

ACM for segmenting intensity inhomogeneity images corrupted by a mixture of AGMG noise

Nurhuda Ismail^{1,2}, Abdul Kadir Jumaat^{2,3}, Rizauddin Saian⁴, Akmal Shafiq Badarul Azam⁵

¹Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Johor Branch, Pasir Gudang Campus, Masai, Malaysia

²Institute for Big Data Analytics and Artificial Intelligence, Universiti Teknologi MARA, Shah Alam, Malaysia

³Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Shah Alam, Malaysia

⁴Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Arau, Malaysia

⁵Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA, Sarawak Branch, Mukah Campus, Mukah, Malaysia

Article Info

Article history:

Received Feb 12, 2026

Revised Aug 20, 2026

Accepted Aug 27, 2026

Keywords:

Active contour model

Gamma noise

Gaussian noise

Image segmentation

Intensity inhomogeneity

Mixed noise

ABSTRACT

In practice, active contour models (ACMs) are effective in segmenting images, which are mostly applied in computer vision and medical imaging. However, the result may be challenged when the images contain noise and intensity inhomogeneity. Images are commonly deteriorated by a mixture of noise involving additive Gaussian and multiplicative Gamma (AGMG). To the best of our knowledge, this is the first study to formulate an ACM for segmenting intensity inhomogeneity images corrupted by AGMG noise. Our proposed mathematical model was developed using a combination of dual denoising terms to recover the noisy image and Laplacian of Gaussian (LoG) terms to smooth the targeted object. The model is solved using calculus of variations and finite difference methods. The performance of the suggested model for segmenting images was evaluated in MATLAB software and compared with current models using low and severe AGMG noise levels. Quantitative evaluations demonstrated that the proposed model outperformed existing models, achieving an average Jaccard similarity coefficient (JSC) improvement of roughly 2.5% and a 16.00% reduction in average relative error. Consequently, the proposed model proves significantly more effective for the segmentation of images degraded by intensity inhomogeneity and mixed AGMG noise.

This is an open access article under the [CC BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.



Corresponding Author:

Abdul Kadir Jumaat

Institute for Big Data Analytics and Artificial Intelligence (IBDAAI), Universiti Teknologi MARA

Kompleks Al-Khawarizmi, 40450, Shah Alam, Selangor, Malaysia

Email: abdulkadir@tmsk.uitm.edu.my

1. INTRODUCTION

Manipulating images using electronic devices such as scanners, computers or cameras is often implemented in digital image processing. The most implemented procedure in digital image processing is image segmentation, that being utilized in several real-world applications, including medical imaging [1], [2], image classification [3], and image recognition [4]. The crucial goal of segmentation is to split images into distinct components.

Numerous image segmentation models are classified into non-variational and variational. Machine learning (ML) models are the examples models in the class of non-variational models. The robustness of deploying ML models is dependent on the quality of the databases [5]. For the ML-based model, the research [6] proposed the AlexNet framework based on an automated deep convolutional neural network (D-CNN) model, while [7], [8] investigated convolutional neural network (CNN) for medical imaging. However,

processing the databases of ML models requires high computational costs, additional time, and other resources [9]. Other non-variational models, such as thresholding [10] and region-expanding [11], provided uncomplicated calculation and fast computation. However, the models may lead to poor segmentation results. Additionally, they are effective for non-complex images but unsuitable for treating complicated images. As an alternative, variational models are often utilized in image segmentation because of their ease of computation and flexibility.

In the variational model, the mathematical optimization problem is a treatment of the segmentation problem. This variational model enables the segmentation curve to maintain continuity and smoothness throughout its evolution. One of them is the model of active contour model (ACM), often applied for segmenting the images, which is based on energy functionals [12]. The researchers classify the variational ACMs as edge-based and region-based. For instance, Caselles *et al.* [12] introduced edge-based ACMs by implementing edge detectors to bring the curve towards the targeted edge. These classical models did not fix any global constraint in the image, so that the splitting regions, which is segmentation, would happen quickly and achieve accurate segmentation. However, the models are sensitive to noisy images and rely more on contour initialization.

Noise is unwanted information that appears in images and affects the important features in the images. Two kinds of noise models can be used to represent noise in the images: additive noise and multiplicative noise. These noises are not always additive and multiplicative, but these two types could be mixed [13]. Practically, in medical imaging, the mixed noise model is the most suitable model that can describe noise in the images [14]. Mixed noise means that any type of noise is added to the images after the other [15], and a well-known mixed noise model is the mix of two noises, including additive Gaussian and multiplicative Gamma (AGMG) noise. Many researchers, including [13], [16], [17], have looked into this type of mixed noise for the method of restoring (denoising) images that are implemented in a variety of sectors, for instance, medical imaging, remote sensing, and radar imaging. In addition to image noise, intensity inhomogeneity also directly impacts the accuracy of image segmentation [18]. Image processing techniques can be less accurate due to intensity inhomogeneity, which occurs when the intensity varies uniformly across an originally homogeneous region; this issue commonly affects medical imaging modalities such as computerized tomography (CT), sonography, and mammograms.

Conversely, variational region-based ACM is a model that divides the pixel's foreground from background pixels by collecting similar intensities, thereby determining the edges between regions with different characteristics. Region-based ACMs may produce better segmentation because the models do not rely on an edge detector. This makes them less sensitive to initialization and capable of handling complex shapes and changes in the topology. These factors lead us to focus our research on variational region-based ACMs. The variational region-based ACM enables the division of entire objects captured in the images [19]. Some critical challenges for the model are noise and intensity inhomogeneity, which commonly occur in medical imaging [20]. The model of Mumford-Shah (MS) is a pioneer of region-based ACM [21], and has limitations on piecewise smooth (PS) approximation and constant (PC), so researchers [22], [23] implemented a PS function called the PS model into the MS model to develop their models. Both models in [22], [23] are suitable for segmenting grayscale images, including those with noise and complex regions. Nevertheless, their models produce segmentation with high computational costs, and other issues may also impact image segmentation [24]. Meanwhile, the model proposed by [25] has been tested with speckle noise presented in ultrasound images and is well suited for segmentation with noise. Zhang *et al.* [26] have proposed an algorithm for multiscale segmentation using [27] and total variation (TV) energy. The model [26] can handle images contaminated with two types of noise: i) salt and pepper and ii) Gaussian.

Recent hybrid segmentation methods can be broadly divided into two categories: model-driven hybrid variational methods and methods that integrate variational models with deep learning. In the first category, Tang *et al.* [28] proposed a variational level-set model that combines local and global Gaussian information to segment images affected by noise and intensity inhomogeneity. Similarly, Hsieh and Yang [29] used image decomposition and hybrid regularization to separate the bias field, important image structures, and residual noise. Although these methods are called hybrid, they combine different components within a variational framework and do not use deep learning. The second category combines the mathematical interpretability of variational models with the feature-learning capability of deep neural networks. For example, Qi *et al.* [30] combined variational physical priors, edge information, and mean-curvature regularization with a tailored U-Net. This method uses the interpretability of variational models and the feature-learning ability of deep neural networks to segment noisy images with weak or broken boundaries. Despite their advantages, such deep-learning-based hybrid methods generally require sufficiently large annotated datasets, substantial computational resources, and careful network training. Kumar *et al.* [31] proposed CDM, a coupled deformable model that jointly performs image restoration and segmentation in the presence of speckle noise and severe intensity inhomogeneity. However, CDM may have difficulty locating weak or discontinuous boundaries. To overcome this limitation, Kumar and Jain [32] developed ECDM by adding an edge-enhancement component. This extension improves boundary localization and numerical stability while maintaining robustness to speckle noise and intensity variation.

Nevertheless, the existing variational ACM, hybrid, and deep-learning-based segmentation methods discussed above were not specifically designed or evaluated to segment intensity inhomogeneity images corrupted by a combination of AGMG noise. Our paper proposed a new variational ACM for grayscale intensity inhomogeneity images interrupted by a combination of AGMG noise, termed the global segmentation with noise (GSWN) model, a problem that has not been explicitly addressed in existing segmentation models. To do so, we will be incorporating the dual denoising term involved in [1] to treat mixed AGMG noise suppression and the optimized Laplacian of Gaussian (LoG) term [2], [33] to enhance the object's boundary. Next, our paper will discuss how to formulate the proposed GSWN model. We will also compare and explain the experimental findings presented in the third section. Extensive experiments on natural and medical images with varying levels of mixed AGMG noise demonstrate that the proposed model achieves better segmentation results compared with the comparison models. Finally, we will conclude and recommend for our future study.

From an engineering perspective, reliable segmentation of images affected by mixed AGMG noise is important for accurate measurement, classification, and decision-making. In medical imaging, the proposed method can help identify anatomical structures or abnormal regions when noise and uneven intensity make their boundaries difficult to detect. In radar and remote-sensing images, it can help separate targets, terrain, and land-cover regions affected by sensor noise. By reducing noise and enhancing object boundaries, the proposed method can produce more reliable segmentation results for degraded images.

2. METHOD

Our research aims to formulate the proposed GSWN model for segmenting grayscale intensity inhomogeneity images exposed to a mixture of AGMG noise. Initially, the AGMG noise is defined, followed by a detailed explanation of formulating the new GSWN model. Finally, the algorithm to implement the proposed GSWN model is presented.

2.1. The additive Gaussian and multiplicative Gamma noise

In reality, images contaminated with two types of noise, which are AGMG, have frequently affected medical imaging, radar imaging and remote sensing, and they have been studied by many researchers, such as [13], [16], [17] for the image restoration (denoising) problem. However, to date, this type of mixed noise has not been specifically addressed within the image segmentation problem. Zhao *et al.* [13] defined the images disrupted by a mix of Gaussian and Gamma noise with a ratio r where it is normally set to $r = 0.5$. Next, they defined a clean image, referred to $z(x, y)$, observed (noisy) image as $z_0(x, y)$, additive noise η , and multiplicative noise ρ . They also consider Q as the event to the additive noise and $\bar{Q}$ the multiplicative noise. Then, the rate ratio of mixed Gaussian-Gamma noise is $r = P(Q)$. Thus, the mixture of AGMG noise is defined as (1):

$$z_0(x, y) = \begin{cases} z(x, y) + \eta & \text{when event } Q \text{ occurs,} \\ z(x, y)\rho & \text{otherwise.} \end{cases} \quad (1)$$

where η is an additive Gaussian (AG) noise, which follows a Gaussian distribution parameter that considers a mean 0 and a variance represented as σ^2 . Meanwhile, the noise of MG, ρ is referred to Gamma distribution with shape L and rate L , giving mean 1 and variance $\frac{1}{L}$. The Gaussian distribution of additive noise, η at each pixel in the images is obtained in (2), and the Gamma distribution of multiplicative noise, ρ at each pixel in the images is shown in (3):

$$p_1(\eta) = (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{\eta^2}{2\sigma^2}}, \quad (2)$$

$$p_2(\rho) = L^L \rho^{L-1} e^{-L\rho} / \Gamma(L), \{\rho \geq 0\}, \quad (3)$$

where the parameters σ^2 and L are unknown. Here, Γ is a Gamma function parameterized by L defined by (4):

$$\Gamma(L) = \int_0^\infty t^{L-1} e^{-t} dt, (L > 0) \quad (4)$$

here, L is the shape parameter of the Gamma function controlling the distribution's spread, while t is a dummy variable of integration used only for mathematical definition. In the AGMG noise model, each image pixel is affected by either AGMG noise. The selection is controlled by the mixing probability r . AG noise occurs with probability r while multiplicative Gamma (MG) noise occurs with probability $1 - r$. In this study, $r = 0.5$, meaning that both types of noise have an equal probability of occurrence. The Gaussian noise is defined as $\eta \sim N(0, \sigma^2)$, where σ^2 represents its variance; a larger σ^2 indicates stronger additive noise. The multiplicative

noise follows $\rho \sim \text{Gamma}(L, \text{rate} = L)$ with mean 1 and variance $\frac{1}{L}$ [34]. Therefore, a smaller L yields a stronger multiplicative noise. The Gaussian and Gamma noise variables are generated independently for each pixel. In the experiments, two paired of noise settings were considered: $(\sigma^2, L) = (0.0001, 1000), (0.01, 50)$. The corresponding Gamma variance levels $\frac{1}{L}$ are 0.001, 0.02, respectively.

An equal mixing probability was used so that Gaussian and Gamma noise had the same chance of affecting each pixel. Two parameter settings were selected to represent low and severe noise. A larger Gaussian variance produces stronger additive noise, while a smaller Gamma shape parameter produces stronger multiplicative noise. These settings allowed us to evaluate the model under different noise levels. For a fair and reproducible comparison, all models were tested using the same noise parameters, noise-generation procedure, and corrupted images.

2.2. The proposed global segmentation with noise model

Consider $z_0 = z_0(x, y)$ as an observed grayscale image that is corrupted by AGMG noise, and a clean image is $z = z(x, y)$ defined on a bounded region $\Omega \subset \mathbb{R}^2$. The energy minimization functional of the proposed GSWN model is given as a combination of six distinct energy terms such that $F_{\text{GSWN}} = F_{\text{regularization}} + F_{\text{fitting1}} + F_{\text{fitting2}} + F_{\text{denoising(Gaussian)}} + F_{\text{denoising(Gamma)}} + F_{\text{LoG}}$. Here, we defined

$$\begin{aligned} F_{\text{regularization}} &= \mu \int_{\Omega} \delta(\phi) |\nabla \phi| d\Omega, & F_{\text{fitting1}} &= \lambda_1 \int_{\Omega} (z_0(x, y) - c_1)^2 H(\phi) d\Omega, & F_{\text{fitting2}} &= \lambda_2 \int_{\Omega} (z_0(x, y) - c_2)^2 (1 - H(\phi)) d\Omega, \\ F_{\text{denoising(Gaussian)}} &= \beta_1 \int_{\Omega} (z(x, y) - z_0(x, y))^2 d\Omega, & F_{\text{denoising(Gamma)}} &= \beta_2 \int_{\Omega} (\log z(x, y) + z_0(x, y) / z(x, y)) d\Omega, & \text{and} \\ F_{\text{LoG}} &= \alpha \int_{\Omega} L(x, y) H(\phi) d\Omega. \end{aligned}$$

Thus, the full energy minimization functional is defined in (5):

$$\begin{aligned} \min_{\phi, c_1, c_2} F_{\text{GSWN}(\phi, c_1, c_2)} &= \mu \int_{\Omega} \delta(\phi) |\nabla \phi| d\Omega + \lambda_1 \int_{\Omega} (z_0(x, y) - c_1)^2 H(\phi) d\Omega + \lambda_2 \int_{\Omega} (z_0(x, y) - c_2)^2 (1 - H(\phi)) d\Omega \\ &+ \beta_1 \int_{\Omega} (z(x, y) - z_0(x, y))^2 d\Omega + \beta_2 \int_{\Omega} (\log z(x, y) + z_0(x, y) / z(x, y)) d\Omega + \alpha \int_{\Omega} L(x, y) H(\phi) d\Omega, \end{aligned} \quad (5)$$

Refer to (5), all the parameter values for $\mu, \lambda_1, \lambda_2, \beta_1, \beta_2$, and α are positive. The term of regularization as the first term in the model is to adjust the smoothness of the generated segmentation contour ϕ . It consists of the Dirac delta function, δ and the TV function $|\nabla \phi|$. Then, the second and third terms represent fitting terms to adjust the image data driven force inside and outside the contour, where H is the Heaviside function. The terms are generated from the Chan-Vese model [27] where the constants c_1 and c_2 represent the average intensity values associated with the foreground and background, respectively. The fourth and fifth terms are denoising terms, which are essential for image analysis [35]. Specifically, the fourth and fifth terms are denoising terms for two noises: AGMG, respectively, that are formulated by incorporating the idea of utilizing the model of Rudin-Osher-Fatemi (ROF) [36] and Aubert-Aujol (AA) [37]. Additionally, according to [38], the image function z in the fourth and fifth terms can be approximated as $z(x, y) = f_1(x, y)H(\phi) + f_2(x, y)(1 - H(\phi))$ where $f_1(x, y) = \kappa_{\sigma}(x, y) * H(\phi)z_0(x, y) / \kappa_{\sigma}(x, y) * H(\phi)$ and $f_2(x, y) = \kappa_{\sigma}(x, y) * (1 - H(\phi))z_0(x, y) / \kappa_{\sigma}(x, y) * (1 - H(\phi))$. Here, κ_{σ} is a kernel function that contains a Gaussian kernel with $\sigma > 0$. Our research used these settings to address intensity inhomogeneity, and dual denoising terms can treat AGMG noise in the images. The last term refers to the optimized LoG proposed by [2], [33]. The energy functional of optimized LoG is generated as (6):

$$F_{\text{LoG}}(L(x, y)) = \int_{\Omega} \left[e^{-\alpha |\nabla G_{\sigma} * z_0(x, y)|} (L(x, y))^2 + \left(1 - e^{-\alpha |\nabla G_{\sigma} * z_0(x, y)|} \right) (L(x, y) - \beta \nabla^2 (G_{\sigma} * z_0(x, y)))^2 \right] d\Omega. \quad (6)$$

here, $L(x, y)$ is the optimized LoG value of the image, G_{σ} represent as a Gaussian kernel function with variance, σ^2 while $z_0(x, y)$ is an observed (grayscale) image, and the parameters α and β are positive.

Noted that the proposed GSWN model builds on the region-based Chan-Vese framework; therefore, the contour regularization and foreground-background fitting terms in (5) are established components. The classical Chan-Vese model (M5) assumes globally piecewise-constant regions and does not explicitly account for image degradation caused by noise. Although some related variational ACMs combine denoising and segmentation, they generally assume a single noise type or use a general-purpose fidelity term. In contrast, the proposed GSWN model is designed to handle AGMG noise simultaneously within one variational objective. The local representations in (5) and (6) also accommodate intensity inhomogeneity, while the optimized LoG term provides complementary boundary information for weak or noise-corrupted edges. Therefore, the novelty of the GSWN model lies not in its individual components, but in their noise-specific integration within a unified restoration-segmentation framework.

Next, the minimization technique based on Gateaux derivatives (GD) was applied to minimize F_{GSWN} with respect to unknown $\phi(x, y)$. A test function, ψ at the same type as $\phi(x, y)$ was chosen, and the derivative notation for (5) can be written as (7):

$$\begin{aligned} \left. \frac{\partial F_{GSWN}(\phi, \psi)}{\partial \phi} \right|_{h=0} &= \mu \int_{\Omega} \left(\delta(\phi + h\psi) \frac{d}{dh} |\nabla \phi + h\nabla \psi| + |\nabla \phi + h\nabla \psi| \frac{d}{dh} \delta(\phi + h\psi) \right) d\Omega \\ &+ \lambda_1 \int_{\Omega} \left((z_0(x, y) - c_1)^2 \frac{d}{dh} H(\phi + h\psi) \right) d\Omega + \lambda_2 \int_{\Omega} \left((z_0(x, y) - c_2)^2 \frac{d}{dh} (1 - H(\phi + h\psi)) \right) d\Omega \\ &+ \beta_1 \int_{\Omega} \frac{d}{dh} \left((f_1(x, y)H(\phi) + f_2(x, y)(1 - H(\phi)) - z_0(x, y))^2 \right) d\Omega + \alpha \int_{\Omega} \frac{d}{dh} L(x, y)H(\phi + h\psi) d\Omega \\ &+ \beta_2 \int_{\Omega} \frac{d}{dh} \left[\log(f_1(x, y)H(\phi + h\psi) + f_2(x, y)(1 - H(\phi + h\psi))) + \frac{z_0(x, y)}{(f_1(x, y)H(\phi + h\psi) + f_2(x, y)(1 - H(\phi + h\psi)))} \right] d\Omega \end{aligned} \quad (7)$$

By the calculus of variations as done in [39], the functional in (7) can be simplified further as in (8):

$$\begin{aligned} \frac{\partial F_{GSWN}(\phi, \psi)}{\partial \phi} &= -\mu \int_{\Omega} \psi \delta(\phi) \nabla \cdot (\nabla \phi / |\nabla \phi|) d\Omega + \mu \int_{\partial\Omega} \psi (\delta(\phi) / |\nabla \phi|) \frac{\partial \phi}{\partial n} ds + \lambda_1 \int_{\Omega} ((z_0(x, y) - c_1)^2 \delta(\phi) \psi) d\Omega \\ &- \lambda_2 \int_{\Omega} ((z_0(x, y) - c_2)^2 \delta(\phi) \psi) d\Omega + 2\beta_1 \int_{\Omega} [D(z(x, y) - z_0(x, y)) \delta(\phi) \psi] d\Omega \\ &+ \beta_2 \int_{\Omega} [D(z(x, y) - z_0(x, y)) \delta(\phi) \psi / (z(x, y))] d\Omega + \alpha \int_{\Omega} L(x, y) \delta(\phi) \psi d\Omega \end{aligned} \quad (8)$$

with $D = f_1(x, y) - f_2(x, y)$. The Neumann boundary condition $\nabla \phi \cdot n = \frac{\partial \phi}{\partial n} = 0$ was considered, and the corresponding Euler–Lagrange equation is generated when $\frac{\partial F_{GSWN}}{\partial \phi} = 0$ and it is expressed in (9):

$$\begin{aligned} \frac{\partial F_{GSWN}}{\partial \phi} &= \delta(\phi) \left[-\mu \nabla \cdot (\nabla \phi / |\nabla \phi|) + \lambda_1 (z_0(x, y) - c_1)^2 - \lambda_2 (z_0(x, y) - c_2)^2 \right. \\ &\left. + 2D\beta_1 (z(x, y) - z_0(x, y)) + D\beta_2 ((z(x, y) - z_0(x, y)) / z^2(x, y)) + \alpha L(x, y) \right] = 0 \end{aligned} \quad (9)$$

Next, we proceed to approximate (9) by implementing the gradient descent method with an artificial time t and expressing it as (10):

$$\begin{cases} \frac{\partial \phi}{\partial t} = \delta(\phi(x, y)) \left[\mu \nabla \cdot (\nabla \phi / |\nabla \phi|) - \lambda_1 (z_0(x, y) - c_1)^2 + \lambda_2 (z_0(x, y) - c_2)^2 \right. \\ \left. - 2D\beta_1 (z(x, y) - z_0(x, y)) - D\beta_2 ((z(x, y) - z_0(x, y)) / z^2(x, y)) - \alpha L(x, y) \right] = 0 \text{ in } \Omega, \\ \delta(\phi) \nabla \phi / |\nabla \phi| \cdot n = \delta(\phi) / |\nabla \phi| \cdot \frac{\partial \phi}{\partial n} = 0, \text{ or } \nabla \phi \cdot n = 0 \text{ or } \frac{\partial \phi}{\partial n} = 0 \text{ on } \partial\Omega. \end{cases} \quad (10)$$

Then, we used the method of finite difference to solve (10) for the GSWN model by discretization. The solution is defined as (11):

$$\begin{aligned} \phi_{i,j}^{n+1} &= \phi_{i,j}^n + \Delta t \delta(\phi_{i,j}^n) \left[\mu \nabla \cdot (\nabla \phi_{i,j}^n / |\nabla \phi_{i,j}^n|) - \lambda_1 (z_0(x, y) - c_1)^2 + \lambda_2 (z_0(x, y) - c_2)^2 \right. \\ &\left. - 2D\beta_1 (z(x, y) - z_0(x, y)) - D\beta_2 ((z(x, y) - z_0(x, y)) / z^2(x, y)) - \alpha L(x, y) \right], \end{aligned} \quad (11)$$

2.3. Algorithm for the proposed global segmentation with noise model

Algorithm 1 demonstrates the processes needed to implement the new proposed GSWN model. For the implementation, this research used MATLAB R2021a software using the MATLAB Image Processing Toolbox, which was conducted using an Intel Core i7-12650H CPU, and the processor ran at 2.30 GHz with supporting installed memory (RAM) at 16.0 GB. MATLAB programming will automatically stop when the solution reaches the tolerance value $tol = 1 \times 10^{-5}$ or $max\ i\ t = 200$ as the maximum number of iterations.

Algorithm 1. Pseudocode for the GSWN algorithm

1. INPUT: Image I from (1), parameters $(\mu, \lambda_1, \lambda_2, \beta_1, \beta_2, \alpha, \sigma)$, Δt , tol , $maxit$.
2. INITIALIZE: Set initial level set ϕ_0 and compute optimized LoG from (6).
3. FOR $n = 1$ to $maxit$ DO:
 - a. Compute the GSWN model update using (11).
 - b. Update level set: $\phi^{n+1} = \phi^n + \Delta t$ from (11).
 - c. IF $\|\phi^{n+1} - \phi^n\| / \|\phi^n\| \leq tol$ THEN:
 - Exit loop (Converged).
 - d. END IF
- END FOR
4. OUTPUT: Final level set function ϕ .

The implementation of the proposed GSWN model is summarized in Algorithms 1 which provides a structured pseudocode representation of the iterative process. This algorithm provides a clear and reproducible procedure for implementing the proposed GSWN model. The computational complexity of the proposed method is $O(d^2 \times N)$, in which N is the image size and d is the number of iterations, which is deduced from the work by [40]. This is owing to the iterative gradient descent process, where each iteration necessitates pixel-wise updates across the entire image domain.

2.4. Convergence analysis

Based on the proposed functional GSWN in (5), define the stationary solution set as $\Gamma = \{\bar{\phi} \in \mathbb{R}^N : \nabla_{\phi} F_{GSWN}(\bar{\phi}) = 0\}$ where N denotes the number of pixels in the discretized image domain. The gradient descent algorithm can be represented by the composite mapping $A = MP$. The mapping $P: \mathbb{R}^N \rightarrow \mathbb{R}^N \times \mathbb{R}^N$ is defined by $P(\phi) = (\phi, -\nabla_{\phi} F_{GSWN}(\phi))$ whereas $M: \mathbb{R}^N \times \mathbb{R}^N \rightarrow \mathbb{R}^N$ represents the numerical update of the level-set function. Given the current iterate ϕ^k , the mapping P produces the current point and its negative-gradient direction. The negative gradient of GSWN energy with respect to ϕ is presented in (12):

$$\begin{aligned} -\nabla_{\phi} F_{GSWN}(\phi^k) = \delta_{\varepsilon}(\phi(x, y)) \left[\mu \nabla \cdot \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \lambda_1 (z_0(x, y) - c_1)^2 + \lambda_2 (z_0(x, y) - c_2)^2 - 2D\beta_1 (z(x, y) - z_0(x, y)) \right. \\ \left. - D\beta_2 ((z(x, y) - z_0(x, y)) / z^2(x, y)) - \alpha L(x, y) \right]. \end{aligned} \quad (12)$$

To establish convergence of the GSWN algorithm, the following five sufficient conditions are considered: i) M is a closed mapping; ii) P is continuous; iii) A is closed at every $\phi \notin \Gamma$; iv) (ϕ) is a descent function for A and ϕ ; and v) the sequence $\{\phi^k\}$ is contained in a compact set T .

First, consider the mapping M . Given the current level-set function ϕ^k and the direction $d^k = -\nabla_{\phi} F_{GSWN}(\phi^k)$, then the level-set function is subsequently updated by $\phi^{k+1} = \phi^k + \Delta t_k d^k$ where $\Delta t_k > 0$ is the time step and $F_{GSWN}(\phi^{k+1}) < F_{GSWN}(\phi^k)$. Therefore, whenever $\nabla_{\phi} F_{GSWN}(\phi^k) \neq 0$, thus M is a closed mapping by [41]. Next, the mapping P is continuous because the energy functional GSWN, $F_{GSWN}(\phi)$ is continuously differentiable with respect to ϕ . Because P is continuous and M is closed, the A is closed at $\phi(\nabla_{\phi} F_{GSWN}(\phi) \neq 0)$ [41]. When $\phi \notin \Gamma$, it gives $d = -\nabla_{\phi} F_{GSWN}(\phi) \neq 0$ and $\nabla_{\phi} F_{GSWN}(\phi)^T d < 0$ to present that $F_{GSWN}(\phi)$ is a descent function for the mapping A and ϕ . To ensure that the level-set sequence is bounded, the updated function is set to $\phi(x, y)$ if $|\phi(x, y)| \leq B$, $-B$ if $\phi(x, y) < -B$ and B if $\phi(x, y) > B$, where $B > 0$ is a fixed constant. The bounded feasible set is therefore defined as $T = \{\phi = (\phi_1, \dots, \phi_N) \in \mathbb{R}^N : |\phi_i| \leq B, i = 1, \dots, N\}$. Obviously, it is proved that the compact set T consists of the sequence $\{\phi^k\}$. Therefore, Algorithm 1 is convergent [41].

3. RESULTS AND DISCUSSION

The proposed GSWN model (M4) was compared with three existing models: M1 [42], M2 [1], and M3 [33]. These models were selected because they are region-based variational ACMs designed to handle noisy images. M1 and M3 were developed for specific noise types, whereas M2 was designed for general mixed noise. Therefore, they provide suitable baselines for evaluating whether explicitly modelling AGMG noise improves segmentation performance. For a fair comparison, all models were tested using the same corrupted images, initial contours, and stopping criteria. The parameter values recommended in the original publications were used as starting points. Where necessary, these values were adjusted through preliminary experiments to achieve stable performance and were then kept fixed for each experimental condition.

All parameter values and initialization settings are reported to support reproducibility. According to our numerical evaluation, we set the following parameters: $\Delta t = 0.0001$, $\varepsilon = 1$, $tol = 0.00001$, $maxit = 200$,

$\lambda_1 = 1, \lambda_2 = 1, \beta_1 = 1, \beta_2 = 1, \alpha = 1$, and $\sigma = 10$. Meanwhile, the parameter μ varies across experiments. A larger μ value is assigned when the target object is large, and the object cannot be detected due to the presence of noise. In contrast, for detecting smaller objects, μ should be set to a smaller value. The specific value of μ used in each case will be provided along with the corresponding initial level set function. This study used 20 grayscale intensity inhomogeneity images with different noise intensity levels as input images obtained from various open-source databases, including natural images from [43], [44], microscopic bone marrow images from [45], skin cancer images obtained from [46] and nucleus fluorescence micrograph retrieved from [47]. The same database also provides its ground truth images containing the actual segmented regions. In the present experiments, the AGMG noise parameters were assumed to be known a priori and were not estimated from the observed images. As a standard procedure in the image processing community, all the images were corrupted with mixed noise: AGMG noise for the simulation purpose of testing our proposed model. No preprocessing was applied to the input images. The same synthetically corrupted images were used for all evaluated models.

Figures 1 and 2 show the chosen two sample intensity inhomogeneity input images (out of 20 input images) with square initial contours in red, corrupted by a mixture of AGMG noise under the different true noise parameters σ^2 and L represent different noise intensity levels. The figures below also illustrate two selected sample segmented images for comparison studies with 3 existing models (M1, M2, and M3) under different noise intensity levels of a mixture of AGMG noise as low and severe noise, as illustrated in Figures 1 and 2, respectively. In detail, the AG noise uses a Gaussian distribution with mean 0 and different variance σ^2 . Meanwhile, the MG noise refers to the Gamma distribution with mean 1 and different shape parameters L . In the experiments, the input images are corrupted by mixed AGMG noise with a fixed mixture ratio of Gaussian-Gamma $r = 0.5$ under the following combination of true parameters: $\sigma^2 = 0.0001, L = 1000$ and $\sigma^2 = 0.01, L = 50$. The larger σ^2 indicates severe Gaussian noise, while lower L indicates severe Gamma noise [17]. With these settings, the images produced different noise levels: low and severe noise, respectively.

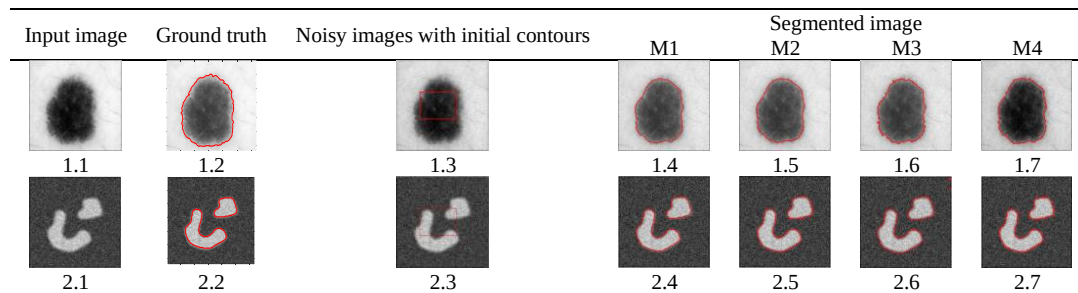


Figure 1. Intensity inhomogeneity input images, binary ground truth masks, noisy images with low noise of mixture of AGMG ($\sigma^2 = 0.0001, L = 1000$) and segmented images for comparison models

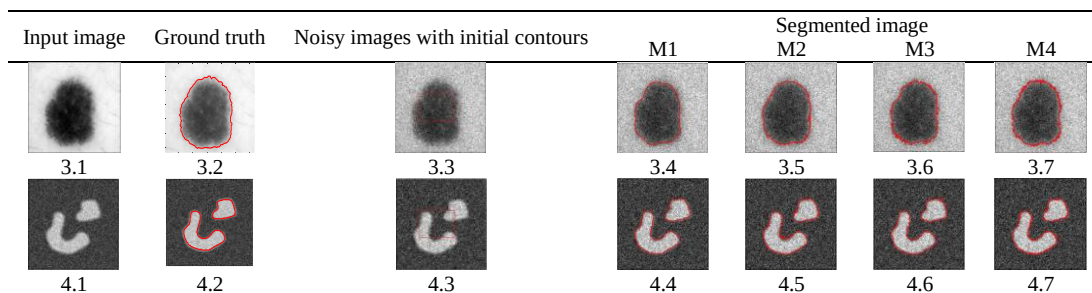


Figure 2. Intensity inhomogeneity input images, binary ground truth masks, noisy images with severe noise of mixture of AGMG ($\sigma^2 = 0.01, L = 50$) and segmented images for comparison models

Figures 1 and 2 depict intensity inhomogeneity input images, ground truth segmentation, noisy images with different noise intensity levels of a mixture of AGMG and segmented images for comparison models for the chosen two sample intensity inhomogeneity input images (out of 20 input images). The initial segmentation contours, ϕ is indicated in red. For all experiments, the initial contour was placed at the same predetermined location in each image. The same initial contour was used for all compared models to ensure a fair evaluation. The dimensions of the input images are 128 by 128 pixels and were utilized for all images. According to Figure 1, Images 1.4 to 1.7 and Images

2.4 to 2.7 show the segmentation results for M1-M4, which were tested with a low mixture of AGMG noise. As a result, for M3, we obtained Image 2.6, which shows a little over-segmentation at the top-right of the image. The other images show good segmentation for M1-M4. Meanwhile, Figure 2 shows the segmentation results on input images which were tested with severe mixture of AGMG noise. The image segmentation was successful and smooth, except for the output from M3, as shown in image 4.6, which produced a less smooth result.

In addition to qualitative visual assessment, these experiments also were examined through numerical analysis utilizing the Dice similarity coefficient (DSC), Jaccard similarity coefficient (JSC), error rate metric, accuracy metric, and processing time (seconds). The value of JSC, DSC, and the accuracy metric approaching 1 represents the model achieving accurate segmentation. Conversely, a smaller error value of the Error Rate metric, approaching 0, indicates better segmentation. The range value for each accuracy test is from 0 to 1. PSNR and SSIM were not included because they primarily measure image reconstruction quality, while this study focuses on segmentation accuracy. Boundary deviations were further examined using pixel-level error maps and boundary overlays in subsection 4.5. Thus, Table 1 displays the average value of each quantitative metric under different noise intensity levels of mixed AGMG noise in segmenting 20 input images for a comprehensive understanding of the numerical analysis.

Table 1. The average quantitative result for all input images for all models

Noise level/model	Jaccard/Dice/accuracy/error/processing time (seconds)			
	M1	M2	M3	M4
Low $\sigma^2 = 0.0001$, $L = 1000$	0.8851/0.9377/ 0.96/0.04/1.1141	0.9001/0.9465/ 0.9638/0.0362/48.9645	0.8958/0.9440/ 0.9632/0.0368/8.7230	0.9069/0.9503/ 0.9643/0.0336/12.1628
Severe $\sigma^2 = 0.01$, $L = 50$	0.8406/0.9133/ 0.9476/0.0524/2.1753	0.8894/0.9403/ 0.9589/0.0412/56.475	0.8912/0.9416/ 0.9614/0.0386/10.4081	0.9045/0.9491/ 0.9657/0.0343/12.4059

According to Table 1, image segmentation with a low mixture of AGMG noise, the numerical values, including DSC, JSC, accuracy, and error, indicate that the M4 (GSWN model) outperforms in segmentation. The results obtained are compared with other models and level of noise. Referring to M4 in low AGMG noise, the value of the average JSC is 0.9069, which is 2.46%, 0.76%, and 1.24% higher than the values for M1, M2, and M3, respectively. According to the average DSC for the same experiment with low mixture of AGMG noise, M4 recorded 0.9503, which is 1.34%, 0.40%, and 0.67% higher than M1, M2, and M3, respectively. Similarly, for average accuracy in the low noise setting, M4 recorded 0.9643, which is 0.45%, 0.05%, and 0.11% higher than M1, M2, and M3, respectively. Evaluating the average error rate under the same conditions, M4 achieved the lowest value of 0.0336. This demonstrates a notable relative error reduction, registering 16.00%, 7.18%, and 8.70% lower than the error rates produced by M1, M2, and M3, respectively.

To summarize, our M4 is an effective model to segment intensity inhomogeneity images with a mix of AGMG noise. This is due to the implementation of dual denoising terms integrated with local image information to treat intensity inhomogeneity images with mixed AGMG noise, and the segmentation is made more accurate by incorporating the term of optimized LoG to smooth the homogeneous regions and enhance the object's boundary. In contrast to the comparative models, several potential reasons for less accurate segmentation are based on the accuracy assessment for intensity inhomogeneity images corrupted by a mixture of AGMG noise. Refer to [1] for M2, the model can segment images interrupted by a general mixed noise, and it might not be able to segment the images corrupted with a specific mixture of noise: AGMG noise, while for M1 [42] and M3 [33], the models were designed to handle a particular single noise.

Meanwhile, according to the efficiency assessment result through the processing time between M1-M4, M2 had the longest average processing time in segmenting intensity inhomogeneity images interrupted by a severe mixture of AGMG noise, with a processing time of 56.475 seconds, followed by M4 at 12.4059 seconds, M3 at 10.4081 seconds, and M1 at 2.1753 seconds. The most likely reason for M2 producing the highest computational cost, followed by M4 (proposed GSWN model), is that both models implemented a dual denoising term, which consists of the fitting image equation in the energy functional in (5). This dual denoising term contains a Gaussian kernel and involves a convolution operation, which can significantly increase computational load and cause these models to take the longest time to complete the segmentation.

The proposed M4 model did not achieve the shortest processing time, but it provided the best segmentation accuracy under severe noise. M4 obtained an average Jaccard score of 0.9045, compared with 0.8406, 0.8894, and 0.8912 for M1, M2, and M3, respectively. It required 12.4059 seconds to process each 128×128 image. M4 was approximately 5.7 times slower than M1 and 1.2 times slower than M3, but approximately 4.6 times faster than M2. Therefore, M4 is more suitable for offline applications in which accuracy is more important than processing speed. The current MATLAB implementation is not suitable for near-real-time or large-scale processing without improvements such as parallel computing, graphics card (GPU) acceleration, or a faster numerical solver.

Table 2 presents the mean±standard deviation of the Jaccard, Dice, accuracy, error, and processing time (seconds) obtained from 20 input images under low and severe noise conditions. The mean values represent the overall segmentation performance, while the standard deviations indicate the consistency and reliability of each model. Higher Jaccard, Dice, and accuracy values indicate better performance, whereas lower error and processing time values are preferred.

Table 2. Mean±standard deviation of segmentation performance metrics and processing time (seconds)

Model	Jaccard	Dice	Accuracy	Error	Processing time (seconds)
			Low/severe noise		
M1	0.8851±0.0700/	0.9377±0.0402/	0.9600±0.0238/	0.0400±0.0238/	1.1141±1.4415/
	0.8406±0.1125	0.9133±0.0640	0.9476±0.0337	0.0524±0.0337	2.1753±2.833
M2	0.9001±0.0580/	0.9465±0.0328/	0.9638±0.0226/	0.0362±0.0226/	48.9645±5.1336/
	0.8894±0.0631	0.9403±0.0363	0.9588±0.0309	0.0411±0.0309	56.4750±22.5925
M3	0.8958±0.0615/	0.9440±0.0347/	0.9632±0.0225/	0.0368±0.0225/	8.7230±1.3483/
	0.8912±0.0572	0.9415±0.0322	0.9614±0.0216	0.0386±0.0216	10.4081±4.6354
M4	0.9069±0.0560/	0.9503±0.0313/	0.9643±0.0229/	0.0336±0.0218/	12.1628±1.7885/
	0.9045±0.0521	0.9491±0.0291	0.9657±0.0205	0.0343±0.0205	12.4059±2.1321

The standard deviations indicate that M4 (GSWN model) produced the most reliable segmentation results. Across all noise conditions, M4 generally had the lowest variability in Jaccard, Dice, accuracy, and error. Its performance also remained nearly unchanged from low to severe noise. M3 showed the most stable processing time, while M1 was the fastest but had greater performance variability, particularly under severe noise. M2 had the longest and most variable processing time. Overall, M4 achieved the best and most consistent segmentation performance across all noise levels, demonstrating strong robustness and reliability. Therefore, the GSWN model offers the best balance of segmentation accuracy, robustness, reliability, and computational efficiency.

4. ADDITIONAL VALIDATION AND ANALYSIS

This section presents additional experiments to comprehensively validate the performance, robustness, and practical applicability of the proposed GSWN model. The analysis includes an ablation study, evaluation at different image resolutions, and selection of the regularization and mixed AGMG noise parameters. The convergence behavior and pixel-level segmentation errors are also investigated. Statistical significance is assessed using the Wilcoxon signed-rank test. Finally, the performance of the GSWN model is compared with classical ACMs and a CNN model under severe noise conditions.

4.1. Ablation study

An ablation study was conducted using an input image corrupted by severe noise to evaluate the individual contribution of each term in the proposed GSWN model in (5). Each term was removed separately while the parameters $(\mu, \lambda_1, \lambda_2, \beta_1, \beta_2, \alpha, \sigma)$ were kept fixed. The resulting segmentation performance was evaluated qualitatively and quantitatively using Jaccard, Dice, accuracy, error, and processing time (seconds). Figure 3 presents the segmentation results, while Table 3 reports the corresponding quantitative measurements.

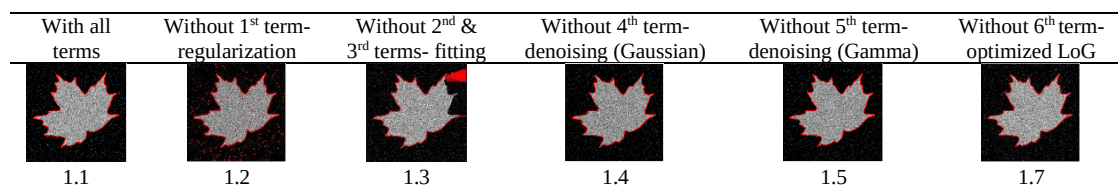


Figure 3. Segmentation result for sample segmented images under severe noise by removing specific terms from (5) while keeping all other conditions identical

Figure 3 and Table 3 quantify the contribution of each component relative to the complete GSWN model. Removing the regularization term reduced Jaccard, Dice, and accuracy by 0.0187, 0.0096, and 0.0062, respectively, and increased error from 0.0071 to 0.0133. Removing the two fitting terms caused the largest deterioration: Jaccard decreased by 0.1708, Dice by 0.0956, and accuracy by 0.0680, while error increased by 0.0680. Removing the Gaussian-denoising term produced smaller changes of -0.0004 in Jaccard, -0.0002 in Dice, -0.0001 in accuracy, and +0.0001 in error. Removing the Gamma-denoising term changed the same metrics by -0.0011, -0.0005, -0.0003, and +0.0003, respectively. Removing the optimized LoG term decreased

Jaccard, Dice, and accuracy by 0.0015, 0.0007, and 0.0005 and increased error by 0.0005. The reduced models were 1.73-2.86 seconds faster (approximately 9.9%-16.4%), confirming that the full formulation trades additional computation for the best segmentation result. The fitting and regularization terms make the largest measurable contributions in this example, while the two noise-specific terms and optimized LoG provide smaller but complementary improvements.

Table 3. Quantitative evaluation metrics by removing specific terms from (5) while keeping all other conditions identical

With all terms	Without 1 st term- regularization	Without 2 nd & 3 rd terms- fitting	Without 4 th term- denoising (Gaussian)	Without 5 th term- denoising (Gamma)	Without 6 th term- optimized LoG
Jaccard/Dice/accuracy/error/processing time (seconds)					
0.9779/0.9888/0.9929/0.0071/17.416	0.9592/0.9792/0.9867/0.0133/15.686	0.8071/0.8932/0.9249/0.0751/15.6451	0.9775/0.9886/0.9928/0.0072/14.9410	0.9768/0.9883/0.9926/0.0074/14.5541	0.9764/0.9881/0.9924/0.0076/15.5780
302	898	86	55	83	68

4.2. Performance of the global segmentation with noise model at different image resolutions

An experiment was conducted to evaluate the performance of the proposed GSWN model at different image resolutions under severe noise. Three resolutions, namely 128 by 128, 256 by 256, and 512 by 512, were evaluated. To assess the convergence behavior, the residual curves are plotted as presented in Figure 4, while the quantitative results are reported in Table 4.

The decreasing residual curves in Figure 4 indicate that the model converges consistently for every resolution. As shown in Table 4, increasing the resolution improves the segmentation results. The 512 by 512 image achieves the highest Jaccard, Dice, and accuracy values and the lowest error. However, its processing time is longer. Therefore, higher image resolution improves segmentation accuracy but increases computational cost.

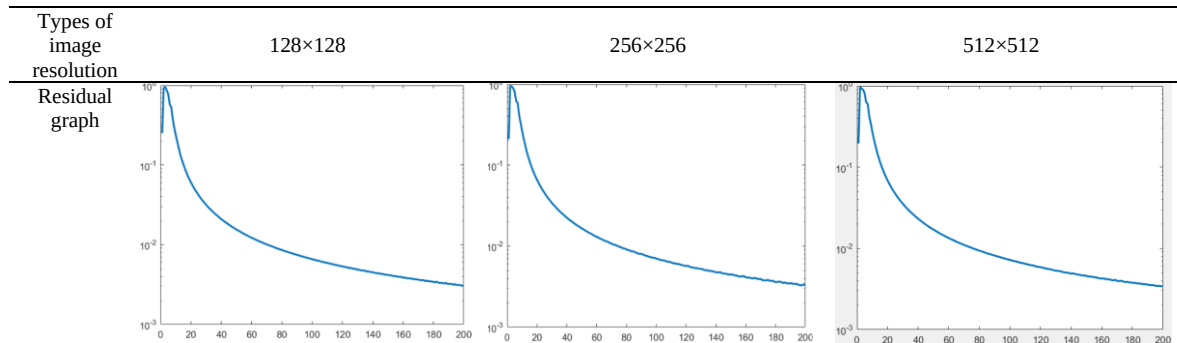


Figure 4. Residual curves of the GSWN model for different image resolutions under severe noise

Table 4. Quantitative performance of the GSWN model for different image resolutions under severe noise

Types of image resolution	Jaccard	Dice	Accuracy	Error	Processing time (seconds)
128×128	0.8596	0.9245	0.9628	0.0372	15.190584
256×256	0.9007	0.9478	0.9748	0.0252	20.621966
512×512	0.9211	0.9589	0.9803	0.0197	55.987701

4.3. Selection of the regularization parameter μ

In this study, the regularization parameter μ is critical for achieving successful segmentation. An experiment was conducted under severe noise to determine a suitable range value of the regularization parameter μ for the proposed GSWN model. Three values, $\mu = 0$, 15,000 and 40,000, were evaluated under identical experimental conditions. Table 5 reports the quantitative performance.

Table 5 confirms that $\mu = 15,000$ provides the best performance, with the highest Jaccard (0.8508), Dice (0.9194), and accuracy (0.9467), as well as the lowest error (0.0533) and processing time (14.2862 seconds). This is because, when $\mu = 0$ many false-positive regions appear because regularization is absent. Increasing μ to 15,000 produces a smooth contour that closely follows the object boundary. A higher value of $\mu = 40,000$ produces an overly smooth contour and slightly reduces boundary accuracy. Therefore, $\mu = 15,000$ was selected as the optimal value for this experiment.

Table 5. Performance of the GSWN model using different values of the regularization parameter μ

Varieties of μ value	Jaccard	Dice	Accuracy	Error	Processing time (seconds)
0	0.7821	0.8777	0.9138	0.0862	16.598040
15000 (optimal)	0.8508	0.9194	0.9467	0.0533	14.282602
40000 (high)	0.8367	0.9111	0.9416	0.0584	14.387904

4.4. Selection of the mixed additive Gaussian and multiplicative Gamma noise parameters r and L

For this section, experiments were conducted to examine the effects of the mixed AGMG noise parameters r and L on the performance of the proposed GSWN model. The parameter r was evaluated at 0.1, 0.5, and 0.9 using $\sigma^2 = 0.01$, $L = 50$ represented severe noise. Similarly, L was evaluated at 10, 50, and 100 by considering the parameter $\sigma^2 = 0.01$, $r = 0.5$. The quantitative results are presented in Tables 6 and 7, respectively.

Tables 6 and 7 show only small variations in Jaccard, Dice, accuracy, and error. This implies that the GSWN model successfully identifies the object boundaries for all tested values of r and L . The contours remain stable despite changes in the noise parameters. For the tested settings, $r = 0.9$ produces the best results among the r values, whereas $L = 100$ provides the best results among the L values. Overall, these findings indicate that the GSWN model is robust to variations in the mixed AGMG noise parameters.

Table 6. Quantitative performance of the GSWN model for different values of r , with $\sigma^2 = 0.01$, $L = 50$

Varieties of r value with $\sigma^2 = 0.01$, $L = 50$	0.1	0.5	0.9
	Jaccard/Dice/accuracy/error/processing time (seconds)		
	0.9406/0.9694/0.9655/ 0.0345/15.646471	0.9389/0.9685/0.9645 /0.0355/15.420067	0.9411/0.9697/0.9658/ 0.0342/5.878302

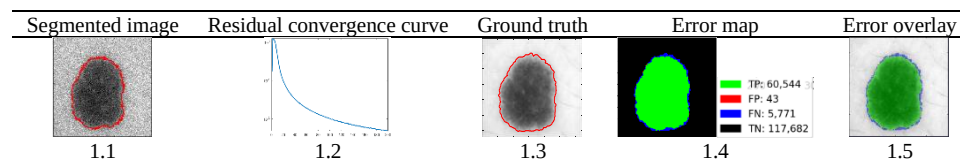
Table 7. Quantitative performance of the GSWN model for different values of parameter L , with $\sigma^2 = 0.01$, $r = 0.5$

Varieties of r value with $\sigma^2 = 0.01$, $r = 0.5$	10	50	100
	Jaccard/Dice/accuracy/error/processing time (seconds)		
	0.9388/0.9685/0.9645/ 0.0355/16.021527	0.9401/0.9691/0.9652/ 0.0348/15.036470	0.9404/0.9693/0.9654/ 0.0346/15.645713

4.5. Convergence and pixel-level segmentation error analysis

The convergence behavior and pixel-level segmentation accuracy of the proposed GSWN model were evaluated under severe AGMG noise level. The results include segmented images, residual convergence curves, ground-truth masks, error maps, and overlays shown in Figure 5. The error maps classify pixels as true positive (TP), false positive (FP), false negative (FN), and true negative (TN).

Figure 5 shows that the residual values for GSWN model decrease steadily with increasing iterations, indicating stable convergence. The error maps are dominated by black (background) TN and green TP regions, while only small red FP and blue FN regions appear near the object boundaries. Although boundary errors slightly increase at higher noise levels, the segmented regions remain close to the ground-truth masks.

Figure 5. Segmentation results, residual curves, ground-truth masks, error maps, and error overlays produced by the GSWN model under severe AGMG noise ($\sigma^2 = 0.01$, $L = 50$)

4.6. Statistical comparison using the Wilcoxon signed-rank test

A two-sided Wilcoxon signed-rank test was used to compare M4 with M1, M2, and M3 under low and severe noise. Each comparison included 20 pairs of results from the same 20 images. For each image, the models used the same corrupted input, ground-truth mask, and initial contour. This pairing reduced the effect of differences in image difficulty. The Wilcoxon test was selected because it can compare paired results without

assuming that the differences follow a normal distribution. A two-sided test was used because the direction of the performance difference was not assumed beforehand. The results are presented in Table 8.

Based on Table 8, values are presented as raw p-values, with Holm-adjusted p-values in parentheses. L and S denote low and severe noise, respectively. Statistical significance was determined using Holm-adjusted $p < 0.05$. “Higher,” “Lower,” and “Mixed” indicate the direction of GSWN’s performance relative to the benchmark models. The Wilcoxon test showed that the GSWN model achieved significantly higher Jaccard, Dice, and accuracy values and significantly lower errors than all comparison models at every noise level ($p_{Holm} < 0.05$) indicating M4 has better segmentation performance. For processing time, M4 was significantly faster than M2 but slower than M1 and M3. Overall, the results confirm that the GSWN model provides significantly better segmentation accuracy, although this improvement involves a moderate computational-time trade-off compared with M1 and M3. The Holm adjustment was applied to reduce the risk of false significant findings caused by conducting multiple model comparisons. Thus, every statistical comparison used 20 image pairs ($n = 20$).

Table 8. Wilcoxon signed-rank test results for the GSWN model versus comparison models under different noise levels

Metric	M1 vs. M4, raw p (Holm p)	M2 vs. M4, raw p (Holm p)	M2 vs. M4, raw p (Holm p)	Direction of M4
Jaccard	L: <0.0001 (<0.0001); S: <0.0001 (<0.0001)	L: <0.0001 (<0.0001); S: 0.0001 (0.0001)	L: <0.0001 (<0.0001); S: <0.0001 (<0.0001)	Higher
Dice	L: <0.0001 (0.0003); S: <0.0001 (<0.0001)	L: <0.0001 (0.0003); S: 0.0001 (0.0002)	L: <0.0001 (0.0003); S: <0.0001 (0.0002)	Higher
Accuracy	L: 0.0022 (0.0045); S: <0.0001 (<0.0001)	L: 0.0015 (0.0045); S: 0.0001 (0.0002)	L: 0.0015 (0.0045); S: <0.0001 (0.0002)	Higher
Error	L: 0.0001 (0.0002); S: <0.0001 (<0.0001)	L: <0.0001 (<0.0001); S: <0.0001 (<0.0001)	L: <0.0001 (0.0002); S: <0.0001 (0.0001)	Lower
Processing time (seconds)	L: <0.0001 (<0.0001); S: <0.0001 (<0.0001)	L: <0.0001 (<0.0001); S: <0.0001 (<0.0001)	L: <0.0001 (<0.0001); S: <0.0136 (0.0136)	Mixed

4.7. Comparison of the proposed global segmentation with noise model with classical active contour models under severe noise

This subsection compares the segmentation performance of the proposed GSWN model (M4) with two classical ACMs: the Chan–Vese model (M5) [27] and the geodesic ACM (M6) [48]. A medical test image affected by severe noise is used to evaluate the models qualitatively and quantitatively. The segmentation results are visually compared in Figure 6, while Table 9 reports the metrics using Jaccard, Dice, accuracy, error, and processing time (seconds).

Figure 6 shows that M4 (proposed GSWN model) produces contours that are closer to the actual object boundaries under severe noise. In contrast, M5 and M6 generate inaccurate contours and incorrectly segment noisy background regions. The quantitative results in Table 9 support these observations. M4 achieves the highest Jaccard, Dice, and accuracy values and the lowest segmentation error for both images. It also requires less processing time than M5 and M6. These results demonstrate that the GSWN model is more accurate, efficient, and robust to severe noise than the Chan–Vese and geodesic ACMs. This is expected because M5 and M6 are not formulated to handle images with noise.

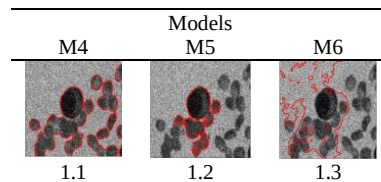


Figure 6. Segmentation results of the GSWN model (M4), Chan–Vese model (M5), and geodesic ACM (M6) under severe noise

Table 9. Quantitative performance of the GSWN model (M4), Chan–Vese model (M5), and geodesic ACM (M6) under severe noise

M4	M5	M6
Jaccard/Dice/accuracy/error/processing time (seconds)		
0.8556/0.9222/0.9428/0.0572/12.728935	0.3309/0.4973/0.7373/0.2627/42.517385	0.2926/0.4527/0.5280/0.4720/74.556719

4.8. Comparison of the proposed global segmentation with noise model with a convolutional neural network under severe noise

This subsection compares the proposed GSWN model (M4) with a CNN model using SegNet (M7) [8] under severe noise conditions. 80 noisy images are used to assess their segmentation performance. The average Jaccard, Dice, accuracy, error, and processing time (seconds) are reported in Table 10.

Table 10. Quantitative performance of the GSWN model (M4) and CNN model using SegNet (M7) under severe noise

Noise level/model	M4	M7
	Jaccard/Dice/accuracy/error/processing time (seconds)	
Low $\sigma^2=0.0001$, $L=1000$	0.9069/0.9503/0.9643/0.0336/12.1628	0.8407/0.9114/0.9403/0.0597/0.1191
Severe $\sigma^2=0.01$, $L=50$	0.9045/0.9491/0.9657/0.0343/12.4059	0.7751/0.8681/0.9096/0.0904/0.1032

The proposed M4 significantly outperforms M7 across Jaccard, Dice, accuracy, and error segmentation metrics. However, this precision comes at a computational cost. M7, being a purely feed-forward neural network at inference, requires only 0.1032 seconds to process a severe noise image. In contrast, the iterative energy minimization process of M4 requires 12.4059 seconds. This highlights a clear trade-off: M4 is better for applications prioritizing accuracy and robustness in severely corrupted environments, whereas M7 is strictly preferable when real-time processing speeds are the primary constraint.

The current CPU implementation of the proposed model is suitable for offline medical, radar, remote-sensing, and industrial inspection applications where segmentation accuracy is more important than processing speed. However, it is not yet suitable for near-real-time or large-scale applications. Its processing speed could be improved using parallel computing, GPU acceleration, adaptive stopping criteria, and batch processing.

5. CONCLUSION

A new variational global ACM for grayscale intensity inhomogeneity images exposed to a mixture of AGMG noise has been proposed. Our proposed GSWN model incorporates dual denoising terms that contain local image information to smooth the images and reduce noise. Our model also integrates with the optimized LoG to ensure the homogeneous regions are smooth and improve the object's boundary. These approaches enable our model to handle intensity inhomogeneity and a mixture of AGMG noise in the images. The segmentation process involves minimizing the GSWN model to obtain the Euler–Lagrange equation and solving it by the finite difference method.

We assess the accuracy and efficiency of models by conducting experiments on the segmentation of intensity inhomogeneity images with low and severe mixtures of AGMG noise. As a result, our proposed GSWN model outperforms existing models in image segmentation, achieving higher numerical values, including JSC, DSC, and accuracy values, while the lowest error value. However, our model has a limitation in segmentation processing time, ranking second-highest, while the existing model, M2, exhibits the longest segmentation time due to the complexity of the models. In reality, most digital images produced by medical imaging modalities widely used in healthcare settings, such as ultrasound, CT, magnetic resonance imaging (MRI), and mammography, often have issues with intensity inhomogeneity and mixed noise, specifically AGMG noise; thus, our proposed GSWN model is recommended and supports digital healthcare policies objectives that at improving diagnosis accuracy and minimize the number of errors in interpretation. Beyond medical imaging, the proposed model may also be applicable to remote sensing and industrial inspection, where image quality is often affected by intensity inhomogeneity and mixed noise. Its robustness to AGMG noise may improve segmentation reliability in low-quality acquisition environments by reducing noise while preserving object boundaries.

Furthermore, the model has the potential to serve as a segmentation component in automated diagnostic pipelines. However, further validation using real clinical data is required before its practical implementation. Nevertheless, the proposed model has other limitations. First, the experiments mainly rely on synthetically generated noise, with limited validation on real-world images. Second, as a non-convex model, its segmentation performance may be sensitive to contour initialization and parameter settings. However, in the present experiments, the same initialization setting was used for all image types and comparison models to ensure a fair evaluation. All model parameters were also kept fixed, except for the regularization parameter which was adjusted according to the noise conditions. Therefore, applying the model to images with noise characteristics, different object sizes or resolutions may require appropriate selection of parameter. The model has greater computational overhead than lightweight segmentation methods.

Future work will investigate GPU-based acceleration and other efficient algorithms to reduce computational time. Hybrid approaches combining ACMs with deep learning will also be explored to improve generalization. In addition, automatic noise-parameter estimation and adaptive parameter selection using reinforcement learning will be investigated to reduce manual parameter tuning. The proposed model will also be extended to convex, color and vector-valued intensity-inhomogeneous images corrupted by different noise levels of a mixture of AGMG noise. Finally, it will be evaluated on diverse real-world and clinical datasets with varying noise characteristics to assess its robustness and practical applicability.

ACKNOWLEDGMENTS

The authors thank Universiti Teknologi MARA, Shah Alam, for providing institutional and infrastructural support that facilitated the conduct of this research.

FUNDING INFORMATION

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Nurhuda Ismail	✓	✓			✓	✓	✓	✓	✓	✓				✓
Abdul Kadir Jumaat	✓	✓		✓		✓	✓	✓		✓		✓		✓
Rizauddin Saian					✓					✓		✓		✓
Akmal Shafiq Badarul Azam					✓		✓			✓				✓

C : Conceptualization

M : Methodology

So : Software

Va : Validation

Fo : Formal analysis

I : Investigation

R : Resources

D : Data Curation

O : Writing - Original Draft

E : Writing - Review & Editing

Vi : Visualization

Su : Supervision

P : Project administration

Fu : Funding acquisition

CONFLICT OF INTEREST STATEMENT

The authors state no conflict of interest.

DATA AVAILABILITY

Data supporting the findings of this study are available upon reasonable request from the corresponding author.

REFERENCES

- [1] H. Ali, L. Rada, and N. Badshah, "Image segmentation for intensity inhomogeneity in presence of high noise," *IEEE Transactions on Image Processing*, vol. 27, no. 8, pp. 3729–3738, Aug. 2018, doi: 10.1109/TIP.2018.2825101.
- [2] N. Badshah, H. Rabbani, and H. Atta, "On local active contour model for automatic detection of tumor in MRI and mammogram images," *Biomedical Signal Processing and Control*, vol. 60, Jul. 2020, doi: 10.1016/j.bspc.2020.101993.
- [3] N. Mohanty and M. Sarmadi, "Brain tumor MRI classification and identification using an image classification model via Convolutional Neural Networks," *medRxiv*, Sep. 25, 2024, doi: 10.1101/2024.09.13.23299832.
- [4] M. Othman, S. L. S. Abdullah, K. A. Ahmad, M. N. A. Bakar, and A. R. Mansor, "The fusion of edge detection and mathematical morphology algorithm for shape boundary recognition," *Journal of Information and Communication Technology*, vol. 15, no. 1, pp. 133–144, 2016.
- [5] T. M. Noguerol, F. Paulano-Godino, M. T. Martín-Valdivia, C. O. Menias, and A. Luna, "Strengths, weaknesses, opportunities, and threats analysis of artificial intelligence and machine learning applications in radiology," *Journal of the American College of Radiology*, vol. 16, no. 9, pp. 1239–1247, Sep. 2019, doi: 10.1016/j.jacr.2019.05.047.
- [6] M. A. Iqbal and K. H. Talukder, "Detection of potato disease using image segmentation and machine learning," in *2020 International Conference on Wireless Communications Signal Processing and Networking (WiSPNET)*, Aug. 2020, pp. 43–47, doi: 10.1109/WiSPNET48689.2020.9198563.
- [7] A. Bhandari, J. Koppen, and M. Agzarian, "Convolutional neural networks for brain tumour segmentation," *Insights into Imaging*,

ACM for segmenting intensity inhomogeneity images corrupted by a mixture of AGMG ... (Nurhuda Ismail)

- vol. 11, no. 1, Dec. 2020, doi: 10.1186/s13244-020-00869-4.
- [8] A. S. B. Azam, A. K. Jumaat, and S. Ibrahim, "Comparison of various colorization techniques for MRI brain tumor segmentation using convolutional neural networks," in *2023 IEEE 8th International Conference on Recent Advances and Innovations in Engineering (ICRAIE)*, Dec. 2023, pp. 1–6, doi: 10.1109/ICRAIE59459.2023.10468529.
 - [9] G. Litjens *et al.*, "A survey on deep learning in medical image analysis," *Medical Image Analysis*, vol. 42, pp. 60–88, Dec. 2017, doi: 10.1016/j.media.2017.07.005.
 - [10] A. Xu, L. Wang, S. Feng, and Y. Qu, "Threshold-based level set method of image segmentation," in *2010 Third International Conference on Intelligent Networks and Intelligent Systems*, Nov. 2010, pp. 703–706, doi: 10.1109/ICINIS.2010.181.
 - [11] C. Cigla and A. A. Alatan, "Region-based image segmentation via graph cuts," in *2008 15th IEEE International Conference on Image Processing*, 2008, pp. 2272–2275, doi: 10.1109/ICIP.2008.4712244.
 - [12] V. Caselles, R. Kimmel, and G. Sapiro, "Geodesic active contours," in *Proceedings of IEEE International Conference on Computer Vision*, 1995, pp. 694–699, doi: 10.1109/ICCV.1995.466871.
 - [13] C. Zhao, J. Liu, and J. Zhang, "A dual model for restoring images corrupted by mixture of additive and multiplicative noise," *IEEE Access*, vol. 9, pp. 168869–168888, 2021, doi: 10.1109/ACCESS.2021.3137995.
 - [14] F. Wang, H. Huang, and J. Liu, "Variational-based mixed noise removal with CNN deep learning regularization," *IEEE Transactions on Image Processing*, vol. 29, pp. 1246–1258, 2020, doi: 10.1109/TIP.2019.2940496.
 - [15] K. Chithra and T. Santhanam, "A novel denoising technique for mixed noise removal from grayscale and color images," *Journal of Theoretical and Applied Information Technology*, vol. 96, no. 3, pp. 626–642, 2018.
 - [16] N. Chumchob, K. Chen, and C. Brito-Loeza, "A new variational model for removal of combined additive and multiplicative noise and a fast algorithm for its numerical approximation," *International Journal of Computer Mathematics*, vol. 90, no. 1, pp. 140–161, Jan. 2013, doi: 10.1080/00207160.2012.709625.
 - [17] C. Li and Q. Fan, "A modified variational model for restoring blurred images with additive noise and multiplicative noise," *Circuits, Systems, and Signal Processing*, vol. 37, no. 6, pp. 2511–2534, Jun. 2018, doi: 10.1007/s00034-017-0675-6.
 - [18] L. Mabood *et al.*, "Multi-scale-average-filter-assisted level set segmentation model with local region restoration achievements," *Scientific Reports*, vol. 12, no. 1, Sep. 2022, doi: 10.1038/s41598-022-19893-z.
 - [19] M. Shi and I. Hussain, "Improved region-based active contour segmentation through divergence and convolution techniques," *AIMS Mathematics*, vol. 10, no. 1, pp. 654–671, 2025, doi: 10.3934/math.2025029.
 - [20] S. Soomro, F. Akram, J. H. Kim, T. A. Soomro, and K. N. Choi, "Active contours using additive local and global intensity fitting models for intensity inhomogeneous image segmentation," *Computational and Mathematical Methods in Medicine*, vol. 2016, pp. 1–15, 2016, doi: 10.1155/2016/9675249.
 - [21] D. Mumford and J. Shah, "Optimal approximations by piecewise smooth functions and associated variational problems," *Communications on Pure and Applied Mathematics*, vol. 42, no. 5, pp. 577–685, Jul. 1989, doi: 10.1002/cpa.3160420503.
 - [22] A. Tsai, A. Yezzi, and A. S. Willsky, "Curve evolution implementation of the Mumford-Shah functional for image segmentation, denoising, interpolation, and magnification," *IEEE Transactions on Image Processing*, vol. 10, no. 8, pp. 1169–1186, 2001, doi: 10.1109/83.935033.
 - [23] L. A. Vese and T. F. Chan, "A multiphase level set framework for image segmentation using the Mumford and Shah model," *International Journal of Computer Vision*, vol. 50, no. 3, pp. 271–293, Dec. 2002, doi: 10.1023/A:1020874308076.
 - [24] C. Li, C.-Y. Kao, J. C. Gore, and Z. Ding, "Minimization of region-scalable fitting energy for image segmentation," *IEEE Transactions on Image Processing*, vol. 17, no. 10, pp. 1940–1949, Oct. 2008, doi: 10.1109/TIP.2008.2002304.
 - [25] C. Li, C. Xu, C. Gui, and M. D. Fox, "Level set evolution without re-initialization: A new variational formulation," in *2005 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR '05)*, San Diego, CA, USA, 2005, pp. 430–436, doi: 10.1109/CVPR.2005.213.
 - [26] H. Zhang, L. Tang, and C. He, "A variational level set model for multiscale image segmentation," *Information Sciences*, vol. 493, pp. 152–175, Aug. 2019, doi: 10.1016/j.ins.2019.04.048.
 - [27] T. F. Chan and L. A. Vese, "Active contours without edges," *IEEE Transactions on Image Processing*, vol. 10, no. 2, pp. 266–277, 2001, doi: 10.1109/83.902291.
 - [28] L. Tang, H. Zhang, Y. Xu, Y. Ren, and C. Li, "A hybrid variational level set model based on double Gaussian distribution fitting energy for image segmentation and bias correction," *IET Image Processing*, vol. 18, no. 14, pp. 4461–4491, Dec. 2024, doi: 10.1049/ipr2.13248.
 - [29] P.-W. Hsieh and S.-Y. Yang, "Variational image segmentation via additive decomposition and hybrid regularization with an energy-decaying scheme," *Journal of Mathematical Imaging and Vision*, vol. 68, no. 4, Aug. 2026, doi: 10.1007/s10851-026-01315-8.
 - [30] K. Qi, Z. Huang, and W. Yang, "Robust variational model based tailored UNet: Leveraging edge detector and mean curvature for improved image segmentation," *arXiv*, Dec. 2025, doi: 10.48550/arXiv.2512.07590.
 - [31] A. Kumar, S. Majee, and S. K. Jain, "CDM: A coupled deformable model for image segmentation with speckle noise and severe intensity inhomogeneity," *Chaos, Solitons & Fractals*, vol. 172, Jul. 2023, doi: 10.1016/j.chaos.2023.113551.
 - [32] A. Kumar and S. K. Jain, "ECDM: Enhanced edge based coupled deformable model for image segmentation in the presence of speckle noise and severe intensity inhomogeneity," *Applied Mathematical Modelling*, vol. 128, pp. 659–684, Apr. 2024, doi: 10.1016/j.apm.2024.01.039.
 - [33] K. Ding, L. Xiao, and G. Weng, "Active contours driven by region-scalable fitting and optimized Laplacian of Gaussian energy for image segmentation," *Signal Processing*, vol. 134, pp. 224–233, May 2017, doi: 10.1016/j.sigpro.2016.12.021.
 - [34] R. Chan, H. Yang, and T. Zeng, "A two-stage image segmentation method for blurry images with poisson or multiplicative gamma noise," *SIAM Journal on Imaging Sciences*, vol. 7, no. 1, pp. 98–127, Jan. 2014, doi: 10.1137/130920241.
 - [35] S. Kollem, K. R. L. Reddy, and D. S. Rao, "A review of image denoising and segmentation methods based on medical images," *International Journal of Machine Learning and Computing*, vol. 9, no. 3, pp. 288–295, Jun. 2019, doi: 10.18178/ijmlc.2019.9.3.800.
 - [36] L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D: Nonlinear Phenomena*, vol. 60, no. 1–4, pp. 259–268, Nov. 1992, doi: 10.1016/0167-2789(92)90242-F.
 - [37] G. Aubert and J.-F. Aujol, "A variational approach to removing multiplicative noise," *SIAM Journal on Applied Mathematics*, vol. 68, no. 4, pp. 925–946, Jan. 2008, doi: 10.1137/060671814.
 - [38] C. Li, C.-Y. Kao, J. C. Gore, and Z. Ding, "Implicit active contours driven by local binary fitting energy," in *2007 IEEE Conference on Computer Vision and Pattern Recognition*, Jun. 2007, pp. 1–7, doi: 10.1109/CVPR.2007.383014.
 - [39] J. Spencer and K. Chen, "A convex and selective variational model for image segmentation," *Communications in Mathematical Sciences*, vol. 13, no. 6, pp. 1453–1472, 2015, doi: 10.4310/CMS.2015.v13.n6.a5.
 - [40] K. Zhang, H. Song, and L. Zhang, "Active contours driven by local image fitting energy," *Pattern Recognition*, vol. 43, no. 4, pp. 1199–1206, Apr. 2010, doi: 10.1016/j.patcog.2009.10.010.

- [41] B. Chen, *Optimization theory and algorithms (2nd edition)*. Tsinghua University Press, 2005.
- [42] J. Fang, H. Liu, L. Zhang, J. Liu, and H. Liu, "Active contour driven by weighted hybrid signed pressure force for image segmentation," *IEEE Access*, vol. 7, pp. 97492–97504, 2019, doi: 10.1109/ACCESS.2019.2929659.
- [43] M.-Y. Chen *et al.*, "Automatic Chinese food identification and quantity estimation," in *SIGGRAPH Asia 2012 Technical Briefs*, Nov. 2012, pp. 1–4, doi: 10.1145/2407746.2407775.
- [44] T. Dietenbeck, M. Alessandrini, D. Friboulet, and O. Bernard, "Creaseg: A free software for the evaluation of image segmentation algorithms based on level-set," in *2010 IEEE International Conference on Image Processing*, Sep. 2010, pp. 665–668, doi: 10.1109/ICIP.2010.5652991.
- [45] I. Ismahan, "Microscopic bone marrow images," *Kaggle*, 2022. [Online]. Available: <https://www.kaggle.com/datasets/fanouna/microscopic-bone-marrow-images>. (Accessed: Nov. 27, 2022).
- [46] N. C. F. Codella *et al.*, "Skin lesion analysis toward melanoma detection: A challenge at the 2017 International symposium on biomedical imaging (ISBI), hosted by the international skin imaging collaboration (ISIC)," in *2018 IEEE 15th International Symposium on Biomedical Imaging (ISBI 2018)*, Apr. 2018, pp. 168–172, doi: 10.1109/ISBI.2018.8363547.
- [47] L. Kostykin, C. Schnörr, and K. Rohr, "Globally optimal segmentation of cell nuclei in fluorescence microscopy images using shape and intensity information," *Medical Image Analysis*, vol. 58, Dec. 2019, doi: 10.1016/j.media.2019.101536.
- [48] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models," *International Journal of Computer Vision*, vol. 1, no. 4, pp. 321–331, Jan. 1988, doi: 10.1007/BF00133570.

BIOGRAPHIES OF AUTHORS



Nurhuda Ismail    received her Diploma in Science in 2004 from Universiti Teknologi MARA (UiTM), Perlis, Malaysia, B.Sc. degree in Mathematics in 2007 from Universiti Teknologi MARA (UiTM), Shah Alam, Malaysia and M.Sc. degree in Mathematics in 2011 from Universiti Sains Malaysia (USM), Penang, Malaysia. She is now serving as a senior lecturer at the Faculty of Computer and Mathematical Sciences, Johor Branch, Pasir Gudang Campus, Malaysia. Her research interests include mathematical image analysis and image processing. She can be contacted at email: nurhudaismail@uitm.edu.my.



Abdul Kadir Jumaat    received his B.Sc. and M.Sc. degrees in Mathematics from the Universiti Teknologi MARA (UiTM), Shah Alam, Malaysia and the Ph.D. degree in Applied Mathematics (Mathematical Imaging Methods) from the University of Liverpool, United Kingdom. He is now serving as a senior lecturer at the Faculty of Computer and Mathematical Sciences and a research fellow at the Institute for Big Data Analytics and Artificial Intelligence (IBDAAI) in UiTM, Shah Alam, Malaysia. His research interests include applied mathematics, mathematical modelling, image/signal processing and analysis, artificial intelligence, computer vision, and pattern recognition. He can be contacted at email: abdulkadir@tmsk.uitm.edu.my.



Rizauddin Saian    received his B.Sc. degree in Mathematics from Universiti Malaya in 1998, M.Sc. degree in Information Technology in 2005 and the Ph.D. degree in 2013 from Universiti Utara Malaysia. He is now serving as an associate professor at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (Perlis), Malaysia, teaching there since October 1999. His research interests include ant colony optimization, data mining, and optimization algorithms. He can be contacted at email: rizauddin@uitm.edu.my.



Akmal Shafiq Badarul Azam    received his B.Sc. degree in Computational Mathematics and M.Sc. degree in Applied Mathematics (Financial and Management Mathematics) from the Universiti Teknologi MARA (UiTM), Malaysia, in 2021 and 2022, respectively. He is now serving as a lecturer at the Faculty of Computer and Mathematical Sciences, Sarawak Branch, Mukah Campus, Malaysia. His research interests include mathematical image analysis, medical image processing, and machine learning. He can be contacted at email: akmalshafiq@uitm.edu.my.