

Development of stability charts for double salience reluctance machine modeled using hill's equation

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ABSTRACT

The paper presents a novel algorithm for the development of stability charts. The second-order differential homogeneous equation describing a double salient reluctance machine with a capacitance connected to its stator winding is transformed into hill's equation. The circuit components are the stator coil time-varying inductance of a double salient reluctance machine, capacitance and resistance. All these are modeled by hill's equation. The double salient reluctance machine acts as an energy conversion system. The maximum and minimum inductance of the energy conversion system is measured in laboratory by inductance, capacitance, and resistance (LCR) meter. These values help to determine the inductance modulation index. The inductance modulation index, the characteristic constant and the characteristic parameter obtained from modeling equations are used in the MATLAB/Simulink model. The MATLAB/Simulink simulations generate stable and unstable oscillations to form stability charts. The proposed stability charts are in good agreement with the Ince-Strutt stability chart, which is widely applied in physics, mechanics and in electrical engineering, especially where the state of stability of a system or an electric oscillatory circuit is to be determined.

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1. INTRODUCTION

This paper discusses a double-salient reluctance machines whose stator coil of time-varying inductance is connected in parallel to a capacitance and a resistor to form an oscillator circuit. The electric oscillatory circuit is modeled by differential homogeneous equation. This equation is later transformed into hill's equation or mathieu's equation in canonical form [1]. The applications of mathieu's equation are discussed in [2]. These oscillations form a stability chart when characteristic constants are plotted against characteristic parameters or inductance modulation indices respectively. This type of stability chart does not involve voltage fluctuation in electrical system. Typical examples these stability charts using differential homogeneous equation can be observed in mechanical system such as the vibration of an elliptical membrane of the drum head [3]. In an electric oscillatory circuit, one of the energy storage components (resistor, capacitor, and inductor) is modulated with time at a certain frequency. Parametric oscillations could be

observed in the oscillation of current due to the periodic movement of capacitance close and far apart and the movement of inductance in and out of the coil. These are parametric excitations of oscillations owing to the periodic modulation of the parameters in oscillating systems. The frequency at which a system oscillates or vibrates in the absence of damping is known as its natural frequency. Most parametric oscillators generate stable and unstable oscillations to form stability charts. The instability phenomenon in parametric resonance phenomenon results in the development of stability charts known as resonance tongue. The stability chart or Strutt-diagram for the mathieu's equation and the write up programs in MATLAB code was written in [4], [5]. The genetic algorithm of non-linear electric circuit of an oscillator is explained in [6]. The Floquet theory, matrix representation, meeissner equation and the widely use hill's equation resulting in the formation of stable and unstable regions of instability chart are discussed in [7]. A MATLAB/Simulink model is developed to monitor and display reactor parameters in order to protect reactor in triga puspati reactor system from unforseen disastrous consequences during instability phenomenon [8]. The oscillation at the natural frequency develops a simple harmonic motion. Electric circuit components are modeled and analyzed using the hill's equation and the theory of oscillations and systems are put forward in [9]. Parametric oscillation of an electrostatically actuated microbeam using variational iteration method (VIM) leads to the achievement of the stable and unstable regions of the stability chart in [10]. During parametric resonance, the instability phenomena are developed known as resonace tongues which consists of stable and unstable regions that formed stability chart [11].

Improvement in the instability of unsymmetric composite plates with SMA using FEM is carried out in [12]. Prikhodko *et al.* [13] presented the oscillations of modulated in amplitude and frequency as solutions of homogeneous mathieu equations. The results on riccati, Floquet theory and applications leading to the stable and unstable regions are discussed in [14]. An alternating current generator is modeled with RLC of time-varying inductance using Floquet's theory [15]. Floquet theory studies the stability analysis of linear differential equations with periodic coefficients and nonlinear systems of nontrivial periodic solutions [16]. The resonance torques whose stable and unstable regions (tongues) are separated by transition curves are presented by [17]. The power swing oscillation of the nonlinear stability indices was investigated in [18]. The degree of freedom is the number of independent parameters that determine the state of a system is presented in [19]. Nonlinear oscillation was solved using the pioncare method in [20] and instability in autoparametric resonance is discussed in [21]. The results of the mathieu equation explain the energy transfer in dusty plasma, as found in [22]. Resonance due to force oscillations for different damping forces and quality factor is demonstrated resulting into linear and nonlinear damping in [23]. The stability region of Ince-Strutt is analytically expressed in [24]. Shoshani and Shaw [25] presents the dynamic response of a system subjected to parametric resonant excitation in nonlinearities and modulated in a time-periodic manner. The transformation of homogeneous differential equation into hill equation and the features and overview of stability charts are given by [26]. Parametric instability on a shaft is experienced when a disc is attached on the shaft at high speed thereby resulting in stable and unstable region of the stability chart, though not related to Ince-Strutt stability chart [27]. Unlike susceptibe exposed infections recovered model for predicting the covid 19 spread in Jordan [28], this paper deals with parameters causing sinusoidal waveforms and nonsinusoidal waveforms in electric oscillatory circuit. Due to variation in weather conditions, instability of the magnitude of the energy generated by the renewable energy source is reduced or increased but the altenating voltage and current remain sinusoidal [29]. The stability property of periodic solutions of hill's equation is in the non-homogeneous situation supported by floquet lyapunov theory is studied. The stability chart changes due to the external periodic signal through the addition of the resonance lines [30].

However, this paper is organized in 5 sections. Section 1 presents the introduction and literature review. Section 2 presents the proposed algorithm and section 3 present the method used to obtain results. Sections 4 and 5 present results and discussion and conclusion respectively.

2. PROPOSED ALGORITHM

Most published research journals used the write-up programs developed by Ince-Strutt for their stability charts in their journals. These stability charts are bounded by the transition curves. This paper proposes a novel algorithm that will develop stability charts which are simple and understandable. To achieve this, an electric oscillatory circuit consisting of a time-varying inductance, capacitance and resistance are modeled as a homogenous differential equation and transformed into hill's equation. Assumptions are made to transform the modeled equation into mathieu's equation in canonical form. Then, MATLAB/Simulink is developed from hill's equation or mathieu's equation in canonical form. Tables are drawn to evaluate the state of stability for different values of the characteristic constants and characteristic parameters. The values of the characteristic constants are plotted against the characteristic parameter, and vice versa. It is found that the regions of stable and unstable regions of the proposed stability charts are the same as those of the Ince-Strutt. This novel approach enables us further to determine the maximum and minimum inductance of a

fabricated double salient reluctance machine in order to obtain inductance modulation index. This leads to the development another stability chart for inductance modulation index and characteristic constant.

3. METHOD

The equivalent circuit of electric oscillatory circuit consists of the parallel connection of the time-varying inductance of the stator coil, capacitance and resistance. The direction of current flowing through the time-varying inductance, capacitance and the resistance are i_L , i_C , and i_R respectively. These currents are shown in equivalent circuit of the oscillator in Figure 1. This is followed by circuit modeling equations. Some assumptions are made in order to transform second-order homogeneous differential equation into hill's equation or mathieu's equation in canonical form.

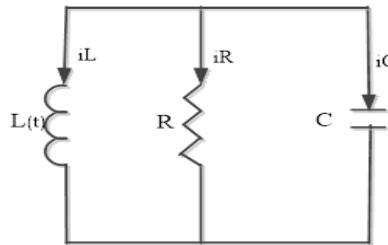


Figure 1. Equivalent circuit of an oscillator

3.1. Modeling equations

The time-varying inductance is given by:

$$L(t) = L_o(1 + m \cos(P_r \theta_r)) \quad (1)$$

where L_o is the average inductance, m is the inductance modulation index which is the ratio of the difference between the maximum and minimum inductance to the sum of the maximum and minimum inductance, P_r is the number of rotor poles and θ_r is the angular rotor position in degree.

The differential equation describing the flux linkage of the equivalent circuit of an oscillator in Figure 1 is given by:

$$\frac{\lambda}{L(t)} + \frac{1}{R} \frac{d\lambda}{dt} + C \frac{d^2\lambda}{dt^2} = 0 \quad (2)$$

where λ is the flux linkage, R is the total stator circuit resistance, and C is the capacitance. In (2) is further transformed and we obtain.

$$\frac{d^2\lambda}{dt^2} + \frac{1}{RC} \frac{d\lambda}{dt} + \omega^2(1 - m \cos P_r \theta_r)\lambda = 0 \quad (3)$$

In parallel connection $\frac{\omega}{Q} = \frac{1}{RC}$, natural frequency $\omega = \frac{1}{\sqrt{L_o C}}$, and Q is the circuit quality factor. Substitute for RC from (3):

$$\frac{d^2\lambda}{dt^2} + \frac{\omega}{Q} \frac{d\lambda}{dt} + \omega^2(1 - m \cos P_r \theta_r)\lambda = 0 \quad (4)$$

where m is the inductance modulation index, which is the ratio of the difference between the maximum and minimum inductance to the sum of maximum and minimum inductance. In this case a double salient reluctance machine is fabricated and wound with coils as shown in Figure 2. The maximum and minimum inductances are investigated and determined experimentally to be 7.89 mH and 0.97 mH respectively.

The hill's equation is expressed as:

$$\frac{d^2y}{dt^2} + Ky = 0 \quad (5)$$

$\frac{\omega}{Q} = \frac{1}{RC}$ and K in (5) is a periodic time-varying parameter $\frac{d\lambda}{dt}$ is changed to another variable $\frac{dy}{dt}$



Figure 2. 2/2 double salient reluctance machine

In (4) is transformed into hill's equation in the form of:

$$\frac{d^2y}{dt^2} + \omega^2 [1 - (\frac{1}{4Q^2}) - m \cos 2z]y = 0 \quad (6)$$

Introduction K_1 into (6):

$$\frac{d^2y}{dz^2} + K_1^2 [\omega^2 - (\frac{\omega}{2Q})^2 - \omega^2 m \cos 2z]y = 0 \quad (7)$$

The values of K_1 and z are defined as (8):

$$\theta = \omega t, \quad 2z = P_r \omega_r t, \quad z = \frac{P_r \omega_r t}{2} \quad \text{and} \quad K_1 = \frac{2}{P_r \omega_r} \quad (8)$$

Where ω_r is the angular speed of the rotor, z is time-dependent variable and this time in seconds

$$\frac{d^2y}{dz^2} + (\frac{2\omega}{P_r \omega_r})^2 [1 - (\frac{1}{4Q^2}) - m \cos 2z]y = 0 \quad (9)$$

In (8), we found out that:

$$a = (\frac{2\omega}{P_r \omega_r})^2 \{1 - \frac{1}{4Q^2}\} \quad \text{and} \quad a = (\frac{2\omega}{P_r \omega_r})^2 \quad \text{for} \quad Q \gg 1 \quad (10)$$

In (9) is finally expressed as (11):

$$\frac{d^2y}{dz^2} + (\frac{2\omega}{P_r \omega_r})^2 [1 - m \cos 2z]y = 0 \quad (11)$$

From (10), $a = (\frac{2\omega}{P_r \omega_r})^2$, so that (10) is transformed into mathieu's equation as (12):

$$\frac{d^2y}{dz^2} + a[1 - m \cos 2z]y = \frac{d^2y}{dz^2} + (a - ma \cos 2z)y = \frac{d^2y}{dz^2} + (a - 2q \cos 2z)y = 0 \quad (12)$$

where $2q = (\frac{2\omega}{P_r \omega_r})^2 = ma$ and if $a \approx 1$, then $2q = m$

3.2. Experimental investigation using characteristic constant a, and characteristic parameter q

The characteristic constant-a and the characteristic parameter-q are investigated through the developed MATLAB/Simulink model. Different values of the characteristic constant and characteristic parameters are inputted in the developed MATLAB/Simulink model. It is simulated each time a characteristic constant and a characteristic parameter are inputted. The state of stability of the two parameters is displayed on the scope. The signal displayed is of the form of parametric oscillation of different types. Each oscillation is the output of the two input parameters. In each case the output signal is either stable oscillation (S) or unstable oscillation (U). A table is made which consists the state of the stability of the corresponding values of characteristic constant and characteristic parameter. Hence, the state of stability of a given characteristic constant and characteristic parameter is determined. The characteristic constant is plotted against characteristic parameter to form a-q stability chart. The quantity z is a time-dependent parameter. In (12), $ma=2q$. The MATLAB/Simulink model developed for a-q stability chart is shown Figure 3.

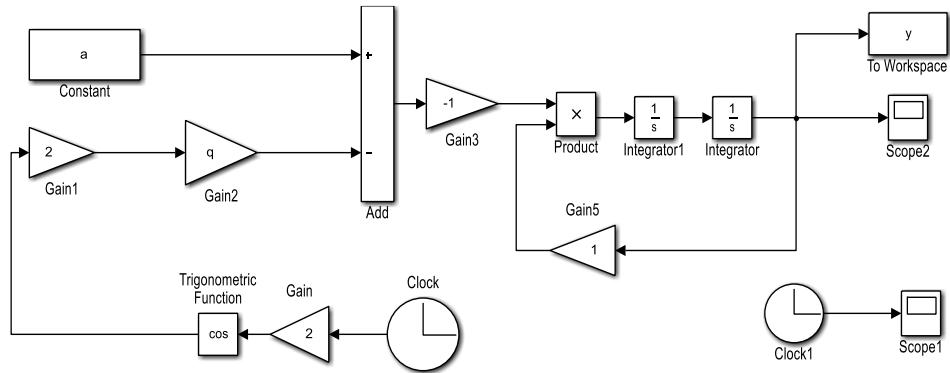


Figure 3. MATLAB/Simulink for a and q

3.3. Experimental investigation using characteristic constant a, and inductance modulation index m

The MATLAB/Simulink model developed for characteristic constant, a and inductance modulation index, m is shown in Figure 4. The inductance modulation index is the ratio of the difference between the maximum and minimum inductance to the sum of the maximum and minimum inductance. For every corresponding characteristic constant and inductance modulation input in the MATLAB/Simulink, the simulink is simulated. The output signal is either stable or unstable oscillation. The values of characteristic constant and the inductance modulation index are plotted against each other to produce a stability chart.

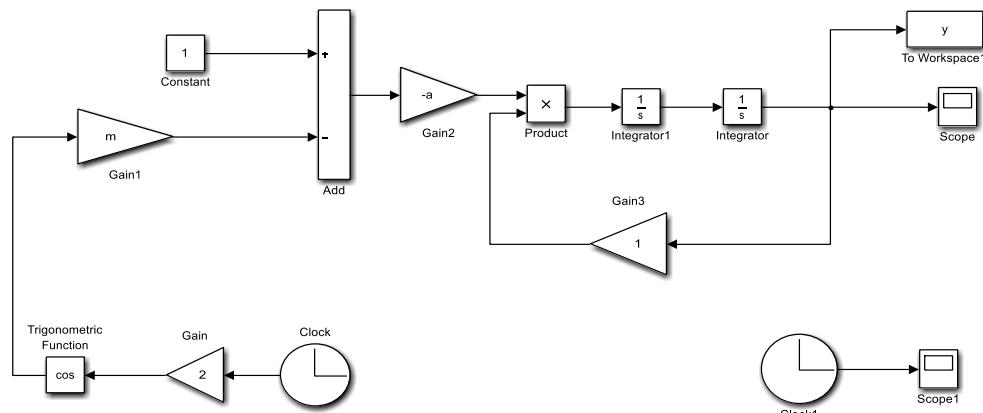


Figure 4. MATLAB/Simulink model for a and m

Stable oscillation is denoted by (S) while the unstable oscillation is denoted by (U) when a ranges from 0 to 1 and q ranges from 0 to 1 (Table 1). Table 2 shows the state of the computed values of a and q respectively as a ranges from 0 to 16 and q ranges from 0 to 14. If the oscillation type is stable, it is denoted by (S), and when it is unstable, it is denoted by (U).

Table 1. Stable oscillation (S) and unstable oscillation (U) for a=0 to 1 and q=0 to 1

a	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	U	U	U	U	U	U	U	U	U	U
0.2	U	U	U	U	U	U	U	U	U	U
0.3	S	U	U	U	U	U	U	U	U	U
0.4	S	S	U	U	U	U	U	U	U	U
0.5	S	S	S	U	U	U	U	U	U	U
0.6	S	S	S	S	U	U	U	U	U	U
0.7	S	S	S	S	S	U	U	U	U	U
0.8	S	S	S	S	S	S	U	U	U	U
0.9	S	S	S	S	S	S	S	U	U	U
1.0	S	S	S	S	S	S	S	S	U	U

Table 2. Stable oscillation (S) and unstable oscillation (U) for $a=0$ to 16 and $q=0$ to 14

q	14	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
13	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
12	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
11	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	S
10	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	S
9	U	U	U	U	U	U	U	U	U	U	U	U	U	U	S	S	S
8	U	U	U	U	U	U	U	U	U	U	U	U	U	U	S	S	S
7	U	U	U	U	U	U	U	U	U	U	U	U	U	U	S	S	S
6	U	U	U	U	U	U	U	S	S	U	U	U	S	S	S	S	S
5	U	U	U	U	U	U	S	S	S	U	S	S	S	S	S	S	S
4	U	U	U	U	U	S	S	S	S	S	S	S	S	S	S	S	S
3	U	U	S	U	U	S	S	S	S	S	S	S	S	S	S	S	S
2	U	U	S	U	U	S	S	S	S	S	S	S	S	S	S	S	S
1	U	S	S	U	S	S	S	S	S	S	S	S	S	S	S	S	S
a	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	

4. RESULTS AND DISCUSSION

When $a=0.6$ and $q=0.1$ the point on the stability chart is stable. The stable oscillation generated from MATLAB/Simulink is shown in Figure 5(a). The oscillation type generated in Figure 5(b). When $a=0.1$ and $q=0.1$ is stable but in Figure 5(c) when $a=0.8$ and $q=0.2$ the oscillation is unstable.

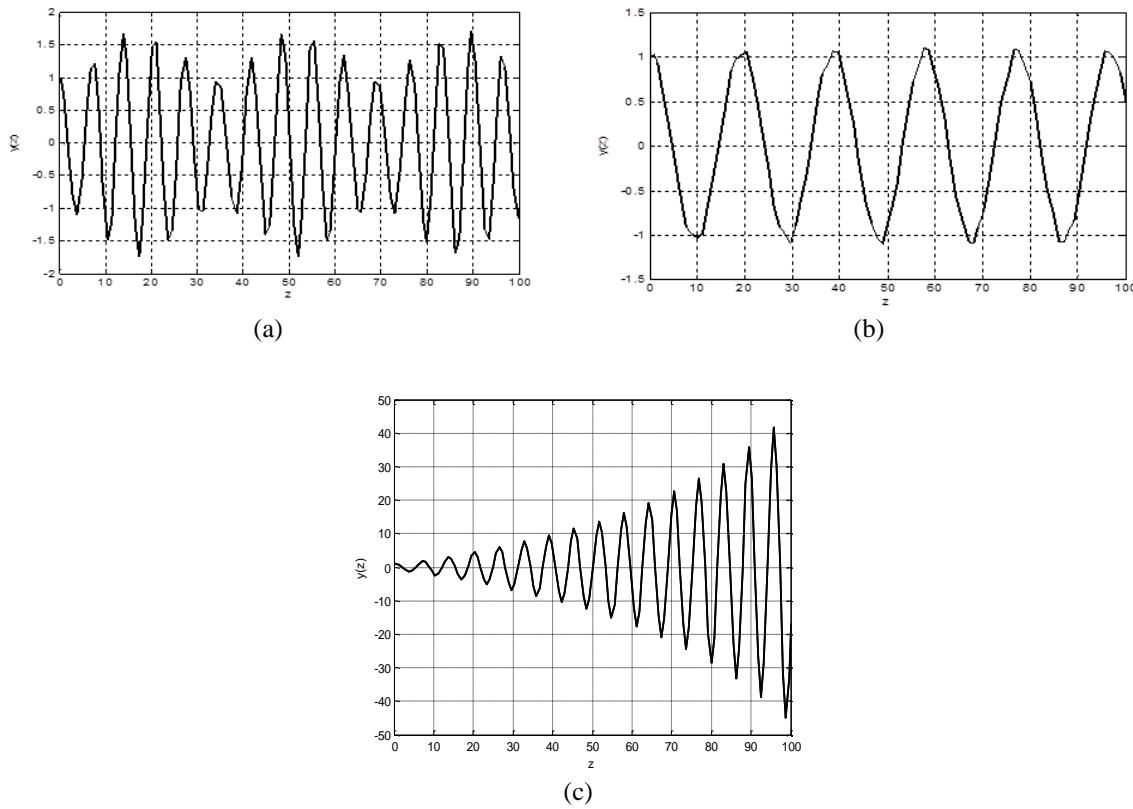


Figure 5. Oscillations; (a) stable, (b) stable when $a=0.1$ and $q=0.1$, and (c) unstable when $a=0.8$ and $q=0.2$

The computed results of the characteristic constants and the characteristic parameters are shown in Figure 6. It shows the proposed stability chart developed by using MATLAB/Simulink and the stability chart developed by Ince-Srutt using a write up MATLAB program. The dotted line is by MATLAB/Simulink while the plane line or the non-dotted line is the stability chart by Ince-Srutt. Figure 7 shows stable and unstab regions of characteristic constant or characteristic number from 1 to 6 and the inductance modulation index ranges from 0.1 to 1.

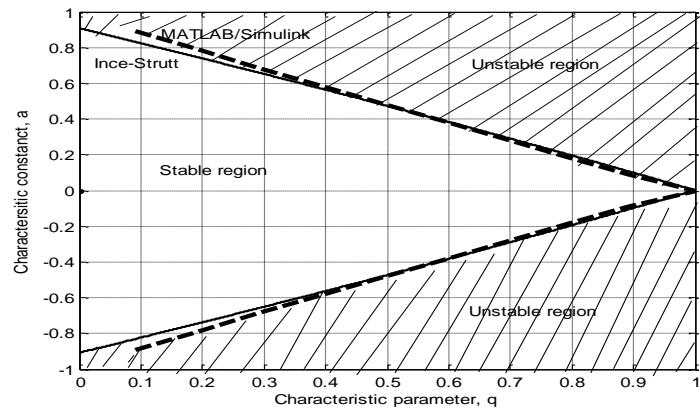
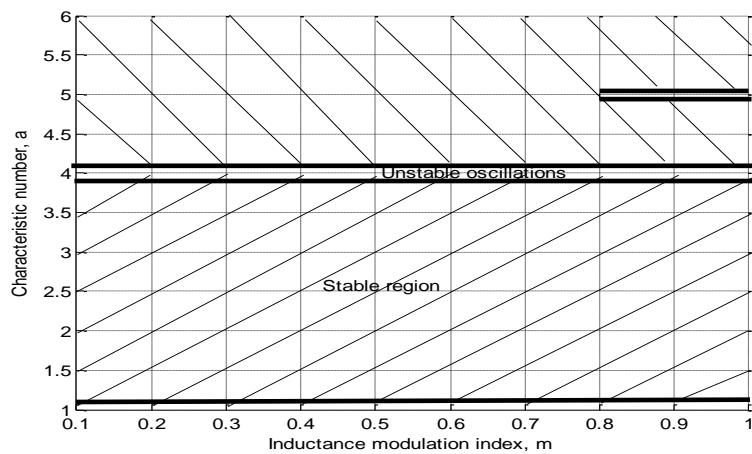
Figure 6. Stability charts when $a=0$ to 1 and $q=0$ to 1

Figure 7. Characteristic constant versus inductance modulation index

Figure 8(a) shows the proposed stability chart of characteristic number versus characteristic parameter using MATLAB/Simulink, the instability regions are indicated by 'x' while the plane region is the stability region. Figure 8(b) shows the stability chart developed from write up MATLAB program of the Ince-Strutt which is generally known as Ince-Strutt diagram. Stable regions are planne while the unstable regions are marked with with lines. The proposed stability chart using MATLAB/Simulink when the characteristic constant a , is plotted against inductance modulation index m , is shown in Figure 9.

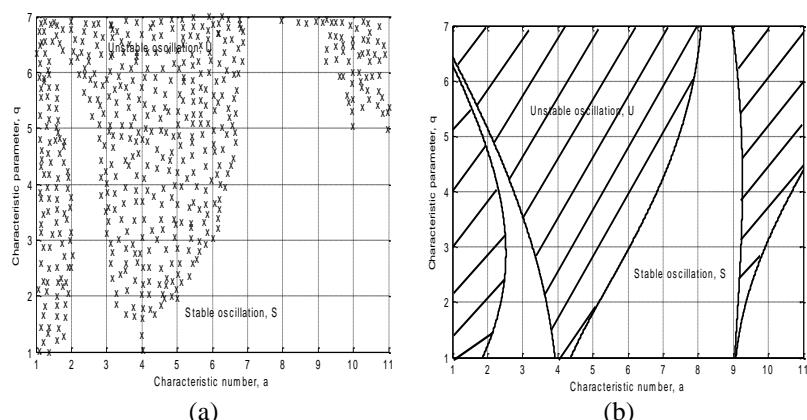
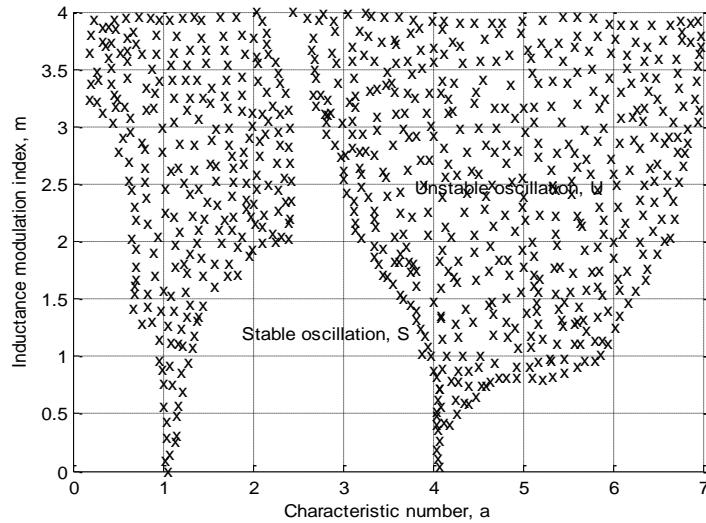


Figure 8. Proposed stability; (a) chart and (b) Ince-Strutt

Figure 9. Proposed stability chart using a and m

5. CONCLUSION

MATLAB/Simulink models are developed for the characteristic constant versus characteristic parameter and the characteristic constant versus inductance modulation index in stability charts. The proposed algorithm provides a clear understanding of stability chart for students whose areas of research are on parametric oscillation or parametric resonance. The state of stability of a given characteristic constant and a characteristic parameter is determined using MATLAB/Simulink, which is absent in the Ince-Strutt write up MATLAB program. The Ince-Strutt diagrams are bounded by the transition curves while in the proposed algorithm, the stability charts are plotted from results developed showing the state of stability of two independent variables (parameters) at a given point. The proposed stability charts are in agreement with those developed by Ince-Strutt known as Ince-Strutt stability charts.

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