

Linear quadratic regulator and pole placement for stabilizing a cart inverted pendulum system

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ABSTRACT

The system of a cart inverted pendulum has many problems such as nonlinearity, complexity, unstable, and underactuated system. It makes this system be a benchmark for testing many control algorithm. This paper presents a comparison between 2 conventional control methods consist of a linear quadratic regulator (LQR) and pole placement. The comparison indicated by the most optimal steps and results in the system performance that obtained from each method for stabilizing a cart inverted pendulum system. A mathematical model of DC motor and mechanical transmission are included in a mathematical model to minimize the realtime implementation problem. From the simulation, the obtained system performance shows that each method has its advantages, and the desired pendulum angle and cart position reached.

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1. INTRODUCTION

Inverted Pendulum is a regular pendulum facing upwards, opposite the direction of gravity. This system is naturally unstable. One way to stabilize this system is to move the pivot point with a certain speed and acceleration in the direction of the vertical axis. The system of inverted pendulum is also nonlinear. By giving the same speed and acceleration to the system, the response from the pendulum angle of the rod varies. The system of an inverted pendulum is also underactuated because it has one input in the form of a vertical motion at the pivot point but has two outputs in the form of rod angles and the position of the pivot point. The nature of this inverted pendulum system makes the researchers make this system the choice of a test or benchmark tool for testing many control techniques since the 1973s to the present [1-2]. The PID control [3-6], fuzzy genetic algorithm [7-10], state-space control with pole placement methods [11-13], linear quadratic regulator (LQR) [14-16], linear quadratic gaussian (LQG) [17-20], observer [21-23], NCTL [24], flower pollination [25], and others have been used in this system.

This paper discusses the LQR and pole placement methods for stabilizing a cart inverted pendulum system. System modeling using the Lagrange equation. The equation of particle motion expressed by the Lagrange equation can be obtained by reviewing the kinetic energy and potential energy of the system without the need to review the forces acting on the system. After deriving the mathematical model using

the Lagrange equation, the system of a cart inverted pendulum is brought to a state-space form for further control using the LQR and pole placement control methods.

2. RESEARCH METHOD

The system of a cart inverted pendulum is illustrated in Figure 1. There is a thrust f which causes the cart to move on the horizontal axis x . This force is generated by a DC motor. ω_1, ω_2 is the rotational speed in the pulley transmission system, τ_1, τ_2 is the variable torque in the pulley transmission system, l is the rod length, and m is the pendulum mass centered on the rod end. This system has one input in the form of f and two outputs, namely the pendulum angle (θ) and the cart position (x).

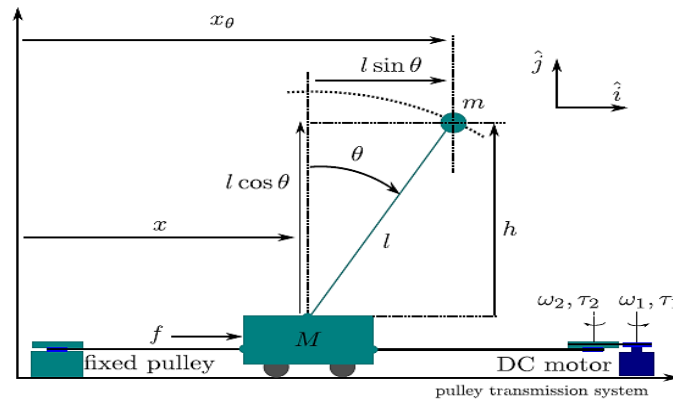


Figure 1. A cart inverted pendulum system

2.1. Mathematical model

2.1.1. Lagrange equation on the system

Mathematical modeling of an inverted pendulum system using the Lagrange equation. Lagrangian function \mathcal{L} is expressed by kinetic energy K and potential energy P as follows:

$$\mathcal{L} = K(\mathbf{q}, \dot{\mathbf{q}}) - P(\mathbf{q}) \tag{1}$$

where $\mathbf{q} = \{q_1, \dots, q_i, \dots, q_n\}$. The desired equation of motion is derived by using:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i \tag{2}$$

where Q_i indicates the external force at the q_i coordinate. The cart moves in the direction i only, so the kinetic energy on the cart is expressed as:

$$K_M = \frac{1}{2} M \dot{x}^2 \tag{3}$$

The pendulum moves in the direction i and j , so that the kinetic energy in the pendulum is expressed as:

$$K_m = \frac{1}{2} m (\dot{x}_\theta^2 + \dot{h}^2) \tag{4}$$

$x_\theta = x + l \sin \theta$ and $h = l \cos \theta$, then the equation becomes:

$$K_m = \frac{1}{2} m (\dot{x}^2 + 2\dot{x}l\dot{\theta} \cos \theta + l^2 \dot{\theta}^2) \tag{5}$$

So that the total kinetic energy of the inverted pendulum cart system is:

$$K = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + 2\dot{x}l\dot{\theta} \cos \theta + l^2 \dot{\theta}^2) \tag{6}$$

The potential energy in the cart inverted pendulum system is only found in the pendulum mass (m).

$$\begin{aligned} P &= mgh \\ P &= mgl \cos \theta \end{aligned} \tag{7}$$

Lagrange equation is obtained as follows:

$$\mathcal{L} = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m (\dot{x}^2 + 2\dot{x}l\dot{\theta} \cos \theta + l^2 \dot{\theta}^2) - mgl \cos \theta \tag{8}$$

The cart inverted pendulum system has two generalised coordinates namely x and θ , so the Lagrange equation for each common coordinate is described as follows:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}} \right) - \frac{\partial \mathcal{L}}{\partial x} = f \tag{9}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0 \tag{10}$$

After being derived, the general coordinates from the Lagrange equation of the system of a cart inverted pendulum can be rewritten to:

$$f = (M + m)\ddot{x} + ml\ddot{\theta} \cos \theta - ml\dot{\theta}^2 \sin \theta \tag{11}$$

$$0 = ml(\ddot{x} \cos \theta - \dot{\theta} \dot{x} \sin \theta) + ml^2 \ddot{\theta} - (-m\dot{x}l\dot{\theta} \sin \theta + mgl \sin \theta) \tag{12}$$

Equations (11) and (12) are nonlinear equations because they have trigonometric terms $\sin ()$ and $\cos ()$. The controller is designed to make θ as small as possible. With this assumption, the linear Lagrange equation of the system of a cart inverted pendulum becomes:

$$f = (M + m)\ddot{x} + ml\ddot{\theta} \tag{13}$$

$$0 = \ddot{x} + l\ddot{\theta} - g\theta \tag{14}$$

2.1.2. DC motor

The armature DC motor equivalent circuit is shown in Figure 2. The open-loop system diagram in the form of state space is shown in Figure 3.

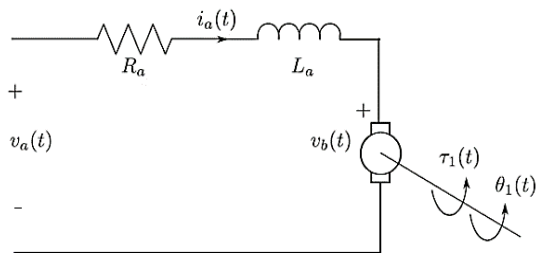


Figure 2. Armature DC motor

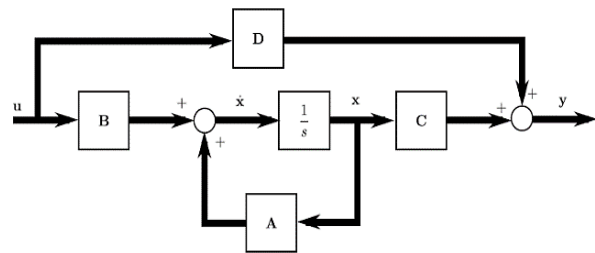


Figure 3. Open-loop system

The equation of the armature DC motor circuit is

$$i_a R_a + L_a \frac{di_a}{dt} + v_b = v_a \tag{15}$$

where $v_b = K_b \frac{d\theta_1}{dt}$, $\omega_1 = \frac{d\theta_1}{dt}$, $\tau_1 = K_t i_a$. So the equation of the armature DC motor circuit becomes:

$$\frac{\tau_1}{K_t} R_a + L_a \frac{d^2 \tau_1}{(dt)^2} + K_b \frac{d\theta_1}{dt} = v_a \quad (16)$$

Generally, inductors in small rotors so that the armature circuit equation becomes:

$$\tau_1 = -K_t \frac{K_b}{R_a} \omega_1 + \frac{K_t}{R_a} v_a \quad (17)$$

The angular velocity of the motor ω_1 is expressed in the speed of the cart \dot{x} to be $\omega_1 = \frac{\dot{x}}{r_1}$, and $K_r = \frac{K_t}{R_a}$ so the equation becomes:

$$f = \frac{K_r}{r_1} \left(-K_b \frac{\dot{x}}{r_1} + v_a \right) \quad (18)$$

So the Lagrange equation at the x coordinate becomes:

$$\frac{K_r}{r_1} v_a = (M + m) \ddot{x} + \frac{K_r K_b}{(r_1)^2} \dot{x} - ml \ddot{\theta} \quad (19)$$

Simplified equations with $c_1 = \frac{K_r K_b}{(r_1)^2}$ and $c_2 = \frac{K_r}{r_1}$, becomes:

$$\dot{x} = \frac{1}{M} (c_2 v_a - c_1 \dot{x} - mg\theta) \quad (20)$$

From (14), it can be obtained:

$$\ddot{\theta} = \frac{1}{l} (g\theta - \dot{x}) \quad (21)$$

Equation (20) is substituted to (21) so that it becomes:

$$\ddot{\theta} = \frac{1}{l} \left(g\theta - \frac{1}{M} (c_2 v_a - c_1 \dot{x} - mg\theta) \right) \quad (22)$$

$$\ddot{\theta} = -\frac{c_2}{Ml} v_a - \frac{c_1}{Ml} \dot{x} + \frac{(M+m)g}{Ml} \theta \quad (23)$$

The Lagrange equation for the system of a cart inverted pendulum has been obtained and subsequently converted into state space.

2.1.2. State space representation

The form of state space of a system is represented as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (24)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \quad (25)$$

where $\mathbf{x} \in R^j$ is a vector state, $\mathbf{u} \in R^k$ is a control input, $\mathbf{A} \in R^{j \times j}$ is a system matrix, $\mathbf{B} \in R^{j \times k}$ input matrix, $\mathbf{C} \in R^{p \times j}$ is an output matrix, and $\mathbf{y} \in R^p$ is an output vector. In a cart inverted pendulum system, the state vector is defined as $\mathbf{x} = [x \ \theta \ \dot{x} \ \dot{\theta}]^T$, derived from the state vector as $\dot{\mathbf{x}} = [\dot{x} \ \ddot{x} \ \ddot{\theta}]^T$, and the control input $u = v_a$. From the Lagrange equation that has been obtained previously, the state space form of the system of a cart inverted pendulum is;

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mg}{M} & -\frac{c_1}{M} & 0 \\ 0 & \frac{(M+m)g}{Ml} & \frac{c_1}{Ml} & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \frac{c_2}{M} \\ -\frac{c_2}{Ml} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{y} = [x \ \theta]^T, \mathbf{D} = \emptyset \quad (26)$$

2.2. Controller design

2.2.1. Linear quadratic regulator

In the LQR method, the weight matrix Q and R are determined to get the most optimal response. LQR is one method of designing a modern control system. The purpose of LQR is to regulate or make the output of y be zero with minimum input. In order to bring the system state from the initial state $x(t) = x(0)$ to the desired state $x(t)$ by minimizing J . In general, LQR problems are:

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)] \tag{27}$$

where $Q \in R^{n \times n}$ and $R \in R^{n \times m}$. Q and R each determine the performance and energy control needed. The optimal gain of feedback K can be obtained by;

$$K = R^{-1}B^TP \tag{28}$$

where P is a positive definite matrix which is a solution of the Riccati equation with:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{29}$$

2.2.2. Pole placement

Pole placement is a control method that is also used to get the most optimal response. Pole placement is also a modern control method. The location of the pole can be used as a reference to find out whether a system is stable or not. A stable system has a pole to the left of the imaginary axis, while a system that has a pole to the right of the imaginary axis is an unstable system. Therefore, there needs to be a control method to bring the pole which is to the right of the imaginary axis to the left of the imaginary axis so that the system becomes stable. Pole placement is a control method that can bring the pole to the right position. By using the gain feedback K , the desired design criteria of the system can be met. The polynomial characteristics can be described as:

$$|sI - A| = s^n + a_1s^{n-1} + \dots + a_{n-1}s + a_n \tag{30}$$

where I is the identity matrix and a is the polynomial coefficient. If the desired eigenvalues $[\mu_1, \dots, \mu_n]$ can be considered, then the desired characteristics of the polynomial are;

$$\prod_{i=1}^n (s - \mu_i) = s^n + \alpha_1 s^{n-1} + \dots + \alpha_{n-1} s + \alpha_n \tag{31}$$

The feedback matrix gain required by K can be calculated using;

$$K = [\alpha_n - a_n \mid \alpha_{n-1} - a_{n-1} \mid \dots \mid \alpha_2 - a_2 \mid \alpha_1 - a_1]T^{-1} \tag{32}$$

The LQR method and pole placement use the feedback matrix. The feedback matrix in question can be described as the matrix K in Figure 4. To support the performance of the system, the pre-gain N matrix is also shown in Figure 5.

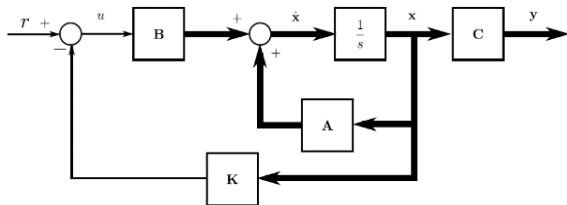


Figure 4. Closed-loop system

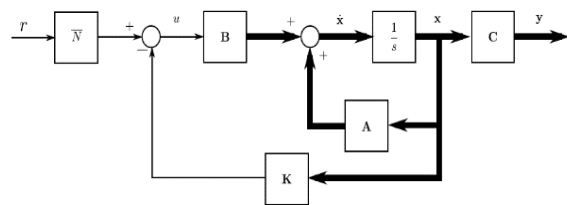


Figure 5. A closed-loop system with pre-gain

Controllability and observability of the system is something that must be met before the control is done. The controllability of the system is indicated by $\text{rank}(C) = n$, where n is equal to the number

of state variables for all initial state values. The rank of observability (\mathcal{O}) must also be the same as the number of states,

$$\mathcal{C} = [B|AB|A^2B|\dots|A^{n-1}B] \tag{33}$$

$$\mathcal{O} = [C|CA|CA^2|\dots|CA^{n-1}] \tag{34}$$

3. RESULT AND ANALYSIS

The parameters of the system of a cart inverted pendulum are determined for simulations in the Linear Quadratic Regulator and the pole placement methods as in Table 1.

Table 1. Cart inverted pendulum parameters

Parameters	Value
R_a	1.25 Ω
r_1	0.02 m
K_b	0.015 V.s/rad
M	1.2 Kg
K_t	0.015 N.m/A
m	0.2 Kg
l	0.3 m

The controllability and observability of the system of a cart inverted pendulum is obtained namely $\text{rank}(\mathcal{C}) = \text{rank}(\mathcal{O}) = 4$ so that control can be done with;

$$\mathcal{C} = \begin{bmatrix} 0 & 0.5 & -0.1875 & 2.795 \\ 0 & -1.667 & 0.625 & -63.81 \\ 0.5 & -0.187 & 2.795 & -2.07 \\ -1.667 & 0.625 & -63.81 & 27.34 \end{bmatrix} \quad \mathcal{O} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.635 & -0.375 & 0 \\ 0 & 38.15 & 1.25 & 0 \\ 0 & 0.6131 & 0.1406 & -1.635 \\ 0 & -2.0438 & -0.4688 & 38.15 \end{bmatrix}$$

It is known that one of the poles of the system is to the right of the imaginary axis. The pole is worth (0.6.1512, -6.2049, -0.3213) so control is needed. Control is done with the desired output in the form of a pendulum angle at the balance point and the position of the cart is at 0.3 meters. The pole-zero mapping from the system of a cart inverted pendulum is shown in Figure 6.

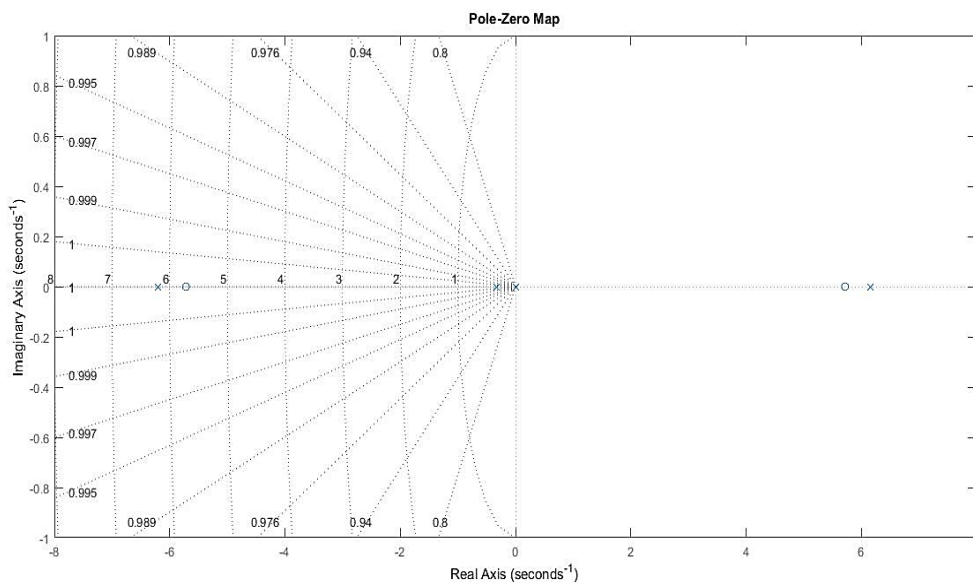


Figure 6. Pole-zero mapping

3.1. Simulation of the LQR method

The simulation using the LQR method uses 5 variations of the weight matrix Q and R as in Table 2. The weighting matrix $Q = [q_1, q_2, q_3, q_4]$ that is changed is q_1, q_2 because that affects state x, θ directly. The combination produces a feedback matrix (K) and pre-gain (N) as Table 3. From the 5 configurations in Table 2 and Table 3, the system response was obtained as shown in Figures 7, 8, and 9.

Table 2. LQR parameter configuration

No	q_1	q_2	R
1	0.1	1	1
2	1	1	1
3	1	2	1
4	1	2	10
5	2	2	10

Table 3. N, K parameters

No	N	$K1$	$K2$	$K3$	$K4$
1	-0.3162	-0.3162	-50.5224	-2.2810	-8.2416
2	-1.0000	-1.0000	-53.6364	-3.3636	-8.7875
3	-1.0000	-1.0000	-53.6561	-3.3644	-8.7892
4	-0.3162	-0.3162	-50.5062	-2.2807	-8.2402
5	-0.4472	-0.4472	-51.2154	-2.5234	-8.3645

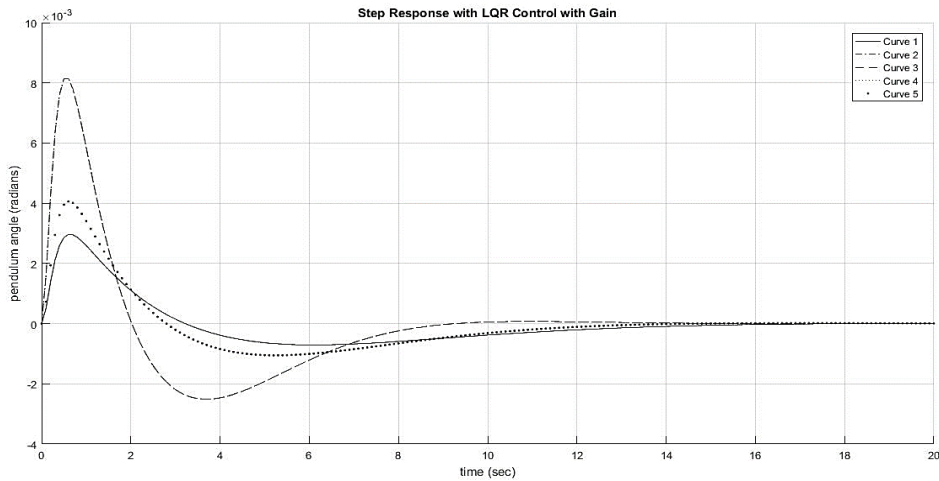


Figure 7. The angle response of the pendulum with the LQR method

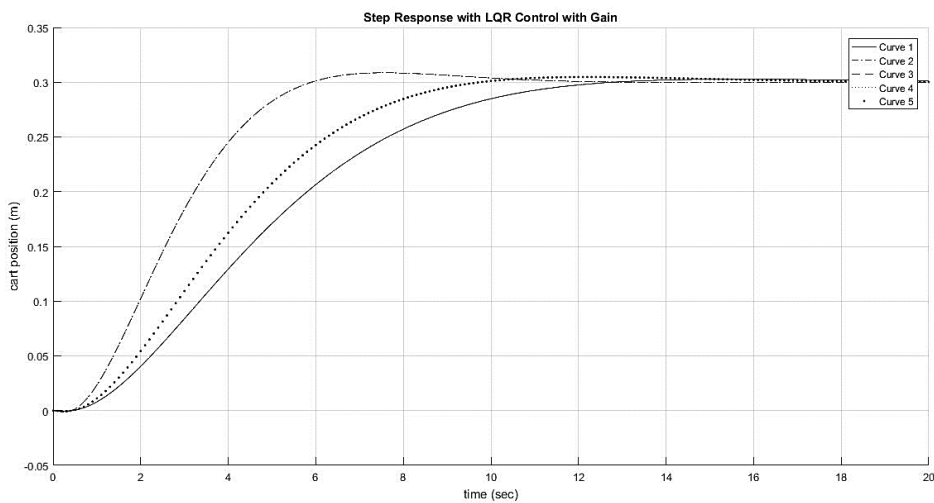


Figure 8. Cart position response with the LQR method

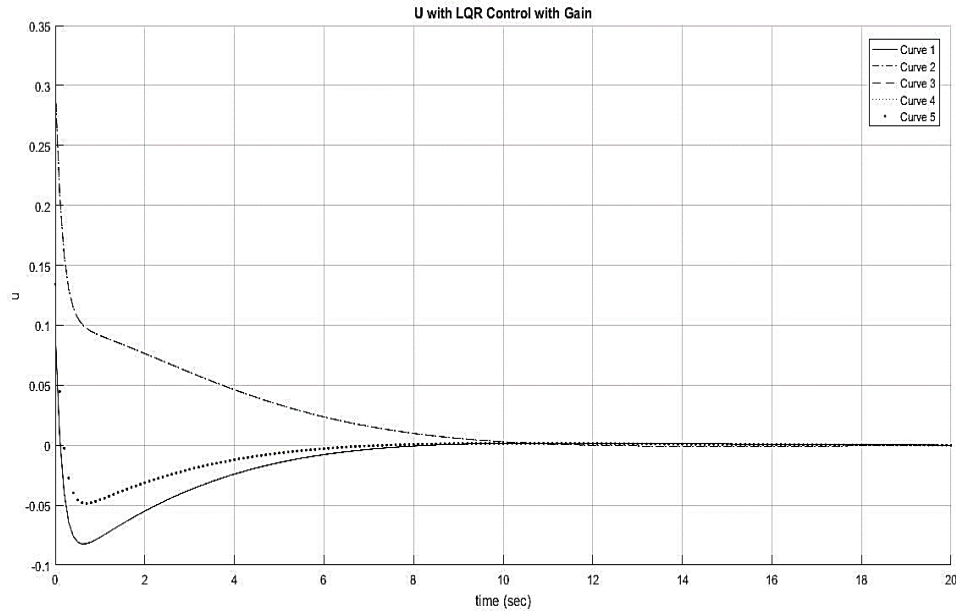


Figure 9. Control signal with the LQR method

From Figure 7, 8, and 9 it can be seen that the greater the Q and R weighting matrix, the faster the system response to reach the desired state and the greater the control signal. However, by choosing a small R , the control signal can be minimized. Therefore, choosing the right Q and R matrix will produce an optimal controller.

3.2. Pole placement method simulation

Simulation using the pole placement method uses 5 variations of the pole matrix s_1, s_2, s_3, s_4 as in Table 4. Poles are chosen randomly with negative values. The combination produces a feedback matrix (K) and pre-gain (N) as in Table 5.

Table 4. Pole placement parameters

No	s_1	s_2	s_3	s_4
1	-6.18+0.19i	-6.18-0.19i	-0.48+0.43i	-0.48-0.43i
2	-6.18+0.19i	-6.18-0.19i	-0.48+0.43i	-0.489
3	-6.18+0.19i	-6.18-0.19i	-0.48+0.43i	-0.189
4	-6.18+0.19i	-6.18-0.19i	-0.48+0.43i	-0.089
5	-6.18+0.19i	-6.18-0.19i	-0.48	-0.089

Table 5. N, K parameters

No	N	$K1$	$K2$	$K3$	$K4$
1	-1.0000	-1.0000	-53.65	-3.3644	-8.7891
2	-0.56+0.49i	-0.56+0.49i	-53.41+3.48i	-3.22+1.17i	-8.74+0.61i
3	-0.21+0.19i	-0.24+0.19i	-50.99+3.32i	-2.41+1.07i	-8.32+0.58i
4	-0.10+0.09i	-0.10+0.09i	-50.18+3.26i	-2.13+1.04i	-8.18+0.57i
5	-0.1029	-0.1029	-50.1898	-2.1388	-8.1815

From the 5 configurations in Table 4 and Table 5, the system response is obtained as shown in Figures 10, 11 and 12. From Figures 10, 11, and 12 it can be seen that the increasingly negative pole position produces an increasingly rapid response at the angle of the pendulum, but the overshoot also gets bigger. By looking at the control signal from each pole, the optimal controller can be selected.

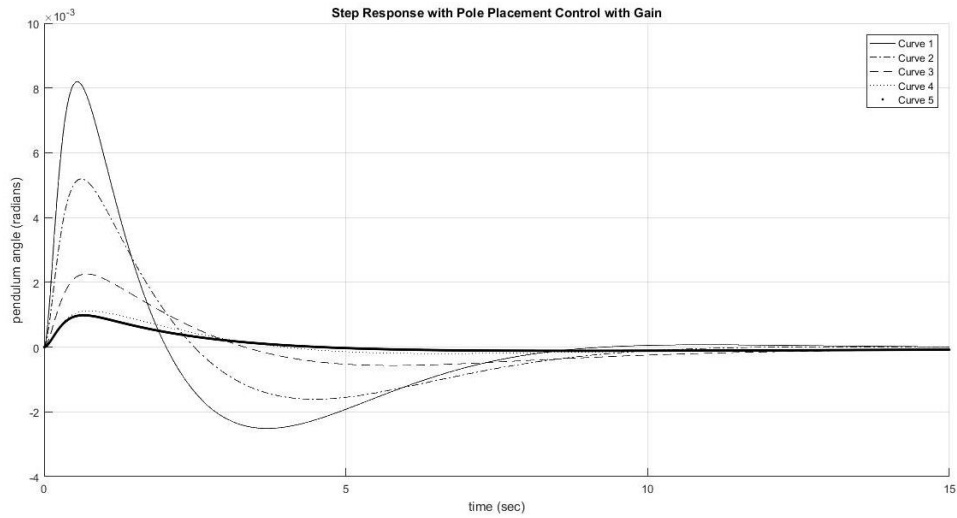


Figure 10. The angle response of the pendulum using the pole placement method

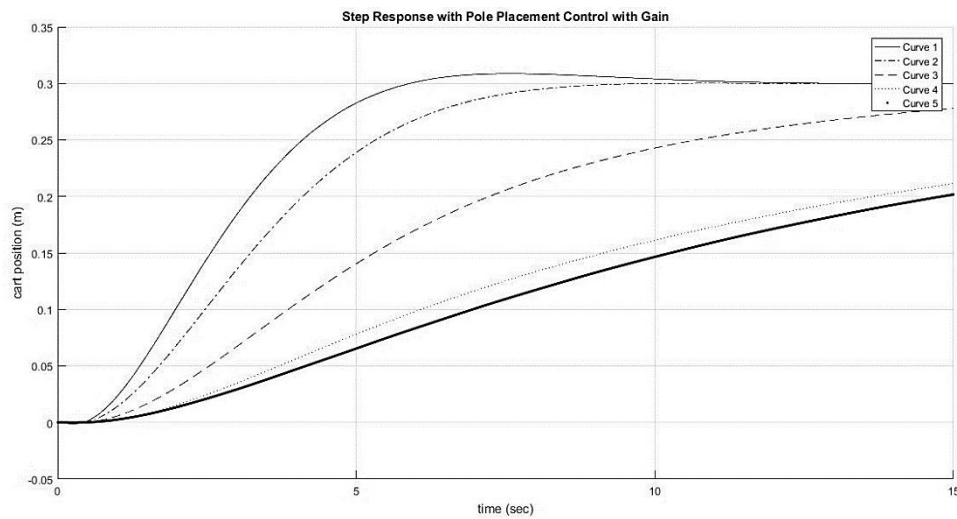


Figure 11. Response cart with the pole placement method

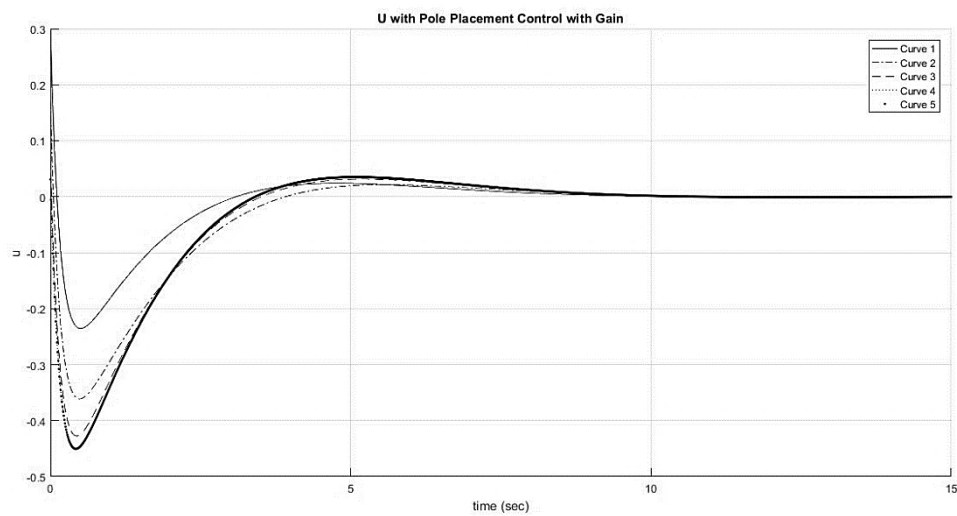


Figure 12. Control signal with the pole placement method

4. CONCLUSION

Linear quadratic regulator and pole placement for stabilizing a cart inverted pendulum system has been successfully conducted. By using LQR, the system response can be obtained by selecting the weight matrix Q and R which is then continued with the calculation to get the desired pole. With the pole placement method, the system response can be chosen by directly selecting the desired pole, although it is necessary to repeatedly determine the pole to get the most optimal results. Both methods can produce optimal system response with fast system response and minimal control signals.

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