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Design of Quadratic Optimal Regulator System for State Space Model of Single phase Inverter both in Standalone and Gridtie Modes

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Abstract

Since in 3-phase systems, the reliability of 3-phase Inverter is not good, under such situations Paralleled Inverter Systems are used. Parallel Inverter operation has a major role in uninterruptible power system (UPS) applications. Most of the standalone inverter systems use a LC filter and proportional-integral (PI) controller in their control loops. When connecting the paralleled inverters to utility grids, the capacitor becomes redundant and thus either a pure inductor or an LCL filter can be used as inverter output stage. Compared with the L filter, the LCL filter is more attractive because it cannot only provide higher harmonics attenuation with same inductance value, but also allow inverter to operate both in standalone and grid-tie modes, which makes it a universal inverter for distributed generation applications. Output of Inverter should always be checked. In this paper State Space Model of Single phase Inverter is analysis in two modes which are: a) Grid-tie Mode b) Standalone Mode.A Quadratic Optimal Regulator System is designed and its unit step response is obtained to validate whether the whole system (inverter system along with designed QOR) is a stable system or not. This is done using MATLAB Program.

***Keywords**: Single phase Inverter, Standalone Mode, Grid tie Mode, State Space Model, Quadratic Optimal Regulator*

1. Introduction

Inverters are predominantly used as interface link to process and convert it to the form suitable for utility because of the control flexibility they provide. Inverters convert power from dc to ac with controllable voltage and variable frequency. For battery interface applications, inverters are used as a bidirectional link to charge batteries during surplus periods and to discharge them in absence of power. Hence inverters take power from different modules and deliver it to common grid, which is synchronized with local utility grid or in islanded mode.

The stability of large-scale distributed generation systems was analyzed by state space model. This model is relatively easy to expand by combining single inverter state space equations to build state space equations for paralleled inverter systems.

In this paper, the state space model is adopted to investigate the stability of single inverter in both standalone and grid-tie mode. [1], [2], [3] This can be extended to n number of inverters connected in parallel. Here the observer estimates state variables and the difference between the estimated state variables and the actual state variables must tend to zero. Feedback gain matrix K is to be determined to yield desired characteristic equation. In order to find feedback gain matrix, we have to choose Locations of Desired Closed loop poles [4]. There are two approaches by which we can determined it, First Approach is by Pole placement Method.[5] But the problem with this method is that if we place dominant poles far away from $j\omega$ axis, system response becomes fast, the signals in the system become very large, as a result the system may becomes non linear which should be avoided. Alternative approach is based on quadratic optimal control approach. [5] This method will determine desired closed loop loop poles such that it balances between accepted response and the amount of control energy required. An advantage of quadratic optimal control method over pole placement is that former provides a systemic way of computing state feedback control gain matrix.

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2. State Space Modelling

2.1 Standalone Mode

Figure 1 shows the Block Diagram of Single phase inverter system in Standalone mode. A LCL filter is commonly used for filtering the output current ripple in standalone mode.

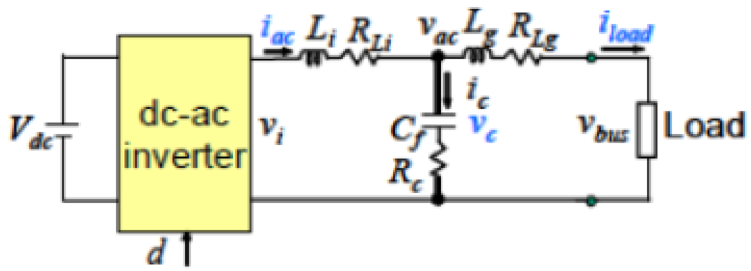

Figure 1. Block diagram of single phase inverter in standalone mode

Three state variables: v_c, i_{load} and i_{ac} are inverter side, capacitor voltage load current and inductor current respectively. d is the excitation signal. If the load is only a pure resistor with resistance Rload, then the plant equation can be expressed as: [12]

$$\begin{bmatrix} 0 & 0 & C_f \\ 0 & L_g & -R_f C_f \\ L_i & 0 & R_f C_f \end{bmatrix} \begin{bmatrix} \frac{dV_c}{dt} \\ \frac{di_{load}}{dt} \\ \frac{diac}{\frac{di_{ac}}{dt}} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -R_{Lg}-R_{load} & 1 \\ R_{Li} & 0 & -1 \end{bmatrix} \begin{bmatrix} V_c \\ i_{load} \\ i_{ac} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ V_{dc} \end{bmatrix} [d] \quad (1)$$

The state equation of Single Inverter model in Standalone mode can be obtained by assuming:
L_g = 40mH, L_i = 50mH, C_f = 0.02F, R_c =10Ω, , R_{Lg} = 6Ω, R_{Li} = 5Ω, R_{Load} = 13.5Ω

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{di_{load}}{dt} \\ \frac{diac}{\frac{di_{ac}}{dt}} \end{bmatrix} = \begin{bmatrix} 20 & -25 & -450 \\ 0 & 487.5 & -482.5 \\ 100 & 0 & -1050 \end{bmatrix} \begin{bmatrix} V_c \\ i_{load} \\ i_{ac} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 8700 \end{bmatrix} [d] \quad (2)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_{load} \\ i_{ac} \end{bmatrix} \quad (3)$$

where,

$$A = \begin{bmatrix} 20 & -25 & -450 \\ 0 & 487.5 & -482.5 \\ 100 & 0 & -1050 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 8700 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

2.2 Grid-tie Mode

Here the inverter is feeding an L-C filter. State variables iL and vc are inverter side inductor current, and capacitor voltage respectively. VE is the excitation source.

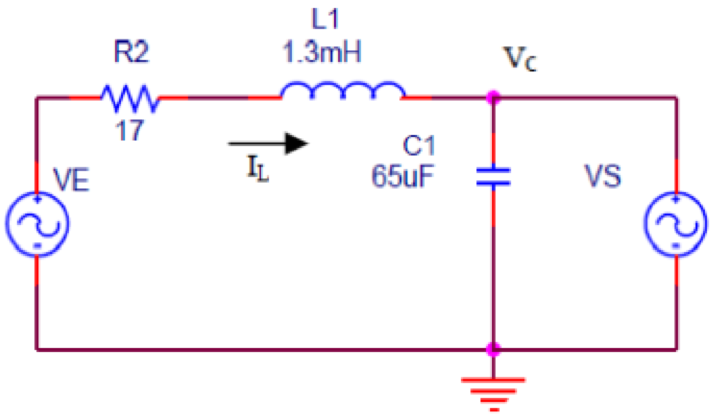

Figure 2. Block diagram of single phase inverter in standalone mode

The General State equation of Single phase Inverter in Grid-tie mode can be expressed as: [14]

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{d_{iL}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{-1}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} [V_E] \quad (4)$$

The state equation of Single Inverter model in Grid-tie mode can be obtained by assuming:

L = 1.3mH, R =1Ω, C = 65μF

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{d_{iL}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 15380 \\ -769.2 & -769.2 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 769.2 \end{bmatrix} [V_E] \quad (5)$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} \quad (6)$$

where,

$$A = \begin{bmatrix} 0 & 15380 \\ -769.2 & -769.2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 769.2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

3. Quadratic Optimal Regulator Systems

Considering optimal regulator problem, given system equation

$$\dot{x} = Ax + Bu \tag{7}$$

determines the matrix K of the optimal control vector

$$u(t) = -Kx(t) \tag{8}$$

so as to minimize the performance index

$$J = \int_0^\infty (x * Qx + u * Ru)dt \tag{9}$$

where
Q is a positive definite Hermitian or real systemmetric matrix and R is positive definite Hermitian or real symmetric matrix.
Note that the second term on RHS of above equation accounts for expenditure of the energy of control signals. The matrixes Q and R determines the relative importance of error and the expenditure of this energy.
Solving the optimization problem

$$\dot{x} = Ax = BKx = (A - BK)x \tag{10}$$

Assuming that the matrix A-BK is stable,

$$\begin{aligned} J &= \int_0^\infty (x * Qx + x * K * RKx)dt \\ &= \int_0^\infty x * (Q + K * RK)xdt \end{aligned} \tag{11}$$

Setting

$$x * (Q + K * RK)x = \frac{d}{dt}(x * Px) \tag{12}$$

where P is positive definite Hermitian or real symmetric matrix.

$$x * (Q + K * RK)x = -\dot{x} * Px - x * P\dot{x} = -x * [(A - BK) * P + P(A - BK)]x \tag{13}$$

Comparing both sides of above equation and noting that this equation must hold good for any x,

$$(A - BK) * P + P(A - BK) = -(Q + K * RK) \tag{14}$$

Now the performance index J can be evaluated as

$$J = \int_0^\infty x * (Q + K * RK)x\, dt = -x * Px \tag{15}$$

Since all eigen values of A-BK are assumed to have negative real parts, we have x(∞) =0.
Therefore we obtain

$$J = x(0)*Px(0)$$

Thus, the performance index can be obtained in terms of initial condition x(0) and P.
The solution to the quadratic optimal control problem can be obtained as follows:
Since r has been asuumed to be positive definite Hermitian or real symmetric matrix, we can write R= T*T
where T is non singular matrix. Then equation can be written as

$$(A - K * B) * P + P(A - BK) + Q + K * T * TK = 0$$

Which can be re written as

$$A*P+PA+[TK-(T)^{-1}BP]*[TK-(T)^{-1}B*P]-PBR^{-1}B*P+Q=0 \quad (16)$$

The minimization of J with respect to K requires minimization of

$$x*[TK-(T)^{-1}B*P]*TK-(T)^{-1}B*P]x$$

with respect to K. Since this equation is non negative, the miminum occurs when it is zero, or when

$$TK = (T)^{-1}BP$$

Hence

$$K=(T)^{-1}(T)^{-1}B*P=R^{-1}B*P \quad (17)$$

This equation gives the optimal matrix K.
The optimal control law to quadratic optimal control problem when the performance index is given by equation

$$u(t)=-Kx(t)=R^{-1}B*Px(t) \quad (18)$$

The matrix P in above equation must satisfy Equation or following reduced equation:

$$A*P+PA-PBR^{-1}B*P+Q=0 \quad (19)$$

Note, if performance index is given in terms of output vector rather than state vector,i.e

$$J = \int_0^\infty (y * Qy + u * Ru)dt$$

Then the index can be modified by using output equation Y=Cx to

$$J = \int_0^\infty x * (Q + K * RK)x \, dt = -x * Px \quad (20)$$

And the design steps presented above can be applied to obtain optimal matrix K.

4. Design of Quadratic Optimal Regulator System with MATLAB

In MATLAB, the command for designing Quadratic Optimal Regulator System is

lqr(A,B,C,D)

This solves continuous time linear quadratic regulator problem and associated Riccati equation.
Standalone Mode:

In the previous session, the state space model of single phase inverter has already been derived in Standalone Mode:

$$A = \begin{bmatrix} 20 & -25 & -450 \\ 0 & 487.5 & -482.5 \\ 100 & 0 & -1050 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 8700 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Using the above MATLAB command, Quadratic Optimal Regulator System is designed for Single phase Inverter system in Standalone Mode as shown:

```
A=[20 -25 -450;0 -487.5 -482.5;100 0 -1050];
B=[0;0;8700];
C=[1 0 0];
D=[0];
Q=[100 0 0;0 1 0;0 0 1];
R=[0.01];
K=lqr(A,B,Q,R)
k1=K(1);k2=K(2);k3=K(3);
AA=A-B*K;
BB=B*k1;
CC=C;
DD=D;
step(AA,BB,CC,DD)
```

Grid tie Mode:

In the previous session, the state space model of single phase inverter has already been derived in Grid tie Mode:

$$A = \begin{bmatrix} 0 & 15380 \\ -769.2 & -769.2 \end{bmatrix} B = \begin{bmatrix} 0 \\ 769.2 \end{bmatrix} C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Using the above MATLAB command, Quadratic Optimal Regulator System is designed for Single phase Inverter system in Grid tie Mode as shown:

```
A=[0 15380; -769.2 -769.2];
B=[0;769.2];
C=[1 0];
D=[0];
Q=[100 0;0 1];
R=[0.01];
K=lqr(A,B,Q,R)
k1=K(1);
AA=A-B*K;
BB=B*k1;
CC=C;
DD=D;
step(AA,BB,CC,DD)
```

5. Simulation Results

The unit step response of the Designed Quadratic Optimal system is plotted. We can observe in both cases that the designed system is stable both in Standalone mode and Grid-tie mode.

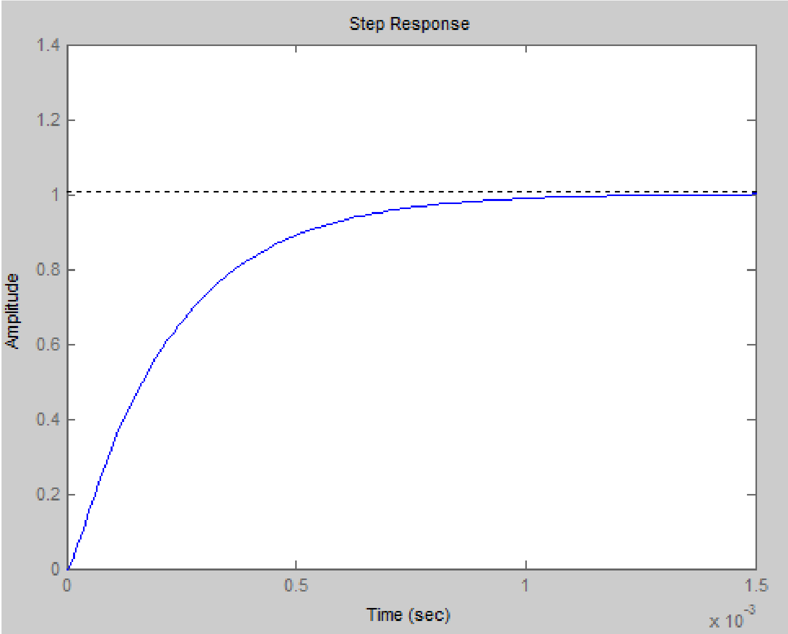

Figure 3. Unit Step of Designed Quadratic Optimal Regulator System in Standalone Mode

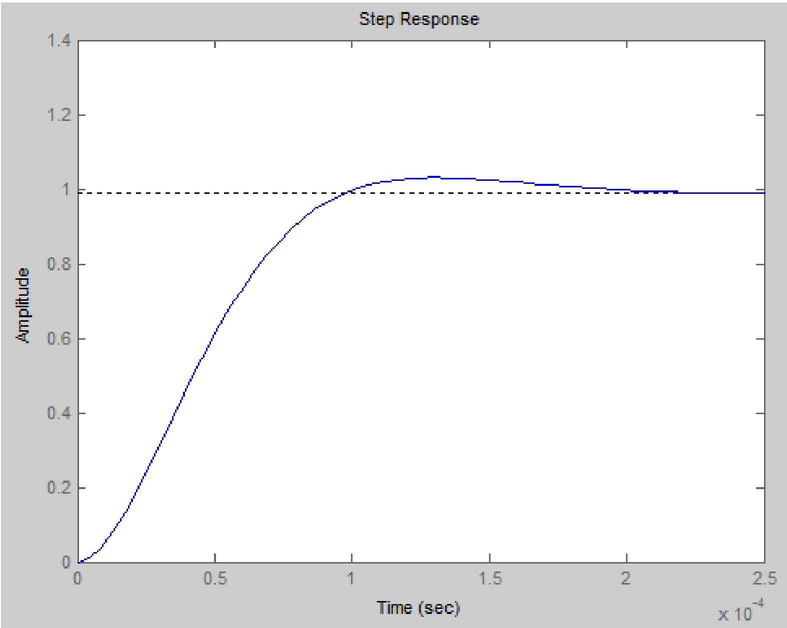

Figure 4. Unit Step of Designed Quadratic Optimal Regulator System in Grid tie Mode

5. Conclusion

For any given any initial state x(t), the optimal regulator problem is to find an allowable control vector u(t) that transfers the state to desired region of state space and for which the performance index is minimized. For existence of an optimal control vector u(t), the system must be completely state controllable. The system that minimizes the selected performance index is optimal. Although the controller may have nothing to do with optimality in many practical applications, design is based on quadratic performance index yields a stable control system. The characteristic of an optimal control law based on quadratic performance index is that it is a linear function of state variables i.e to feedback all state variables. This requires that all such variables be available for feedback. If not all state variables are available for feedback, then we need to employ a state observer to estimate ummeasurable state variables and use estimated values to generate optimal control signals.

In this paper State space model of single phase inverter is designed both in Standalone and Grid tie mode. An Quadratic Optimal Regulator system is designed for the inverter system both in Standalone mode and Grid tie mode.

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