at least certain amount of Nyquist sampling coverage is a safe design principle: it balances incoherence with adequate low-frequency information, and aligns with current practices where 30–50% of central lines are retained in routine brain imaging [6]. This provides meaningful scan time reduction without detrimentally compromising diagnostic quality. In our work, maintaining at least 40% Nyquist sampling around the k-space center while under-sampling its periphery can still achieve 5–7 times acceleration (R=0.15 and R1<06), which directly improves patient throughput and reduces motion-related artifacts. However, the trade-off between the image quality and the scan time should be carefully examined by practical protocol designers to ensure diagnostic reliability, especially in subtle pathologies. More importantly, our empirical threshold provides a quantitative guide for such decisions, offering a balance between efficiency and reliability that can be integrated into clinical sequence development. Moreover, from electrical engineering aspects, our results also inform MRI hardware. The ability of multi-coil arrays in CS-pMRI to outperform single-coil CS suggests that coil geometry and sensitivity encoding are crucial in exploiting our proposed hybrid under-sampling approach. Optimizing RF coil configurations to maximize central k-space encoding efficiency could further enhance acceleration performance, especially when Nyquist coverage is constrained by hardware or time limitations.

Finally, these findings are especially relevant for scanners or institutions without advanced DL reconstruction frameworks, where sampling pattern design remains the primary lever for acceleration. Even in DL-enabled settings, this study's empirical threshold can serve as a starting point for mask initialization before fine-tuning via learning-based methods. Our ongoing work is currently focused on developing DL-based reconstruction from under-sampled k-space (not presented in this study). Using this practical guideline of the empirical threshold for hybrid under-sampling can result in much better reconstruction image quality, even at high acceleration factors in the range of 7–10.

4. CONCLUSION

This work investigated the effect of varying proportions between random and Nyquist sampling in a hybrid CS-MRI and parallel MRI (CS-pMRI). Using systematic simulations and evaluations, we demonstrated that image reconstruction quality strongly depends on maintaining sufficient sampling density at the center of k-space. When random sampling exceeds approximately 60% of the total under-sampled data, reconstruction performance degrades sharply, leading to visible aliasing and reduced image fidelity. These findings confirm that while random sampling ensures incoherence, dense Nyquist coverage of low-frequency components is indispensable for preserving image contrast and structural integrity. We have also found that CS-pMRI consistently outperformed CS-MRI, as validated by quantitative image quality metric measurements. This synergy highlights the potential of CS-pMRI as a more robust and reliable strategy for accelerated MRI, particularly in this hybrid under-sampling regimes. The empirical threshold identified in this study offers practical guidance for sampling pattern design in Cartesian MRI. Specifically, ensuring that at least 40% of k-space acquisition is allocated to Nyquist sampling can safeguard diagnostic image quality while still achieving significant reduction of scan time. Importantly, the proposed hybrid approach is computationally feasible on standard hardware and can be readily integrated into existing clinical protocols, making it suitable for environments without access to DL-based reconstruction. Overall, the results provide a simple yet effective guideline for balancing random and deterministic sampling to optimize MRI acceleration, which was not quantitatively addressed in earlier works. This contribution offers both theoretical insight and practical utility, ensuring that hybrid under-sampling strategies can be possibly applied in real-world MRI protocols with predictable image quality outcomes.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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Duc-Tan Tran	✓			✓	✓	✓		✓		✓	✓	✓		
Anh Quang Tran	✓		✓	✓		✓		✓	✓	✓		✓	✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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