

# Robust optimization model for capacitated vehicle routing problem in waste transportation under demand uncertainty

Hendra Cipta, Rina Widyasari, Raisha Zuhaira Dongoran

Department of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Sumatera Utara, Medan, Indonesia

## Article Info

### Article history:

Received Aug 25, 2025

Revised Apr 18, 2026

Accepted Jul 14, 2026

### Keywords:

Capacitated vehicle routing problem

Clarke-Wright savings

Demand uncertainty

Price of robustness

Robust optimization

Waste transportation

## ABSTRACT

Urban waste transportation systems often experience inefficiencies due to uncertainty in daily waste generation, leading to vehicle overloads, increased operational costs, and environmental impacts. This study proposes a robust optimization model for the capacitated vehicle routing problem (RO-CVRP) to explicitly address demand uncertainty in municipal waste collection. A budgeted uncertainty parameter  $\gamma$  is incorporated to control the level of protection against worst-case deviations. Initial routes are generated using the Clarke-Wright savings (CWS) algorithm and subsequently evaluated within a robust optimization framework. Computational experiments are conducted using real data from temporary disposal sites (TPS) in Tanah Enam Ratus Subdistrict Medan City, with a vehicle capacity of 10 m<sup>3</sup>. The results show that higher  $\gamma$  values produce more conservative routing solutions, increasing the number of vehicles while reducing the risk of capacity violations. Price of robustness (PoR) analysis highlights the trade-off between transportation cost and reliability, confirming the model's effectiveness for resilient waste logistics planning.

*This is an open access article under the [CC BY-SA](#) license.*



## Corresponding Author:

Hendra Cipta

Department of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Sumatera Utara  
Medan, Indonesia

Email: [hendracipta@uinsu.ac.id](mailto:hendracipta@uinsu.ac.id)

## 1. INTRODUCTION

Indonesia grapples with ongoing municipal solid waste management (MSWM) issues fueled by rapid urbanization, population expansion, and rising consumption. Major cities like Jakarta, Surabaya, Bandung, Medan, and Makassar produce thousands of tons of waste daily, overwhelming strained transportation and disposal systems, which results in waste buildup at temporary disposal sites (TPS) and elevates environmental, health, and social hazards [1], [2]. Conventional collection routes planned statically based on experience underutilize vehicle capacity, driving up travel distances, fuel use, operational expenses, and service delays [3].

Municipal waste transportation in cities like Medan, Indonesia's third-largest urban center generating 1,700-2,000 tons daily, faces major inefficiencies from fluctuating waste volumes driven by household patterns, weather, public events, and socio-economic shifts [4], [5]. These uncertainties often cause vehicle overloads, disrupted routes, or missed collection points, undermining the 20–30% gains in distance, costs, fuel, and emissions promised by optimized routing [6], [7]. Deterministic plans falter amid demographic pressures, rainfall, festivals, and urban growth, leading to extra trips, congestion, higher labor and fuel demands, and elevated emissions, while straining the nearing-capacity Terjun landfill.

From an optimization perspective, demand uncertainty in waste transportation often leads to infeasible routing plans, vehicle overloading, and service disruptions when deterministic models are

employed [8], [9]. Robust optimization provides a systematic framework to address this issue by generating solutions that remain feasible across a predefined range of demand realizations. In particular, the budgeted uncertainty approach proposed by Bertsimas and Sim introduces a robustness parameter  $\Gamma$  that allows decision-makers to explicitly control the trade-off between cost efficiency and protection against worst-case demand deviations [10], [11].

Numerous studies have examined vehicle routing problem (VRP) and capacitated VRP (CVRP) models for waste transport [12]. Peña *et al.* [13] evaluated the Clarke-Wright savings (CWS) heuristic as a benchmark for optimizing electric waste collection routes under vehicle-capacity and battery constraints. Shao *et al.* [14] developed a hybrid variable neighborhood search and Tabu Search method to optimize synchronized commercial waste collection operations. Furthermore, Caramia *et al.* [15] proposed a two-stage optimization framework that integrates facility location, clustering, and vehicle routing for regional waste management. Evolutionary metaheuristics and dynamic programming variants offered superior performance over traditional heuristics but demanded intensive parameter calibration and lengthy runtimes, hindering municipal adoption [16]–[18]. While these approaches improved efficiency, most assumed deterministic demand, failing to address fluctuations in waste volume. Moreover, advanced heuristics often required heavy parameter tuning or long computation times, limiting their practical use in municipal operations. Given these limitations, the CWS algorithm has been widely recognized as a computationally efficient heuristic that provides near-optimal baseline routes under moderate data sizes [19]. This algorithm is well-regarded for its computational simplicity, speed, and ability to produce near-optimal routing solutions by leveraging a distance-based savings principle [20], [21]. The generated baseline routes are then expanded within a robust optimization framework to incorporate demand uncertainty.

Existing Indonesian research predominantly employs deterministic models for waste management, neglecting demand uncertainty, while global studies on uncertainty modeling favor commercial logistics over municipal waste transport. This study bridges these shortcomings through a robust optimization capacitated vehicle routing problem (RO-CVRP) framework customized for uncertain waste collection demands in Medan. Its innovation centers on integrating the Bertsimas-Sim budgeted uncertainty scheme directly into CVRP constraints operationalized via a Clarke-Wright solution heuristic ensuring robustness at the mathematical level rather than heuristically, thereby enabling controlled adjustments to feasible routes based on demand fluctuations at transfer points.

This study integrates the CWS algorithm [22], [23] with the Bertsimas-Sim robust framework [24] to advance municipal waste routing under demand uncertainty. Key contributions include a robust CVRP model tailored for variable waste loads, an efficient hybrid solution merging heuristic savings with budgeted optimization, real-world validation using Medan operational data, sensitivity analysis of costs against robustness, and a pathway for green extensions incorporating emissions ultimately equipping planners with tools to mitigate overloading and boost collection reliability.

## 2. THE COMPREHENSIVE THEORETICAL

### 2.1. Deterministic capacitated vehicle routing problem

The deterministic CVRP aims to determine a set of routes that minimize total travel cost while ensuring that each customer is visited exactly once and that vehicle capacity constraints are not violated [25], [26]. The standard formulation CVRP given:

Objective function:

$$\min \sum_{k \in K} \sum_{i \in V} \sum_{j \in V, j \neq i} c_{ij} x_{ij}^k \quad (1)$$

Constraints:

$$\sum_{k \in K} \sum_{j \in V, j \neq i} x_{ij}^k = 1, \quad \forall i \in V \setminus \{0\} \quad (2)$$

$$\sum_{j \in V, j \neq i} x_{ij}^k = \sum_{j \in V, j \neq i} x_{ji}^k, \quad \forall k \in K, \quad \forall i \in V \quad (3)$$

$$\sum_{k \in K} \sum_{j \in V \setminus \{0\}} x_{0j}^k = \sum_{k \in K} \sum_{j \in V \setminus \{0\}} x_{j0}^k \quad (4)$$

$$u_i^k - u_j^k + Q_k x_{ij}^k \leq Q_k - q_j, \quad \forall k \in K, \quad \forall i \neq j, (i, j) \in V \setminus \{0\} \quad (5)$$

$$q_i \leq u_i^k \leq Q_k, \quad \forall k \in K, \quad \forall i \in V \setminus \{0\}; \quad u_0^k, \quad \forall k \quad (6)$$

$$\sum_{i \in V \setminus \{0\}} q_i a_i^k \leq Q_k, \forall k \in K \quad (7)$$

$$x_{ij}^k \in \{0,1\}, a_i^k \in \{0,1\}, u_i^k \geq 0 \quad (8)$$

The objective function (1) minimizes the total transportation cost across all vehicles and routes. Constraint (2) ensures that each customer is visited exactly once by one vehicle. Constraint (3) maintains flow conservation, meaning each vehicle entering a node must also leave it. Constraint (4) guarantees that vehicles depart from and return to the depot consistently. Constraint (5) eliminates subtours using the MTZ formulation to enforce route continuity. Constraint (6) enforces vehicle capacity limits at each visited customer. Constraint (7) ensures that the total demand served by each vehicle does not exceed its capacity. Constraint (8) defines the decision variables as binary and non-negative continuous values.

## 2.2. Robust optimization under demand uncertainty

Robust optimization has been widely applied to decision-making problems involving uncertain parameters [27]. The budgeted uncertainty framework proposed by Bertsimas and Sim allows uncertain parameters to deviate within predefined bounds, while limiting the number of parameters that can simultaneously reach their worst-case values through a robustness budget  $\Gamma$  [28], [29].

In this study, demand uncertainty at TPS  $i$  is modeled as (9):

$$d_i = \bar{d}_i + \tilde{d}_i z_i, z_i \in [0,1], \sum_i z_i \geq \Gamma \quad (9)$$

where  $\bar{d}_i$  is nominal demand,  $\tilde{d}_i$  is maximum deviation from the nominal value,  $z_i \in [0,1]$  indicates the realization of uncertainty, and  $\Gamma$  controls the maximum number of TPS demands that may simultaneously experience their worst-case deviations.

## 2.3. Clarke-Wright savings algorithm

The CWS algorithm is a classical heuristic for solving CVRP instances and remains highly relevant due to its computational efficiency and simplicity [30]. The core idea of the algorithm is to merge two routes into a single route when such a merger yields a positive distance saving.

The saving associated with combining customers  $i$  and  $j$  is defined as (10):

$$s_{ij} = c_{i0} + s_{c0j} - c_{ij} \quad (10)$$

where  $c_{i0}$  and  $c_{0j}$  denote the distances from the depot to customers  $i$  and  $j$ , respectively, and  $c_{ij}$  is the distance between the two customers.

# 3. METHOD

## 3.1. Problem description

This research employs a quantitative modeling approach to tackle routing inefficiencies in municipal waste transport amid demand uncertainty, focusing on in Tanah Enam Ratus Subdistrict Medan City, where TPS vary spatially and waste volumes fluctuate daily. The system features one central depot and multiple TPS, with fixed-capacity vehicles following routes from depot to assigned sites and back. Using Bertsimas-Sim budgeted uncertainty sets, demand deviations around nominal values are controlled via an uncertainty budget, balancing robustness against conservatism. The solution unfolds in two stages: an initial deterministic CVRP solved via CWS heuristic for nominal routes, followed by robust capacity checks to guarantee feasibility under variability.

## 3.2. Research framework

This study employs a structured methodology that starts with gathering primary data from field observations and official records covering TPS sites, inter-site distances, and vehicle capacities supplemented by secondary sources from municipal bodies and prior studies. These inputs feed into building a distance matrix, baseline TPS demands, demand fluctuation bounds, and capacity limits, enabling formulation of both deterministic and robust CVRP models to tackle demand uncertainty. The Clarke-Wright algorithm generates initial routes via savings computations, refined through iterative feasibility assessments against standard and robust constraints, with uncertainty budgets tested across scenarios. Outcomes are evaluated on metrics like total distance, vehicle count, solution viability, and robustness cost, culminating in directed graph visualizations that clarify route patterns under diverse uncertainty conditions.

### 3.3. Robust optimization model for capacitated vehicle routing problem

To incorporate demand uncertainty into the CVRP formulation, the robust optimization framework proposed by Bertsimas and Sim [31], [32] is adopted. In this framework, uncertain demand parameters are allowed to deviate from their nominal values within predefined bounds, while the total deviation across customers on a route is limited by the uncertainty budget  $\Gamma_k$ . The deterministic vehicle capacity constraint (6)-(8) is replaced by a robust counterpart that accounts for worst-case demand realizations. Specifically, the total load assigned to each vehicle must not exceed its capacity even when up to  $\Gamma$  TPS demands simultaneously experience their maximum deviations. This ensures that vehicle overload is prevented under adverse but plausible demand scenarios.

$$\sum_{i \in V} \bar{d}_i y_i^k + \max_{\xi \in U} \sum_{i \in V} \xi_i \hat{d}_i y_i^k \leq Q_k, \forall k \in K \quad (11)$$

where  $V_k$  is the set of nodes (TPS) served by vehicle  $k$ .

### 3.4. Robust optimization-capacitated vehicle routing problem model for demand uncertainty

In municipal waste transportation, TPS demand varies due to daily waste generation, weather, and socio-economic factors. The proposed RO-CVRP model addresses this by assuming demand at each TPS  $i$  falls within an uncertainty interval around its nominal value, embedding robust capacity constraints directly into the formulation. This minimizes total travel distance while ensuring resilient routes against demand fluctuations. We assume that the demand at node  $i$  lies within the following interval:

$$q_i \in [\bar{q}_i, \hat{q}_i] \quad (12)$$

where  $\bar{q}_i$  denotes the nominal average demand, while  $\hat{q}_i$  represents the maximum possible deviation of demand at node  $i$ . It is important to note that not all nodes will simultaneously experience their peak demand. To account for this condition, we introduce a budget of uncertainty  $\Gamma_k$ , which limits the number of nodes that may simultaneously reach their maximum deviation.

To obtain the RO-CVRP model under demand uncertainty, the deterministic CVRP formulation is extended by incorporating the robust capacity constraint see (12) and (13) while adhering to the interpretation of  $\Gamma_k$  described earlier. The RO-CVRP model under demand uncertainty is formulated:

Objective function:

$$\min Z = \sum_{k \in K} \sum_{i \in N} \sum_{j \in N, j \neq i} c_{ij} x_{ij}^k \quad (13)$$

with constraints:

$$\sum_{k \in K} a_i^k = 1, \forall i \in V \quad (14)$$

$$\sum_{j \in N, j \neq i} x_{ij}^k = a_i^k, \sum_{j \in N, j \neq i} x_{ji}^k = a_i^k, \forall i \in V, \forall k \in K \quad (15)$$

$$\sum_{j \in V} x_{0j}^k = y_k, \sum_{j \in V} x_{j0}^k = y_k, \sum_{k \in K} y_k \leq m, \forall k \in K \quad (16)$$

$$\bar{q}_i a_i^k \leq u_i^k \leq Q_k a_i^k, \forall i \in V, \forall k \in K \quad (17)$$

$$\sum_{i \in V} \bar{q}_i a_i^k + \underbrace{\max_{S \subseteq V_k, |S| \leq \Gamma_k} \sum_{i \in S} \hat{q}_i a_i^k}_{\text{worst-case demand robust}} \leq Q_k, \forall k \in K \quad (18)$$

Let  $\pi_k \geq 0, s_{ik} \geq 0$ , then replace the above constraint with:

$$\sum_{i \in V} \bar{q}_i a_i^k + \Gamma_k \pi_k + \sum_{i \in V} s_{ik} \leq Q_k, \forall k \in K \quad (19)$$

$$s_{ik} \geq \hat{q}_i a_i^k - \pi_k, s_{ik} \geq 0, \forall i \in V, \forall k \in K \quad (20)$$

$$x_{ij}^k \in \{0,1\}, a_i^k \in \{0,1\}, y_k \in \{0,1\}, u_i^k \geq 0, \pi_k \geq 0, s_{ik} \geq 0 \quad (21)$$

The objective function (13) minimizes total travel distance and transportation costs, while constraints ensure each TPS node is visited once (14), maintain flow balance (15), regulate depot usage and

vehicle flow (16), and limit loads per TPS (17). A robust capacity constraint (18) adapts deterministic limits to uncertain demand by accounting for worst-case deviations (see Table 1) and surge risks, with constraint (19) preventing overloads under variability. Constraint (20) caps simultaneous maximum deviations across nodes and (21) defines MILP-compatible variable types for solvability.

Table 1. Budget of uncertainty  $\Gamma_k$ 

| Notation               | Description                                                                         |
|------------------------|-------------------------------------------------------------------------------------|
| $\Gamma_k = 0$         | All demands are assumed deterministic                                               |
| $\Gamma_k =  V_k $     | All nodes on the route of vehicle $k$ may simultaneously reach their maximum demand |
| $0 < \Gamma_k <  V_k $ | Only $\Gamma$ nodes are assumed to experience maximum demand simultaneously         |

## 4. RESULT AND ANALYSIS

### 4.1. Description data

This section summarizes numerical outcomes from implementing the proposed RO-CVRP model in the Tanah Enam Subdistrict waste transport network, incorporating vehicle capacities alongside uncertainty budgets from  $\Gamma = 0$  to  $\Gamma = 6$ . Findings appear in concise tables, route diagrams, geospatial maps, and robustness cost profiles, highlighting contrasts between deterministic paths (ignoring fluctuations) and robust alternatives (resistant to demand spikes). Table 2 details TPS locations, depot distances, baseline waste volumes (m<sup>3</sup>/day), deviation caps computed as 35% of nominal demand, and rounded to the nearest 0.5 m<sup>3</sup>.

Table 2 provides demand inputs for both deterministic and robust models, with maximum deviation values integrated into the robust formulation via uncertainty parameters  $\Gamma_k$ . Inter-location distances between TPS (in km), derived from official field measurements, appear in Table 3. Table 4 data facilitate the calculation of cost savings and identification of optimal route combinations using the CWS algorithm.

Table 2. Location data TPS and demands

| No. | Name of TPS          | Distance to depot (km) | Nominal demand (m <sup>3</sup> ) | Maximum deviation (m <sup>3</sup> ) |
|-----|----------------------|------------------------|----------------------------------|-------------------------------------|
| 0   | TPA (Depot)          | 0.0                    | 0.0                              | 0.0                                 |
| 1   | Jl. Marelan Raya     | 2.0                    | 5.0                              | 1.75                                |
| 2   | Jl. Baut             | 5.9                    | 4.0                              | 1.4                                 |
| 3   | Jl. Pasar 1 Tengah   | 5.1                    | 6.0                              | 2.1                                 |
| 4   | Komplek Suzuya Plaza | 6.1                    | 7.0                              | 2.45                                |
| 5   | Komplek Marelan 88   | 2.8                    | 5.0                              | 1.75                                |
| 6   | Komplek Deli Indah   | 4.0                    | 4.5                              | 1.6                                 |
| 7   | Komplek Maryland     | 6.0                    | 6.0                              | 2.1                                 |
| 8   | Komplek Sejati       | 5.3                    | 5.0                              | 1.75                                |
| 9   | RS Esmun             | 4.8                    | 3.5                              | 1.25                                |

Table 3. Matrix of distances between TPS (km)

| From/to        | V <sub>0</sub> | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>0</sub> | 0.0            | 2.0            | 5.9            | 5.1            | 6.1            | 2.8            | 4.0            | 6.0            | 5.3            | 4.8            |
| V <sub>1</sub> | 2.0            | 0.0            | 1.6            | 2.1            | 0.35           | 3.9            | 2.0            | 0.65           | 0.9            | 0.65           |
| V <sub>2</sub> | 5.9            | 1.6            | 0.0            | 1.8            | 1.4            | 4.4            | 4.2            | 1.5            | 1.5            | 1.1            |
| V <sub>3</sub> | 5.1            | 2.1            | 1.8            | 0.0            | 2.5            | 3.6            | 2.8            | 2.8            | 2.0            | 1.5            |
| V <sub>4</sub> | 6.1            | 0.35           | 1.4            | 2.5            | 0.0            | 4.6            | 4.4            | 0.7            | 0.75           | 1.3            |
| V <sub>5</sub> | 2.8            | 3.9            | 4.4            | 3.6            | 4.6            | 0.0            | 1.7            | 4.6            | 3.8            | 3.3            |
| V <sub>6</sub> | 4.0            | 2.0            | 4.2            | 2.8            | 4.4            | 1.7            | 0.0            | 4.4            | 3.7            | 3.2            |
| V <sub>7</sub> | 6.0            | 0.65           | 1.5            | 2.8            | 0.7            | 4.6            | 4.4            | 0.0            | 0.7            | 1.3            |
| V <sub>8</sub> | 5.3            | 0.9            | 1.5            | 2.0            | 0.75           | 3.8            | 3.7            | 0.7            | 0.0            | 0.6            |
| V <sub>9</sub> | 4.8            | 0.65           | 1.1            | 1.5            | 1.3            | 3.3            | 3.2            | 1.3            | 0.6            | 0.0            |

Table 4. The savings value for each pairs combination ( $i, j$ )

| No. | Pair ( $i, j$ ) | Savings (km) |
|-----|-----------------|--------------|
| 1   | (1,4)           | 7.75         |
| 2   | (1,9)           | 6.15         |
| 3   | (4,8)           | 5.65         |
| 4   | (1,8)           | 6.4          |
| ... | ...             | ...          |
| 34  | (5,7)           | 4.2          |
| 35  | (1,6)           | 4.0          |
| 36  | (1,5)           | 0.9          |

#### 4.2. Clarke-Wright savings algorithm calculation

Distances between collection points TPS  $i$  and  $j$  (depot as 0) inform savings calculations (11), yielding 36 pairwise combinations from Table 4 data. For instance, linking TPS 1 (Jl. Marelan Raya) and TPS 4 (Komplek Suzuya Plaza) at 7.75 km saved distance versus separate depot trips marks it as a prime route candidate due to high savings. Full results appear in Table 4.

Higher savings values reflect greater route distance reductions achieved by merging two collection points (TPS) into a single path without intermediate depot returns; for example, TPS Jl. Marelan Raya consistently pairs with high savings alongside TPS 4, 8, and 9, positioning it as a key integration node, while TPS Komplek Suzuya Plaza emerges repeatedly as a vital connecting hub in optimized routing configurations.

#### 4.3. Optimal route results

The route construction employs the CWS heuristic, prioritizing TPS pairs with highest savings (Table 4) for merging from individual depot-TPS-depot routes. Merges respect robust capacity constraints (17)-(21) that incorporate demand uncertainty via a tunable budget parameter  $\Gamma$ , balancing routing efficiency against variability protection. Solutions thus minimize travel distance while upholding capacity limits under worst-case demand scenarios.

#### 4.4. Interpretation of deterministic and robust routes

Based on Table 5, it can be interpreted that:

##### a. Deterministic case $\Gamma = 0$

In the deterministic scenario, two trucks depart from depot V0: one serves V1, V4, V8, and V9 before returning, while the other routes through V2, V3, V6, V5, and V7 back to base. The CWS algorithm achieves a minimal total distance of 29.4 km at nominal 10.0 m<sup>3</sup> demand, fully loading capacities without robust adjustments, as no variability applies.

Figure 1 illustrates the deterministic routing solution generated by the CWS algorithm under nominal demand conditions ( $\Gamma = 0$ ). The Figure 1 shows the optimal vehicle routes connecting the depot and all TPS locations without considering demand uncertainty.

Table 5. The optimal route results with  $Q = 10$  and  $\Gamma_k = \{0,3,6\}$

| $\Gamma_k$ | Optimal route                                                                                                                                                                                                                                                                                                         | Total distance (km) | Demand nominal (m <sup>3</sup> ) | Demand robust (m <sup>3</sup> ) |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|----------------------------------|---------------------------------|
| 0          | V <sub>0</sub> -V <sub>1</sub> -V <sub>4</sub> -V <sub>8</sub> -V <sub>9</sub> -V <sub>0</sub> ,<br>V <sub>0</sub> -V <sub>2</sub> -V <sub>3</sub> -V <sub>6</sub> -V <sub>5</sub> -V <sub>7</sub> -V <sub>0</sub>                                                                                                    | 29.4                | 10.0                             | 10.0                            |
| 3          | V <sub>0</sub> -V <sub>1</sub> -V <sub>4</sub> -V <sub>0</sub> , V <sub>0</sub> -V <sub>8</sub> -V <sub>9</sub> -V <sub>0</sub> ,<br>V <sub>0</sub> -V <sub>2</sub> -V <sub>3</sub> -V <sub>6</sub> -V <sub>5</sub> -V <sub>7</sub> -V <sub>0</sub>                                                                   | 34.2                | 10.0                             | 11.8                            |
| 6          | V <sub>0</sub> -V <sub>1</sub> -V <sub>0</sub> , V <sub>0</sub> -V <sub>4</sub> -V <sub>0</sub> , V <sub>0</sub> -V <sub>8</sub> -V <sub>0</sub> ,<br>V <sub>0</sub> -V <sub>9</sub> -V <sub>0</sub> , V <sub>0</sub> -V <sub>2</sub> -V <sub>3</sub> -V <sub>6</sub> -V <sub>5</sub> -V <sub>7</sub> -V <sub>0</sub> | 40.8                | 10.0                             | 13.4                            |

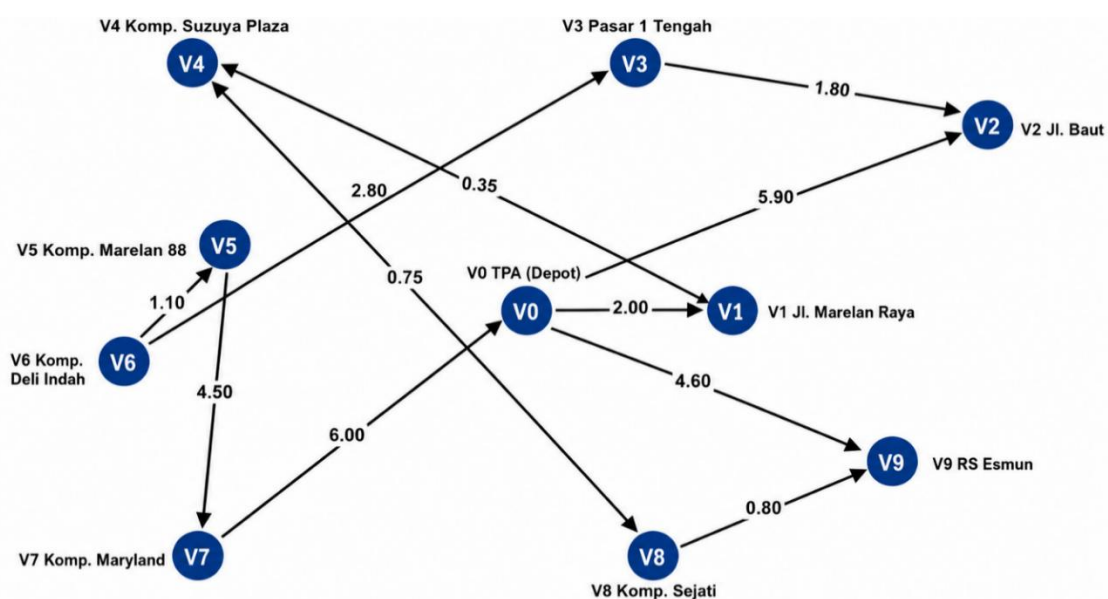


Figure 1. Directed routes for deterministic case

b. Moderate uncertainty  $\Gamma = 3$

Under moderate uncertainty, truck routing adapts by splitting into three paths from depot V0: one truck serves V1 and V4, another covers V8 and V9, the third manages V2, V3, V6, V5, and V7 before returning. This reconfiguration from two to three routes, with two short (two stops each) and one extended (five stops) maintains feasibility as robust demand reaches 11.8 m<sup>3</sup> despite steady 10.0 m<sup>3</sup> nominal levels. Travel distance rises 16.3% to 34.2 km, capturing the trade-off for demand protection.

Figure 2 presents the routing configuration obtained under a moderate uncertainty level ( $\Gamma = 3$ ). Compared with the deterministic solution, additional route adjustments are introduced to maintain vehicle capacity feasibility.

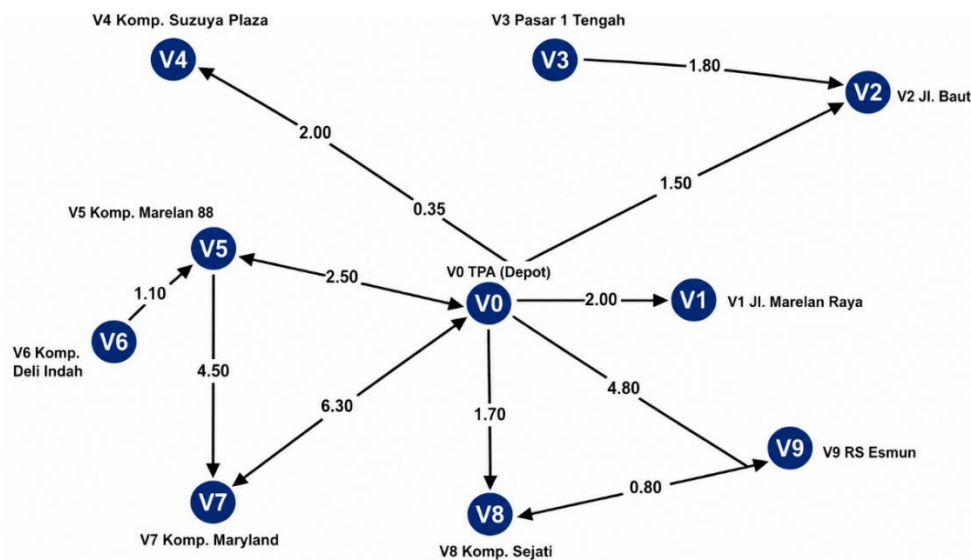


Figure 2. Directed routes under moderate uncertainty

c. High uncertainty  $\Gamma = 6$

Under high uncertainty, robust optimization yields highly conservative routes with extensive fragmentation. Multiple vehicles handle single locations like V1, V4, V8, and V9 while one covers V2-V3-V6-V5-V7, driving up total distance by 38.8% to 40.8 km due to elevated demand at 13.4 m<sup>3</sup>. This tradeoff ensures feasibility against worst-case scenarios but at higher operational cost.

Figure 3 illustrates the routing solution under high demand uncertainty ( $\Gamma = 6$ ). The network becomes more fragmented because additional routes are required to guarantee robustness against worst-case demand realizations.

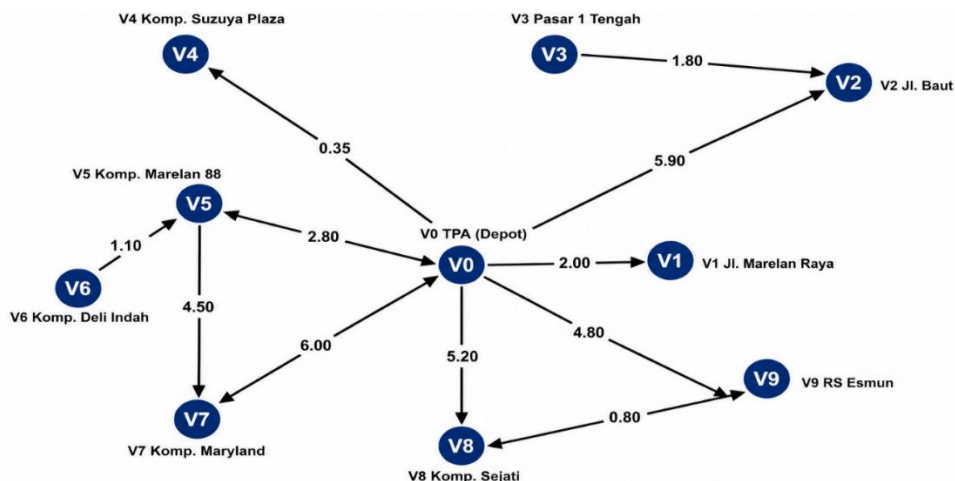


Figure 3. Directed routes under robust high uncertainty

#### 4.5. Price of robustness and sensitivity analysis

The price of robustness (PoR) measures the cost of robustness by comparing the relative growth in total travel distance between robust and deterministic solutions. Findings reveal a steady PoR rise with higher robustness levels, underscoring the efficiency-robustness trade-off. Moderate robustness yields acceptable distance increases for practical use, yet extreme levels impose steep marginal costs, potentially unsuitable for cost-focused operations.

Table 6 shows PoR rising nonlinearly with greater uncertainty levels. Low values enable minor route adjustments to handle demand variations affordably, while high values cause sharp cost increases unsuitable for cost-sensitive logistics. Optimal tuning balances efficiency in stable conditions, cost-reliability trade-offs at moderate levels, and strict safeguards only for critical applications like waste collection. Using Bertsimas-Sim uncertainty sets, the model protects against maximum loads without changing distances, with Clarke-Wright solutions validated for reliable results outperforming deterministic approaches.

Table 6. The comparison of robust total distance and PoR

| $\Gamma_k$ | Total robust distance | PoR (%) vs $\Gamma_k$ |
|------------|-----------------------|-----------------------|
| 0          | 29.4                  | 0.0                   |
| 3          | 34.2                  | 16.3                  |
| 6          | 40.8                  | 38.8                  |

Table 7 compares the performance of deterministic CVRP and the proposed RO-CVRP under different budget uncertainty levels ( $\Gamma = 0, 3, 6$ ). As demand uncertainty increases, the deterministic routing solution rapidly loses feasibility, although it maintains the same nominal travel distance. In contrast, the proposed RO-CVRP consistently preserves route feasibility by adjusting the routing configuration, albeit at the expense of increased travel distance. These results demonstrate the effectiveness of robust optimization in preventing vehicle overloading and maintaining reliable waste collection services under uncertain demand conditions.

Table 7. The comparison of routing strategies under demand uncertainty

| Method             | Budget uncertainty | Number of vehicles | Total distance | Feasibility rate (%) | PoR (%) |
|--------------------|--------------------|--------------------|----------------|----------------------|---------|
| Deterministic CVRP | 0                  | 2                  | 29.4           | 73                   | 0.0     |
|                    | 3                  | 2                  | 29.4           | 41                   | -       |
|                    | 6                  | 2                  | 29.4           | 18                   | -       |
| RO-CVRP (proposed) | 0                  | 2                  | 29.4           | 100                  | 0.0     |
|                    | 3                  | 2                  | 34.2           | 96                   | 16.3    |
|                    | 6                  | 2                  | 40.8           | 100                  | 38.8    |

To further investigate the effect of fleet characteristics on routing performance, a sensitivity analysis was conducted by varying the vehicle capacity while maintaining the same demand uncertainty setting. Three vehicle capacity levels (8 m<sup>3</sup>, 10 m<sup>3</sup>, and 12 m<sup>3</sup>) were evaluated.

Table 8 indicate that larger vehicle capacities significantly reduce route fragmentation under uncertainty. When vehicle capacity increases, fewer vehicles are required to serve the TPS locations while maintaining feasibility. This analysis highlights the interaction between vehicle capacity and demand uncertainty, emphasizing the importance of fleet configuration in robust routing design.

Table 8. The sensitivity analysis

| Vehicle capacity  | Vehicles | Distance (km) |
|-------------------|----------|---------------|
| 8 m <sup>3</sup>  | 4        | 43.5          |
| 10 m <sup>3</sup> | 3        | 34.2          |
| 12 m <sup>3</sup> | 2        | 31.1          |

## 4. CONCLUSION

This study introduces a robust optimization model to the CVRP for municipal solid waste collection amid demand fluctuations, merging the Bertsimas-Sim uncertainty budget with CWS heuristics to yield resilient routes. Real data from Medan City reveal how escalating uncertainty budgets reshape paths, fleet usage, and distances, with PoR metrics highlighting cost-robustness trade-offs for informed policymaking.

Practically, the model equips city planners with a dependable tool to bolster service reliability and avert overloads, adaptable beyond single locales. Future extensions could integrate green factors like emissions and fuels, diverse fleets, or smart sensor data, advancing both robust algorithms, and eco-friendly waste logistics.

ACKNOWLEDGMENTS

We gratefully acknowledge the funding and support from the Ministry of Religious Affairs of the Republic of Indonesia, as well as the essential facilities, resources, and ongoing assistance.

FUNDING INFORMATION

This study received funding from the Ministry of Religious Affairs of the Republic of Indonesia via the BOPTN 2025 scheme, supporting research development in higher education at Universitas Islam Negeri Sumatera Utara, Medan, Indonesia (Grant No. 266, 2025).

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

| Name of Author          | C | M | So | Va | Fo | I | R | D | O | E | Vi | Su | P | Fu |
|-------------------------|---|---|----|----|----|---|---|---|---|---|----|----|---|----|
| Hendra Cipta            | ✓ | ✓ | ✓  | ✓  | ✓  | ✓ |   | ✓ | ✓ | ✓ | ✓  | ✓  | ✓ | ✓  |
| Rina Widyasari          |   | ✓ |    | ✓  | ✓  | ✓ | ✓ | ✓ | ✓ | ✓ | ✓  | ✓  |   | ✓  |
| Raisha Zuhaira Dongoran |   |   | ✓  | ✓  |    | ✓ | ✓ | ✓ |   | ✓ | ✓  |    |   |    |

|                       |                                |                            |
|-----------------------|--------------------------------|----------------------------|
| C : Conceptualization | I : Investigation              | Vi : Visualization         |
| M : Methodology       | R : Resources                  | Su : Supervision           |
| So : Software         | D : Data Curation              | P : Project administration |
| Va : Validation       | O : Writing - Original Draft   | Fu : Funding acquisition   |
| Fo : Formal analysis  | E : Writing - Review & Editing |                            |

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

REFERENCES

[1] Ferdinan, S. W. Utomo, T. E. B. Soesilo, and H. Herdiansyah, "Household Waste Control Index towards Sustainable Waste Management: A Study in Bekasi City, Indonesia," *Sustain.*, vol. 14, no. 21, 2022, doi: 10.3390/su142114403.

[2] M. M. Azis, J. Kristanto, and C. W. Purnomo, "A Techno-Economic Evaluation of Municipal Solid Waste (MSW) Conversion to Energy in Indonesia," *Sustain.*, vol. 13, no. 13, pp. 1-10, 2021, doi: 10.3390/su13137232.

[3] P. Agamuthu and S. Babel, "Waste management developments in the last five decades: Asian perspective," *Waste Manag. Res.*, vol. 41, no. 12, pp. 1699-1716, 2023, doi: 10.1177/0734242X231199938.

[4] A. F. Rahmat, Z. Bujdosó, and L. D. Dávid, "Waste Management Policy in Four ASEAN Contemporary Issues from Research Works Countries : Emerging Contemporary issues from research works," *Forum Geo.*, vol. 39, no. 2, pp. 250-267, 2025, doi: 10.23917/forgeo.v39i2.10086.

[5] F. Hermawan, "Optimization Of Transportation of Municipal Solid Waste from Resource to Intermediate Treatment Facility with Nearest Neighbour Method (Study on six Sub Sub District in DKI Jakarta Province)," *J. Environ. Sci. Sustain. Dev.*, vol. 1, no. 1, pp. 86-99, 2018, doi: 10.7454/jessd.v1i1.21.

[6] Z. Wang, J. Huo, and J. S. L. Lam, "The vehicle routing programming of municipal solid waste collection under cognitive uncertainty," *Transp. J.*, vol. 64, no. 1, pp. 1-22, 2025, doi: 10.1002/tjo3.12025.

[7] E. Yuliza, B. Suprihatin, P. B. J. Bangun, F. M. Puspita, Indrawati, and S. Octarina, "Waste Collection Vehicle Routing Problem with Time Windows for Route Optimization of Garbage Transport Vehicles," *Sci. Technol. Indones.*, vol. 8, no. 1, pp. 66-70, 2023, doi: 10.26554/sti.2023.8.1.66-70.

[8] M. Anityasari, H. C. Rinardi, and I. D. A. A. Warmadewanthi, "Analysing medical waste transportation using periodic vehicle routing problem for Surabaya public health facilities," *J. Mater. Cycles Waste Manag.*, vol. 27, no. 2, pp. 830-847, 2025, doi:

- 10.1007/s10163-024-02124-0.
- [9] I. Harris, D. E. B. Bermejo, T. Crowther, and J. McDonald, "Factors Affecting Truck Payload in Recycling Operations: Towards Sustainable Solutions," *Logistics*, vol. 8, no. 4, pp. 1-19, 2024, doi: 10.3390/logistics8040118.
  - [10] Y. Shen, L. Yu, and J. Li, "Robust Electric Vehicle Routing Problem with Time Windows under Demand Uncertainty and Weight-Related Energy Consumption," *Complex Syst. Model. Simul.*, vol. 2, no. 1, pp. 18-34, 2022, doi: 10.23919/CSMS.2022.0005.
  - [11] A. Subramanyam, F. Mufalli, J. M. Laínez-Aguirre, J. M. Pinto, and C. E. Gounaris, "Robust multiperiod vehicle routing under customer order uncertainty," *Oper. Res.*, vol. 69, no. 1, pp. 30-60, 2021, doi: 10.1287/OPRE.2020.2009.
  - [12] A. Sadok, "A genetic local search algorithm for the capacitated vehicle routing problem," *Int. J. Adv. Comput. Res.*, vol. 10, no. 48, pp. 105-115, 2020, doi: 10.19101/IJACR.2020.1048021.
  - [13] D. Peña, B. Dorronsoro, and P. Ruiz, "Sustainable waste collection optimization using electric vehicles," *Sustain. Cities Soc.*, vol. 105, p. 105343, 2024, doi: 10.1016/j.scs.2024.105343.
  - [14] S. Shao, S. X. Xu, and G. Q. Huang, "Variable neighborhood search and tabu search for auction-based waste collection synchronization," *Transp. Res. Part B Methodol.*, vol. 133, pp. 1-20, 2020, doi: 10.1016/j.trb.2019.12.004.
  - [15] M. Caramia, D. M. Pinto, E. Pizzari, and G. Stecca, "Clustering and routing in waste management: A two-stage optimisation approach," *EURO J. Transp. Logist.*, vol. 12, pp. 1-12, 2023, doi: 10.1016/j.ejtl.2023.100114.
  - [16] M. K. Zuhanda, S. Suwilo, O. S. Sitompul, R. E. Caraka, Y. Kim, and M. Noh, "Optimization of Vehicle Routing Problem in the Context of E-commerce Logistics Distribution," *Eng. Lett.*, vol. 31, no. 1, pp. 279-286, 2023.
  - [17] A. Alwabli, I. Kostanic, and S. Malky, "Dynamic route optimization for waste collection and monitoring smart bins using ant colony algorithm," in *2020 IEEE 2nd Int. Conf. Electron. Control. Optim. Comput. Sci. ICECOCS 2020*, 2020, doi: 10.1109/ICECOCS50124.2020.9314571.
  - [18] J. Yang, F. Tao, and Y. Zhong, "Dynamic routing for waste collection and transportation with multi-compartment electric vehicle using smart waste bins," *Waste Manag. Res.*, vol. 40, no. 8, pp. 1199-1211, 2022, doi: 10.1177/0734242X211069738.
  - [19] A. Tarhini, K. Danach, and A. Harfouche, "Swarm intelligence-based hyper-heuristic for the vehicle routing problem with prioritized customers," *Ann. Oper. Res.*, vol. 308, no. 1-2, pp. 549-570, 2022, doi: 10.1007/s10479-020-03625-5.
  - [20] A. K. Pamosoaji, P. K. Dewa, and J. V. Krisnanta, "Proposed Modified Clarke-Wright Saving Algorithm for Capacitated Vehicle Routing Problem," *Int. J. Ind. Eng. Eng. Manag.*, vol. 1, no. 1, pp. 9-16, 2019, doi: 10.24002/ijeem.v1i1.2292.
  - [21] I. Niki, J. Poda, and K. Som, "Improving of the Clarke and Wright 's Heuristic for Solving Multiobjective Vehicles Routing Problem," *IAENG Int. J. Appl. Math.*, vol. 55, no. 10, pp. 3158-3165, 2025.
  - [22] F. Tunnisaki and Sutarman, "Clarke and Wright Savings Algorithm as Solutions Vehicle Routing Problem with Simultaneous Pickup Delivery (VRPSPD)," *J. Phys. Conf. Ser.*, vol. 2421, no. 1, 2023, doi: 10.1088/1742-6596/2421/1/012045.
  - [23] R. Nurcahyo, D. A. Irawan, and F. Kristanti, "The Effectiveness of the Clarke & Wright Savings Algorithm in Determining Logistics Distribution Routes (case study PT.XYZ)," *E3S Web Conf.*, vol. 426, 2023, doi: 10.1051/e3sconf/202342601107.
  - [24] D. Kuhn, S. Shafiee, and W. Wiesemann, "Distributionally robust optimization," *Acta Numer.*, vol. 34, pp. 579-804, 2025, doi: 10.1017/s0962492924000084.
  - [25] E. Yuliza, F. M. Puspita, and S. S. Supadi, "Heuristic approach for robust counterpart open capacitated vehicle routing problem with time windows," *Sci. Technol. Indones.*, vol. 6, no. 2, pp. 53-57, 2021, doi: 10.26554/STI.2021.6.2.53-57.
  - [26] J. Oyola, H. Arntzen, and D. L. Woodruff, "The stochastic vehicle routing problem, a literature review, part I: models," *EURO J. Transp. Logist.*, vol. 7, no. 3, pp. 193-221, 2018, doi: 10.1007/s13676-016-0100-5.
  - [27] H. Cipta, S. Suwilo, Sutarman, and H. Mawengkang, "Improved Benders decomposition approach to complete robust optimization in box-interval," *Bull. Electr. Eng. Inf.*, vol. 11, no. 5, pp. 2949-2957, 2022, doi: 10.11591/eei.v11i5.4394.
  - [28] S. Singh and M. P. Biswal, "A robust optimization model under uncertain environment: An application in production planning," *Comput. Ind. Eng.*, vol. 155, 2021, doi: 10.1016/j.cie.2021.107169.
  - [29] D. Bertsimas, V. Gupta, and N. Kallus, *Data-driven robust optimization*, vol. 167, no. 2. Springer Berlin Heidelberg, 2018, doi: 10.1007/s10107-017-1125-8.
  - [30] Q. Arnau, A. A. Juan, and I. Serra, "On the use of learnheuristics in vehicle routing optimization problems with dynamic inputs," *Algorithms*, vol. 11, no. 12, pp. 1-14, 2018, doi: 10.3390/a11120208.
  - [31] M. Goerigk and A. Schöbel, "Algorithm engineering in robust optimization," *Algorithm Engineering*, vol. 9220, 2016, doi: 10.1007/978-3-319-49487-6\_8.
  - [32] İ. Yanıkoğlu, B. L. Gorissen, and D. den Hertog, "A survey of adjustable robust optimization," *Eur. J. Oper. Res.*, vol. 277, no. 3, pp. 799-813, 2019, doi: 10.1016/j.ejor.2018.08.031.

## BIOGRAPHIES OF AUTHORS



**Hendra Cipta**     serves as a lecturer and researcher in the Department of Mathematics, Universitas Islam Negeri Sumatera Utara, Medan, Indonesia, holding a Ph.D. in Mathematics from Universitas Sumatera Utara. His research centres on optimization and applied mathematics, particularly robust optimization, linear programming, and nonlinear programming, with numerous publications in Scopus-indexed national and international journals. He can be contacted at email: [hendracipta@uinsu.ac.id](mailto:hendracipta@uinsu.ac.id).



**Rina Widyasari**    serves as a lecturer and researcher in the Department of Mathematics, Universitas Islam Negeri Sumatera Utara, Medan, Indonesia, while pursuing her Ph.D. in Mathematics at Universitas Sumatera Utara. Her work centres on optimization and applied statistics, with several publications in national and Scopus-indexed international journals. She can be contacted at email: rinawidyasari@uinsu.ac.id.



**Raisha Zuhaira Dongoran**    a recent Mathematics graduate from Universitas Islam Negeri Sumatera Utara, Medan, Indonesia, focuses her research on operations research. She actively supports her undergraduate advisors with research initiatives, data processing, and presentations at global conferences. She can be contacted at email: raisha0703212064@uinsu.ac.id.