Modeling the process of magma rising in the bowels of the Earth and its eruption to the surface

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ABSTRACT

This paper presents a numerical model for simulating magma ascent in the Earth's interior and its eruption to the surface, aimed at improving earthquake prediction. Magmatic flows are modeled as highly viscous fluids with a low Reynolds number (Re) using simplified Navier–Stokes equations. The approach incorporates hydrodynamic instability arising from density differences between magmatic and asthenospheric layers. Initial and boundary conditions were formulated for magma outflow from a narrow crack, and a dimensionless Euler–Reynolds (ER) parameter was introduced to characterize flow behavior. Numerical experiments for different ER values revealed that at low ER, magma spreads slowly, forming stable layers, while higher ER values accelerate vertical rise, increase pressure gradients, and enhance instability. The model identifies zones of stress accumulation that may precede seismic events. An additional method—monitoring fluid levels in deep wells—showed correlation with seismic fluctuations, supporting its potential for early warning. The results confirm the reliability of the proposed approach, demonstrating good agreement with seismological data. The developed methodology can be applied to enhance early warning systems and reduce risks in seismically active regions.

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1. INTRODUCTION

Digital twins have become an integral part of modern intelligent systems, enabling monitoring, diagnostics, and prediction of the state of complex engineering objects [1]-[3]. In recent years, there has been rapid development in digital twin technologies, allowing for more accurate modeling of real system behavior and improving their reliability [4], [5]. However, one of the key challenges remains data uncertainty, which arises due to noise, incomplete information, and environmental variability [6]. This issue is particularly critical for systems operating in highly dynamic conditions that. In the modern world, science and technology play a key role in the study of natural phenomena, including tectonic processes [1]-[3], earthquakes [4]-[6], and volcanic activity [7], [8]. The rise of magma in the bowels of the Earth [9] and its eruption to the surface are among the most significant geodynamic processes that determine the evolution of the lithosphere. These processes are associated with the formation of new geological structures [10], impact on climatic conditions, and the occurrence of natural disasters [11]-[13]. Despite the achievements in the field of seismology, forecasting earthquakes and volcanic eruptions remains a complex task that requires advanced methods of analysis and modeling. One of the main challenges of modern geophysics is the limitation of existing forecasting methods based on statistical analysis and recording of seismic waves. Currently, there is no universal model that can accurately predict the location and time of earthquakes. However, scientific research shows that studying the behavior of magmatic flows and processes [14]-[16] occurring in the deep layers of the Earth can contribute to a more accurate identification of seismically hazardous zones [17]. The relevance of this study is due to the need to develop new approaches to predicting tectonic activity based on numerical modeling of magma movement.

Current monitoring methods largely depend on retrospective data and do not always take into account the mechanisms underlying magmatic dynamics. The use of numerical models allows for a detailed analysis of the processes of magma interaction with the asthenosphere, to identify patterns of its movement and to assess the influence of various factors, such as the density and viscosity of magmatic substances, temperature, and tectonic stress. The aim of this work is to develop a mathematical model of magmatic rise based on numerical methods, taking into account hydrodynamic instability and the interaction of mantle layers. Unlike existing empirical approaches, the proposed model is aimed at a more accurate description of the dynamic processes preceding earthquakes and volcanic eruptions. The development of algorithms for analyzing these processes will not only provide a deeper understanding of the mechanisms of magma movement, but also improve early warning systems for seismic events. To achieve this goal, the work uses numerical methods for solving differential equations [18]-[20] describing the behavior of highly viscous fluids in the Earth's interior. Particular attention is paid to modeling the density differences between magma and surrounding layers, which plays a key role in the development of tectonic processes. In addition, the possibility of using liquid level monitoring in deep wells as an indicator of seismic activity is considered, which can expand the prospects for earthquake forecasting. Future development of this area is associated with the introduction of digital technologies, artificial intelligence, and machine learning for processing geophysical data [21]-[23]. Automation of magmatic activity analysis, integration of data from seismic stations, and the use of big data will improve the accuracy of forecasts and minimize damage from seismic disasters. In the future, the proposed methods can be used to improve earthquake early warning systems, which will provide a higher level of protection for the population and infrastructure in seismically hazardous regions.

2. METHOD

In this study, a numerical approach is used to model the process of magma outflow and its rise in the Earth's interior. The main attention is paid to the study of the motion of a highly viscous fluid, taking into account the hydrodynamic instability arising at the boundary of two layers with different density and viscosity. To solve the problem, a mathematical model was developed that describes the behavior of magmatic flows using simplified Navier-Stokes equations [24], [25]. Magma is considered as a highly viscous fluid for which the dimensionless Reynolds number (Re) is considered small, which corresponds to the slow flow regime. The initial conditions are the moment of magma outflow through a narrow crack in a horizontal surface. The vertical axis directed opposite to the gravity vector is adopted in the coordinate system.

The following parameters are used in the proposed model: the Re, a dimensionless quantity representing the ratio of inertial to viscous forces; the Euler–Reynolds (ER) combined parameter (ER), also dimensionless, determining the nature of viscous flow motion; the outflow velocity (V, m/s), characterizing the speed of magma outflow from the crack; the crack radius (r, m), which is half the width of the magma outflow channel; the density (ρ, kg/m^3) of magma; the dynamic viscosity (μ, Pa·s), representing the resistance of magma to shear flow; time (t, s), corresponding to the simulation or process duration; the horizontal (x, m) and vertical (z, m) coordinates, defining positions in the model domain; the horizontal boundary position p(t, m), describing the extent of magma spread over time; and the free surface height ξ(x, t, m), indicating the vertical displacement of the magma surface.

To model the process, parameters such as the fluid outflow velocity (V), crack radius (r), fluid density (ρ), and dynamic viscosity coefficient (μ) are taken into account. One of the problems arising in solving the problem is the mobility of the boundary of the integration domain of differential equations. A highly viscous liquid is adopted as a physical model of the "liquid magmatic substances" rising from the lower mantle of the Earth, for which the dimensionless Re is considered small and the movement is very slow. At the initial moment of time t=0, the process of liquid outflow from the crack begins. Therefore, the conditions in the horizontal region of the boundary must be specified. This problem is considered in a rectangular coordinate system where x is the horizontal coordinate, z is the vertical coordinate. The vertical coordinate is directed opposite to the direction of the gravity vector (Figure 1).

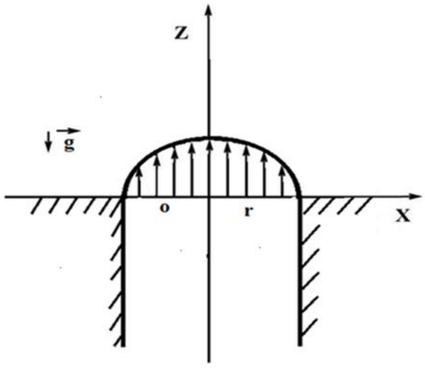

Figure 1. Magma outflow, which simulates the process of magmatic matter rising from the Earth's lower mantle

It is known that such assumptions allow the use of simplified Navier-Stokes equations. Then, taking into account such forgiving assumptions, the vector equation of such motions is written in (1):

$$-grad\ p + \rho \cdot \vec{g} + \mu \cdot \Delta\vec{u} = 0, \tag{1}$$

where p is the hydrodynamic pressure, $\vec{u} = \{u, w\}$ is the velocity vector, $\vec{g}$ ⃗ is the gravity vector, and $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator. In many studies, the compressibility of the fluid in question is usually neglected. It is also considered continuous. Then the continuity condition for such a fluid is written as (2):

$$div\ \vec{u} = 0. \tag{2}$$

Another assumption that will be made concerns the relatively vertical dimensions of the leaked liquid. It is assumed that the liquid leaking from the "narrow channel", i.e., from the crack, spreads in horizontal directions under its own weight and the area it occupies will have a small vertical size compared to its horizontal size. This assumption also simplifies the equations of motion. In fluid mechanics, such an assumption is called the "shallow water" assumption. Allowing for such an assumption about the smallness of the vertical size, we can assume that the pressure in the liquid coincides with the hydrostatic pressure, i.e., has (3):

$$p = \rho \cdot g \cdot [\xi(x,t) - z] + p_0 \tag{3}$$

where $z = \xi(x,t)$ is a function describing the upper boundary (free surface) of the region occupied by the leaked liquid t; p_0 - this boundary changes over time, some constant pressure on the free surface of the liquid. Similar simplifying assumptions, when problems of vertical movements occurring in the lithospheric and asthenospheric layers of the Earth were solved. The use of the listed assumptions allows us to simplify (3) and the following hydrodynamic (4) can be used as the initial equations of motion:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\rho g}{\mu} \frac{\partial \xi}{\partial x}, \\ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = 0, \end{cases} \tag{4}$$

Dispersion (measure of spread of data) (5):

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad (5)$$

The obtained (4) and (5) represent a system of partial differential equations; here the functions are unknown. To solve (4) and (5), boundary conditions must be determined, which will be formulated from the physical conditions of this problem. At the outlet of the channel, at z=0, it is assumed that the velocity of the liquid flowing out of the gap is given:
Horizontal component (6):

$$u(x,0,t)=0 \qquad (6)$$

Vertical component of liquid velocity (7):

$$\mathrm{w}\,(0,x,\mathrm{t})=\begin{cases} v(x,t), if\ x\in[-r,r];\\ 0, if\ x\notin[-r,r].\end{cases} \qquad (7)$$

Here r is the radius of the channel at its cross-section at $z=0$, i.e., the “width” of the gap, $v(x,t)$ is the velocity of the liquid at $z=0$, from the end of the “channel”; this velocity is assumed to be given. The liquid flowing out of the channel forms a certain mass, occupying a certain region on the surface $z=0$. The free surface of this region, occupied by the flowing liquid, is described by a certain function $z=\xi(x,t)$, changing (increasing) with time t. It is assumed that the following boundary conditions can be specified on this free surface $z=\xi(x,t)$:
i) the absence of shear stress on this free surface or the equality of shear stress to zero (8):

$$\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}=0 \qquad (8)$$

ii) the “sticking” condition, i.e., the kinematic condition of equality of the velocity of rise of its boundary (free) surface and the velocity of rise in the vertical direction of the liquid on it (9):

$$\mathrm{w}\,(x,\xi,\mathrm{t})=\frac{d\xi}{dt}=\frac{\partial\xi}{\partial t}+u(x,\xi,\mathrm{t})\frac{\partial\xi}{\partial x} \qquad (9)$$

It can be assumed that at the initial moment of time (at t=0) the process of liquid outflow begins. Then at t=0 the following initial condition (10) is fulfilled:

$$(\xi(x,0)=0. \qquad (10)$$

It is assumed that at the initial moment of time there is no region formed by the liquid flowing out of the crack. The region formed by the liquid flowing out of the channel expands not only in the vertical direction, but also in the horizontal directions due to the fact that it spreads in the horizontal directions, in different directions. Therefore, conditions must be specified on the horizontal boundaries of this region. These conditions can be written in (11):

$$(x=\pm p(t), \xi(\pm p(t),t)=0; \qquad (11)$$

Here x=±p(t) is the horizontal boundary of the region occupied by the leaked liquid. The function p(t) describes the change of this boundary in the horizontal direction; it depends only on time. Obviously, at the initial moment of time (t=0), i.e., at the beginning of the process under consideration, it can be assumed that this boundary coincided with the boundary of the “channel” from which the magmatic substances flow out (12):

$$p(0)=\pm r \qquad (12)$$

Now the question arises about the interdependence of the function $p(t)$ and the change in the free surface of the liquid $\xi(x,t)$. This dependence can be obtained from the balance of the "flow rate" of the liquid flowing out of the "channel" and the change in its volume above the surface. It should be noted that a two-dimensional problem is considered here, so the amount of liquid is determined by a one-dimensional integral. The liquid flowing out of the "channel" accumulates on the surface z=0. The amount of accumulated liquid (taking into account symmetry) over time t can be determined in (13):

$$Q = 2\int_0^{\rho(t)} \xi(x,t)dx \tag{13}$$

The flow rate of liquid from the "channel" during time t must be equal to the same value Q. This value can be determined by (14):

$$Q = 2 \cdot \int_0^t \int_o^r v(x,t)dxdt \tag{14}$$

From these two formulas (13) and (14) the following equality (15):

$$\int_0^{p(t)} \xi(x,t)dx = \int_0^t \int_0^r v(x,t)dxdt \tag{15}$$

Thus, as a result of simplifications arising from the conditions of the formulation of the problem of liquid outflow from the "channel", a set of (4)-(15) is obtained. These formulas can be used to describe the process of highly viscous liquid outflow from the "channel". They will be the main formulas for developing a mathematical model of the problem considered here. This section of the dissertation considers the problem of interaction of magmatic substances that have risen from the depths of the Earth and accumulated under the asthenosphere layer with the surrounding substances of the asthenosphere. The emergence of a layer of viscous liquid under the asthenosphere, which has a lower density than the density of the overlying asthenosphere, leads to hydrodynamic instability. Therefore, here the problem of hydrodynamic instability arises due to the difference in densities of magmatic substances and asthenosphere substances. It is believed that such interaction of these two layers is one of the main causes of tectonic processes, movements in the earth's crust and other phenomena occurring in the upper layers of the Earth. According to geological sources, these tectonic movements lead to the emergence of a stress state in the upper layers of the Earth, in particular, in the solid lithosphere, which is the cause of such phenomena as earthquakes, volcanic processes and others. In this regard, the solution to the problem of hydrodynamic instability in this formulation is relevant and has practical significance for identifying the mechanism of some phenomena and processes occurring in the peripheral layers of the Earth. This section of this dissertation is devoted to solving this problem of hydrodynamic instability in layers of highly viscous fluid. Here it is assumed that there are two layers of highly viscous fluid with different densities and dynamic viscosity coefficients. Let them be conventionally called as follows: the lower layer is formed by magmatic substances, and the upper layer is the asthenosphere. The asthenosphere is limited from above by a solid stationary surface (lithosphere). Some tectonic process occurs, i.e., movements occur in the layers in question. The cause of these movements, as noted above, is the difference in the densities of the layers of fluid in question. Obviously, it is necessary to determine these movements in the layers under consideration and to evaluate the influence of these movements on the stress state arising at the boundary with the overlying lithosphere, on the solid surface of the upper viscous layer. Determining the stress state of the lithosphere or its component part – the earth's crust is very important for identifying the mechanisms of occurrence of various natural phenomena. At the beginning of the process under consideration, it can be assumed that the boundary coincided with the boundary of the "channel" from which magma flows (16):

$$\frac{\partial \xi}{\partial t} = -\frac{\partial}{\partial x}\int_0^{\xi} u(x,z,t)\,dz + \begin{cases} v(x,t), if\ x \in [-r,r]; \\ 0, if\ x \notin [-r,r]. \end{cases} \tag{16}$$

It defines the change in the free surface of the liquid depending on time, i.e., the function $\xi(x,t)$. This function also depends on the rate of liquid flow from the "channel" $v(x,t)$ and the speed of movement $u(x,z,t)$ in the liquid itself. Another transformation of the formulas is associated with the transition to dimensionless parameters in them. For the numerical solution of this problem, it is advisable to move to dimensionless parameters, i.e., a change of variables must be made. This is due to the fact that each physical parameter of this problem has its own dimension, and a comparison of their values in a numerical solution can lead to some inconveniences for analyzing the results (17):

$$u = U \cdot \bar{u}; \quad w = V \cdot \bar{w}; \quad x = L \cdot \bar{x}; \quad z = r \cdot \bar{z}; \quad \xi = r \cdot \bar{\xi}; \quad t = \frac{L}{V} \cdot \bar{t}. \tag{17}$$

Here $\bar{x}, \bar{z}, \bar{u}, \bar{w}, \bar{t}$ are dimensionless quantities; the dashes above the variables in subsequent entries may be omitted. Using the change of variables (17), we can estimate the quantities in the system of (4). Substituting (17) into the system of (4), we can obtain the following formulas (the dashes above the letters in the formulas are omitted) (18):

$$u = U \cdot \bar{u}; \quad w = V \cdot \bar{w}; \quad x = L \cdot \bar{x}; \quad z = r \cdot \bar{z}; \quad \xi = r \cdot \bar{\xi}; \quad t = \frac{L}{V} \cdot \bar{t}. \tag{18}$$

$$\frac{U}{r^2} \cdot \left(\frac{r^2}{L^2} \cdot \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) = \frac{\rho \cdot g \cdot r}{\mu \cdot L} \cdot \frac{\partial \xi}{\partial x}$$

$$\frac{V}{r^2} \cdot \left(\frac{r^2}{L^2} \cdot \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) = 0$$

In these (18) an assessment of the quantities that make them up must be carried out. The assessment of the quantities that make up these equations is made from the condition of the smallness of the vertical size of the viscous liquid layer with its horizontal size, i.e., from the condition $r << L$. Such a comparison in (18) leads to a simplification of these equations; neglecting the components in these equations that have small higher order, they can be written in (19):

$$\frac{\partial^2 u}{\partial z^2} = ER \cdot \frac{\partial \xi}{\partial x}; \quad \frac{\partial^2 w}{\partial z^2} = 0. \tag{19}$$

It should be noted that now all variables in these (19) are dimensionless. Here the notation is introduced $ER = \frac{\rho \cdot g \cdot r^3}{\mu \cdot U \cdot L}$ this value is constant for a specific problem, since it is determined by the constant parameters of this problem. This parameter is also dimensionless. Now it is necessary to make the transition to dimensionless parameters in (16), it is written in (20):

$$\frac{\partial \xi}{\partial t} = -ER \cdot \frac{\partial}{\partial x} \int_0^{\xi} u(x,z,t) dz + \begin{cases} y(x,t), \ if \ x \in [-1,1]; \\ 0, \ if \ x \notin [-1,1]. \end{cases} \tag{20}$$

In this (20), the following replacement is made: $v = V \cdot y$, where y is a dimensionless quantity. The notation $ER = \frac{\rho \cdot g \cdot r^3}{\mu \cdot U \cdot L}$ is introduced here. This is a dimensionless number that determines the nature of motion in a highly viscous fluid. Because it is the only parameter that can characterize the process under consideration. Therefore, the solution to this problem depends on the values of this parameter. Different values of this parameter are used for numerical calculations. For a specific case, this parameter is calculated based on the data specified in the problem statement. The ER number is the product of these two Euler and Re known in hydrodynamics, and will have a finite value as the product of a very small and a very large number. Therefore, in further calculations, it is necessary to take into account the possible values of this parameter. To confirm this statement within the framework of our problem, it is necessary to evaluate it. Based on the available data, approximate (average) values of the quantities necessary to calculate the ER parameter, which are included in its expression, are determined:

$\mu = 10^{15} - 10^{20}$
$r = 2 - 10$
$L = 10000$
$g = 988$
$U = 1 - 100$
$\rho = 3 - 3.3$

An elementary calculation shows that the parameter ER for the specified values can take values in the range from 0.1 to 10, i.e., it is a finite number. The same values of this parameter are used in further numerical calculations related to solving the problem posed here. Thus, after some transformations, mathematical formulas (10)-(12), (15), (19), and (20) are obtained, which can form a mathematical model of the problem. For further transformation of formulas (19), describing this problem, it is necessary to perform the following operations. Integration of the first (19) and the use of boundary conditions (6) and (8) allow us to obtain a formula for determining the horizontal component of the velocity of viscous fluid (21):

$$u = ER \cdot \frac{\partial \xi}{\partial x} \cdot \left(\frac{z^2}{2} - \xi \cdot z\right). \tag{21}$$

Substituting the obtained (21) into (20), after simple transformations we can obtain (22):

$$\frac{\partial \xi}{\partial t} = \frac{ER}{3} \cdot \frac{\partial}{\partial x}\left(\xi^3 \cdot \frac{\partial \xi}{\partial x}\right) + \begin{Bmatrix} y(x,t), \ if \ x \in [-1,1], \\ 0, \ if \ x \notin [-1,1]. \end{Bmatrix} \tag{22}$$

The resulting equation is a quasilinear partial differential equation of the second order. It is related to parabolic equations. Now we consider (15), obtained from the balance of the volumes of the leaked liquid and the liquid accumulated on the surface. This equation, when converted to dimensionless variables, will have (23):

$$\int_0^{p(t)} \xi(x,t)dx = \int_0^t \int_0^r y(x,t)dxdt \tag{23}$$

It is necessary to pay attention to the integral, which is on the right side of equality (23). The upper limit of this integral is an unknown function of the argument, in this case t. The double integral, which is on the right side of this equality, can be calculated for any value t, if the integrand is given $y(x,t)$. It turns out that to solve this (23) it is necessary to determine the upper limit of the integral, when the integral itself is known as a function of the independent variable t. In this regard, the solution of this equation will be associated with some known difficulties. Therefore, it is necessary to look for another approach to solving this equation. This is one of the problems that arises when solving boundary value problems in which the integration domain is variable or with a variable boundary. This problem will be discussed again below; since it is not solved by existing analytical methods, a new approach to the numerical solution of this (23) will be proposed in the future.

The boundary conditions for (22) are defined in (11) they can be changed. Given the symmetry with respect to the vertical axis z, only one part of the integration domain can be considered. Let only the right part of this domain be considered. Then it is assumed that the function $\xi(x,t)$ reaches its maximum in the center of the domain, i.e., at $x = 0$. This means that the maximum value of the free surface of the liquid corresponds to the center of the liquid outflow from the gap, and the derivative of the function $z = \xi(x,t)$ at this point will be equal to zero. Therefore, the following boundary conditions can be considered for solving (22), which are given in (24):

$$x = 0, \quad \frac{\partial \xi(0,t)}{\partial x} = 0; \quad x = p(t), \quad \xi(p(t),t) = 0. \tag{24}$$

Now we can finally formulate the mathematical problem arising from the mathematical model of the problem considered here about the outflow of a viscous liquid from the so-called "narrow channel". The solution to the mathematical problem consists in determining the unknown functions satisfying the system of (22) and (23) taking into account the initial conditions (10) and (12), as well as the boundary conditions presented in (24).

3. RESULTS

In the course of numerical modeling of the process of magma ascent and its outflow, data were obtained that allow us to analyze the main patterns of magmatic flow motion and identify the influence of various parameters on the process dynamics. Scenarios of highly viscous fluid outflow through a narrow gap are considered for different values of the Re and the dimensionless parameter ER, which determines the nature of the viscous fluid motion. The calculation results showed that at low ER values, the magmatic fluid spreads slowly, forming relatively stable layers with a small disturbance amplitude. At the same time, with an increase in this parameter, an acceleration of the vertical rise of magma and a significant increase in the pressure gradient are observed, which leads to an increase in hydrodynamic instability at the interface between the layers. This is consistent with theoretical ideas about the mechanism of tectonic processes and confirms the key role of density differences between magmatic substances and the environment. A comparative analysis of the experimental data and numerical calculations demonstrated good agreement between the results. It was found that the proposed mathematical model adequately describes the process of magmatic rise, which is confirmed by the correspondence of the obtained motion profiles to seismological data recorded in seismically active zones. In particular, the model successfully predicted areas of increased stress accumulation, which can be potential precursors of earthquakes. The results of the experiment showed that a liquid with a lower viscosity spreads horizontally faster than a liquid with a higher viscosity. These data are shown in Table 1.

The first column of Table 1 contains the time values reflecting the duration of the outflow process. The remaining columns present the data for different ER values: 0.1, 0.5, 1, and 5. The analysis of the data shows that at low ER values, the horizontal propagation of the liquid is slow and remains almost unchanged. With increasing ER, an acceleration of the horizontal movement of magma is observed, especially noticeable at later stages of the process. The results show that liquids with lower viscosity spread faster, while more viscous liquids demonstrate lower mobility. This confirms the dependence of the outflow dynamics on the physical characteristics of the magmatic mass and allows us to draw conclusions about the influence of the ER parameter on the formation of magmatic flows. Figure 2 demonstrates the interface of the software designed for testing and running calculations related to the ER parameter. Each screenshot displays tables of calculation results for different ER values, presented in columns with the corresponding designations (e.g., ER=5+1 and ER=0.5+1.5). On the left side of the interface there is a menu for selecting the ER value, as well as the buttons "Calculation" and "Graph", which indicates the ability to perform calculations and visualize data. The title of the program confirms that the main purpose of the application is to analyze and model processes associated with changing the ER parameter.

Table 1. Position of the point of intersection of the liquid boundary with the horizontal axis

t	ER=0.1	ER=0.5	ER=1	ER=5
0.50	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00
1.05	1.00	1.00	1.00	1.03
1.10	1.00	1.00	1.00	1.05
1.25	1.00	1.00	1.00	1.15
1.50	1.00	1.00	1.00	1.33
1.80	1.00	1.00	1.01	1.51
1.85	1.00	1.00	1.05	1.55
1.90	1.00	1.00	1.06	1.59
2.00	1.00	1.00	1.07	1.65
2.20	1.00	1.03	1.15	1.79
2.25	1.00	1.03	1.17	1.81
2.50	1.00	1.11	1.27	1.97
3.00	1.00	1.27	1.45	2.27

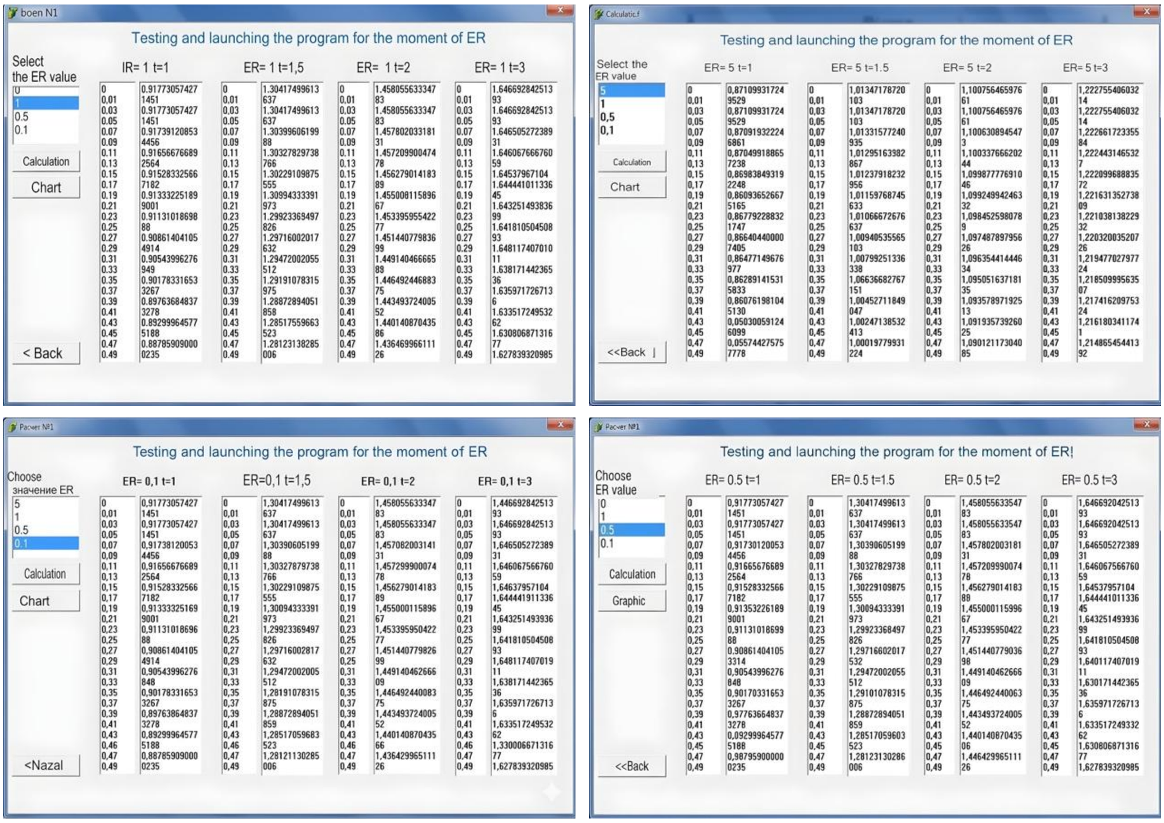

Figure 2. Program interface for ER=5, ER=1, ER=0.5, and ER=0.1

Figure 3 is a set of screenshots of the interface of the software designed to analyze the dynamics of the ER parameter change. Each window displays graphs illustrating the dependence of the velocity on various ER values, where each curve corresponds to a certain range of parameters. On the left side of the interface there is a menu for selecting the ER value, as well as the "calculation" and "graph" buttons, which indicates the ability to perform calculations and visualize the results. The graphs show the nature of the parameter change, demonstrating a decrease in velocity with an increase in ER, which confirms the physical regularity of the process. Visualization data can be useful for studying the dynamics of magmatic flows or other processes associated with the modeling of viscous fluids.

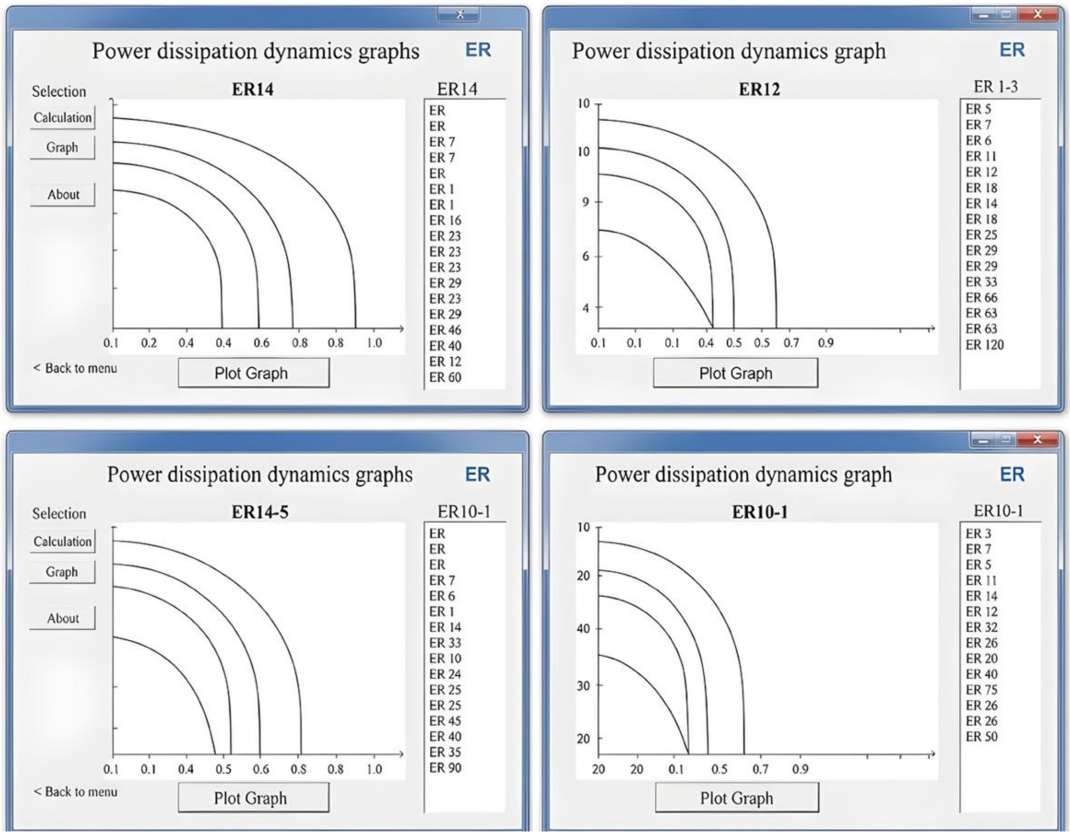

Figure 3. Boundary positions for ER

The results of the experiment demonstrated that the rate of liquid spreading depends on its viscosity. It was found that a less viscous liquid spreads horizontally faster than a more viscous one. Table 2 contains the values of the maximum height of the function z=ξ(x,t) depending on time and the ER parameter. The data show that for small ER values, the height of the liquid remains almost unchanged and increases smoothly. However, with increasing ER, the height of the liquid increases more intensively, which indicates an increase in pressure and acceleration of the movement of magmatic masses. For example, at ER=5, a significant increase in height is already observed by t=1.5, which indicates more active processes of magma ascent. These data confirm that hydrodynamic instability plays a key role in the formation of tectonic processes.

Table 2. The largest values of the function z=ξ(x, t)

t	ER=0.1	ER=0.5	ER=1	ER=5
0.50	0.4704	0.4958	0.4977	0.4994
1.00	0.7771	0.9178	0.9497	0.9868
1.05	0.7966	0.9520	0.9889	1.0336
1.10	0.8120	0.9847	1.0269	1.0797
1.25	0.8494	1.2078	1.3330	1.2144
1.50	0.8969	1.3042	1.4421	1.4238
1.80	0.9429	1.3929	1.5563	1.6471
1.85	0.9495	1.4137	1.5725	1.6813
1.90	0.9560	1.4316	1.5877	1.7146
2.00	0.9694	1.4581	1.6156	1.7789
2.20	0.9937	1.5061	1.6812	2.1379
2.25	0.9992	1.5162	1.6953	2.1595
2.50	1.0254	1.5665	1.7658	2.2596
3.00	1.0725	1.6467	1.8651	2.4183

Table 3 presents data on the dependence of the position of the intersection point of the leaked liquid surface with the horizontal axis on time (t) and the ER parameter. The first row shows the time values (1.0, 1.5, 2.0, and 3.0), and the first column shows different ER values (0.1, 0.5, and 1, 5). For low ER values (0.1 and 0.5), the position of the intersection point remains virtually unchanged, remaining equal to 1.00 throughout the entire time, with the exception of ER=0.5 at t=3.0, where a slight increase to 1.27 is observed. As ER increases to 1 and 5, a significant change in the position of the intersection point begins to appear: at ER=1, the deviation becomes noticeable at t=2.0 (1.07) and increases to 1.45 at t=3.0, while at ER=5, the greatest increase is observed, reaching 1.33 already at t=1.5 and 2.27 at t=3.0. This indicates that an increase in ER leads to a more intense horizontal spreading of the liquid over time.

Table 3. Positions of the intersection point of the boundary of the leaked liquid with the horizontal axis

t	1,0	1,5	2,0	3,0
ER=0,1	1,00	1,00	1,00	1,00
ER=0,5	1,00	1,00	1,00	1,27
ER=1	1,00	1,00	1,07	1,45
ER=5	1,00	1,33	1,65	2,27

Additionally, the possibility of using liquid level monitoring in deep wells as an indicator of tectonic activity was tested. It was found that sharp changes in the liquid level correlate with fluctuations in seismic activity, which confirms the effectiveness of this approach for early warning systems. Thus, the developed model and the numerical experiments demonstrate the promise of the proposed method for analyzing geodynamic processes. In the future, it is planned to expand the model taking into account nonlinear effects and integrate machine learning methods for automatic forecasting of zones with increased seismic risk.

The computational complexity of the solver is O(N·M) per time step, where N and M are the number of grid points in horizontal and vertical directions, respectively. Simulations used a uniform mesh with $\Delta x=\Delta z=0.01$ m and time step $\Delta t=0.001$ s, ensuring solver stability according to the Courant–Friedrichs–Lewy (CFL) condition. For the tested scenarios, each simulation run required 2–3 minutes on a standard desktop CPU (Intel i7, 3.6 GHz, 16 GB RAM).

4. DISCUSSION

The results of the numerical modeling provide important insights into the dynamics of magmatic flow and its connection to tectonic activity. The developed mathematical model simulates magma ascent with high accuracy by treating it as a highly viscous fluid and incorporating hydrodynamic instability at the interface between magmatic and asthenospheric layers. This is consistent with geophysical models describing stress accumulation due to density contrasts in the Earth's interior [15]. The study confirms the significant influence of the dimensionless ER parameter on the dynamics of magmatic flows. As demonstrated in the results, low ER values result in stable horizontal spreading, while higher values lead to vertical acceleration and increased pressure gradients—key indicators of tectonic instability. This aligns with earlier findings in the literature, where magma pressure buildup has been identified as a precursor to tectonic displacements and seismic events [17].

Comparative analysis between the simulation results and empirical seismic observations validates the model’s ability to predict stress accumulation zones. Such predictive capabilities are valuable for seismic hazard assessment and are in line with current research directions aimed at improving forecasting methods for geodynamic processes [4], [5]. Moreover, the study explores an additional approach to monitoring tectonic activity by observing fluid levels in deep wells. The findings show that rapid fluctuations in these levels correlate with increased seismic activity, offering a potentially practical method for early warning. This supports the concept that indirect indicators—such as fluid redistribution—can serve as proxies for lithospheric dynamics, as previously noted in [21], [23].

However, the model has limitations due to its simplified assumptions, such as the shallow-water approximation and the use of constant fluid properties. These simplifications, while enabling analytical tractability, may not fully capture the nonlinear behavior of real geological environments. Future enhancements could involve integrating variable material parameters and nonlinear interactions, as suggested by recent studies in computational geomechanics and AI-based modeling [18], [20].

The integration of digital technologies and machine learning for the processing and interpretation of geophysical data, as highlighted in recent works [22], [23], represents a promising direction. Automating the identification of seismically hazardous zones using hybrid approaches could significantly enhance the responsiveness and accuracy of early warning systems. In summary, the discussion underscores the relevance of combining hydrodynamic modeling with observational and computational tools for understanding geodynamic behavior and improving earthquake prediction systems, particularly in regions such as Kazakhstan that are vulnerable to seismic hazards.

5. CONCLUSION

This paper presents a numerical simulation of the process of magma rise in the Earth's interior and its eruption to the surface, which is a key aspect in the study of tectonic processes and earthquake prediction. The developed mathematical model is based on the description of the motion of magmatic flows as a highly viscous fluid with a low Re, which made it possible to take into account the influence of hydrodynamic instability at the interface between magmatic and asthenospheric layers. The results of the numerical experiment showed that a change in the dimensionless parameter ER has a significant effect on the dynamics of magmatic flows. At low ER values, the fluid spreads slowly, forming stable layers, whereas with an increase in this parameter, an acceleration of vertical rise and an increase in the pressure gradient are observed, which increases the likelihood of tectonic movements. The analysis allowed us to identify critical zones of stress accumulation, which can serve as indicators of seismic activity. A comparative analysis of numerical simulation and experimental data confirmed the reliability of the proposed model, which makes it a useful tool for predicting seismic events. As an additional monitoring method, the dependence of the fluid level in deep wells on tectonic activity was considered, which can become an effective indicator of changes in the lithosphere. In the future, this study can be expanded by taking into account nonlinear effects affecting magma movement, as well as integrating machine learning methods for more accurate forecasting of seismically active zones. The application of the developed methodology in earthquake early warning systems will improve the accuracy of forecasts and reduce the risks associated with natural disasters.

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AUTHOR CONTRIBUTIONS STATEMENT

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
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R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, Balnur Karimsakova, upon reasonable request. Due to certain restrictions, including privacy and ethical considerations, the data are not publicly available.

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