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Stability analysis of a closed non-linear system "FC-BM" of the electric drive of an electric vehicle

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ABSTRACT

The article presents the analysis and stability program of the closed non-linear system frequency converter-brushless motor "(FC-BM)", which differs significantly from the analysis of linear systems. First of all, this is because the stability property of a nonlinear system depends on the initial conditions and external influences: for some input signals, the system will be stable, while for others it becomes unstable. Consequently, the stability criteria developed in the linear theory cannot be applied to their analysis. The stability of a non-linear automatic control system means that small changes in the input signal or disturbances, initial conditions, or plant parameters will not take the output variable beyond a sufficiently small neighborhood of the equilibrium point or limit cycle. Since several equilibrium positions can exist for a non-linear system, stability should be analyzed in the vicinity of each of them. This complicates the task of research.

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1. INTRODUCTION

System stability analysis is one of the most important stages in the design of control systems, however, in the analysis of non-linear systems, strictly speaking, there is no single method that meets the criteria of necessity and sufficiency, and the criteria are, as a rule, only sufficient (for stability). Based on this, for some systems, it is impossible to speak unambiguously about stability. That is why the study of the stability of the closed non-linear system frequency converter-brushless motor "(FC-BM)" is an understudied topic [1].

The problem of stability of non-linear systems has a relatively long and very interesting history of development. It should be noted that the main research topics were formed around the ideas of the Russian mathematician A. M. Lyapunov, who created in 1892 the theory of stability of non-linear systems. The behavior of non-linear systems is described by differential equations. In the future, the study of non-linear systems was developed in articles by domestic and foreign scientists. However, methods for analyzing the stability of non-linear systems usually provide sufficient conditions, so it is impossible to introduce the concept of stability margin for them, which is used in the linear case. In classical control theory, there are two main analytical methods: the first and second Lyapunov methods, as well as a fairly large number of modifications of the second method, which has nothing to do with linearization. The second method of Lyapunov consists of a direct study of the stability of a non-linear system by determining such a function $V(x)$ of the coordinates of a point in the phase space of the given system.

The second theory of Lyapunov on the stability of systems reads: if for an automatic control system with a differential equation of the nth order of perturbed motion, it is possible to choose such a sign-definite function $V(x)$, the derivative of which, by under these equations, would be a sign-definite function of the opposite sign with V, or identically equal to zero, then the unperturbed motion is asymptotically stable.

$$\begin{aligned} &V(x) = 0 \; if \; and \; only \; if \; x = 0 \\ &V(x) > 0 \; if \; and \; only \; if \; x \neq 0 \end{aligned} \tag{1}$$

The requirement for the stability of transient processes of the automatic control system is the main condition for the normal functioning of the control system [2]–[4]. For non-linear control systems, there is no single exact method for solving ordinary differential equations (ODEs) that describe the transient processes of the system, and for each type of non-linearity, one has to find a specific particular method [5]. A non-linear system is a dynamic system in which processes occur that are described by ordinary differential equations. Systems that do not obey the superposition principle are called non-linear systems. The system will be non-linear if it contains at least one link for which the superposition principle does not hold. It is important to emphasize that nonlinear systems, as shown in this paper, can also be studied at a qualitative level. Namely, such classical concepts as the bifurcation point suggest that the theory of qualitative analysis of systems of non-linear equations can give fruitful results even when the system of equations is either not solved at all or can be solved only by numerical methods. In modern conditions, this approach acquires a new sound due to the fact that for the analysis of complex systems, more and more often, methods of many-valued logic, are used. In particular, the works [6]–[8] show that under certain conditions, systems of this kind can be interpreted as information processing systems and, therefore, a new quality can appear in them. This work assumes a primary consideration of fairly simple systems, however, allowing a subsequent transition, including the analysis of complex systems, precisely from the point of view of information processing.

Superposition principle: if several input actions are applied to the input of an object, then the reaction of the object to the sum of the input actions is equal to the sum of the reactions of the object to each action separately. In non-linear systems, the stability of the equilibrium state depends on the initial conditions; therefore, various types of motion are possible depending on the initial conditions. Persistent oscillations during free motion of a linear system are possible only if the system is on the stability boundary, and the oscillation amplitude is determined by the initial conditions. In non-linear systems, undamped oscillations are possible, the amplitude and frequency of which do not depend on the initial conditions and are determined by the properties of the system-self-oscillations (a clock or an electric bell). Self-oscillations are observed under certain conditions in automatic control systems containing non-linear elements.

To determine the stability of the closed system "FC-BM", the phase space method was chosen [9]–[11]. This method makes it possible to obtain a clear picture of the transient processes of the automatic control system and to determine the stability of the control system by phase trajectories. Structural diagram of a closed non-linear system "FC-BM" is shown in Figure 1.

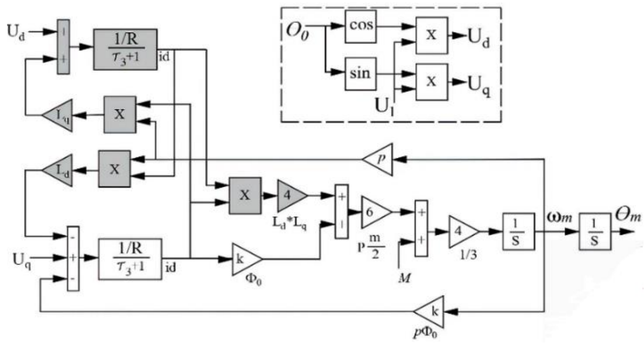

Figure 1. Structural diagram of a closed non-linear system "FC-BM"

2. MATERIALS AND METHOD

The study of non-linear automatic control systems is associated with the need to compose and solve non-linear differential equations. Since there is no single general solution for non-linear differential equations of various types, it is not possible to create a generalized control theory method for studying non-linear systems, similar to the case of ordinary linear systems. To study non-linear systems in the theory of automatic control, private methods have been developed and used, each of which has certain capabilities and is effective in a certain limited area of research problems. The method of phase trajectories and the amplitude-frequency method (method of harmonic linearization) find the greatest application.

One of the main methods for studying non-linear systems is the phase space method introduced into the theory of oscillations by academician A. A. Andronov. A phase space is such a space in which the rectangular coordinates of a point are quantities that determine the instantaneous state of the system. These quantities are called the phase coordinates of the system, their number is equal to the number of degrees of freedom of the system.

Phase coordinates can have any physical meaning (temperature, pressure, and concentration). Often, the output variable $y(t)$ and its time derivatives are chosen as phase coordinates. The movement of the representative point along the phase plane draws a line called the phase trajectory.

The phase trajectory method is a graph-analytical method for studying non-linear systems. The essence of the method lies in describing the behavior of systems using visual geometric representations-phase portraits. The system of equations for the motion dynamics of a brushless motor, compiled based on the transfer functions (TFs) of the brushless motor, is a free motion of a non-linear dynamic control system with three output values $x(t)$.

A differential equation is an equation that, in addition to a function, contains its derivatives. A differential equation based on a higher than the first can be transformed into a system of equations of the first order, in which the number of equations is equal to the order of the original differential equation. The system of equations for the dynamics of a brushless motor, compiled based on the TFs of the brushless motor, has the following form in the form of third-order non-linear differential as (2):

$$\frac{dx_1}{dt} = a_1 x_3 + a_2 x_2 x_3$$
$$\frac{dx_2}{dt} = -a_3 x_2 + a_4 x_1 x_3 \qquad (2)$$
$$\frac{dx_3}{dt} = a_5 u - a_6 x_1 - a_7 x_1 x_2 - a_8 x_3,$$

where $x_1 = \omega$–angular velocity of the brushless motor $x_2 = i_d$, $x_3 = i_q$ projections of the stator currents, u – setting action, $a_1 = 84{,}46$; $a_2 = 1{,}38$; $a_3 = 181{,}8$; $a_4 = 1{,}7$; $a_5 = 444{,}44$; $a_6 = 326{,}33$; $a_7 = 9{,}3$; $a_8 = 427{,}35$.

The authors proposed the following structural diagram of a closed system “FC-BM” (see Figure 2). In Figure 2, a block diagram of a closed system “FC-BM” consists of: a block diagram of a brushless motor, a frequency converter, which in the block diagram is considered as an inertial link with a TF:

$$W_{PR(S)} = \frac{k_{PR(S)}}{T_{PR(S)}+1} \qquad (3)$$

and a speed controller with a non-linear static link of the limitation type and a link for calculating the signal modulus $y = abs(x)$.

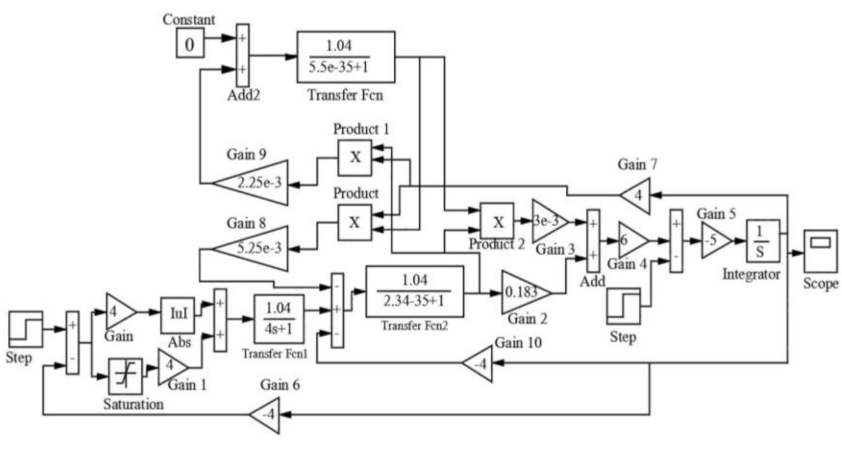

Figure 2. Structural diagram of a closed non-linear electric drive system “FC-BM” on MATLAB/Simulink

The brushless motor of the non-linear system "FC-BM" is the main element of the system; therefore, the stability of its movement should be investigated first of all. To ensure the stability of the closed system "FC-BM". It should be especially noted that the block diagram of a brushless motor contains multiplier links and positive and negative feedback. A structural diagram of the brushless motor model in a rotating coordinate system, with known motor parameters, is shown in Figure 3.

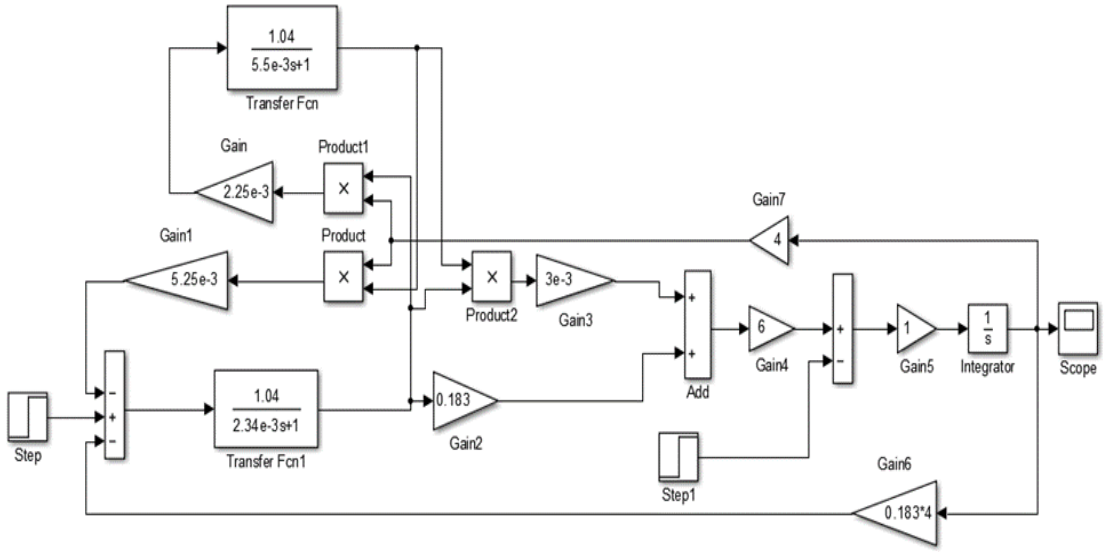

Figure 3. Structural diagram of the brushless motor model in a rotating coordinate system on MATLAB/Simulink

Figure 4 shows the transient curve of the non-linear system "FC-BM". It can be seen from this graph that there is a slight fluctuation. But as can be seen from Figure 5 the speed transient curve of the brushless motor is obtained without overshoot and fluctuations. As described, the formation of the TF of the brushless motor is carried out according to the TFs of the dynamic links. This, in turn, is confirmed mathematically by determining the roots of the characteristic equation of the TF.

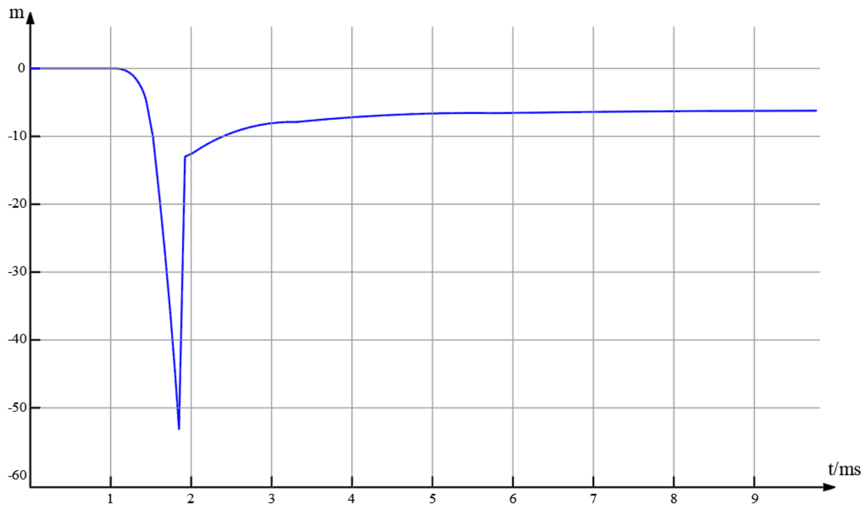

Figure 4. Curve of the transient process of a closed non-linear electric drive system "FC–BM" on MATLAB/Simulink (X-axis - torque of the "FC–BM" system, Y-axis - time in milliseconds)

Let us first consider the mathematical model of the brushless motor for the existence of self-oscillations. The stability of the motion of a brushless motor is determined using the MATLAB/Simulink program using the phase trajectory method. The method of phase trajectories lies in the fact that the behavior of the studied non-linear system is considered and described not in the time domain (in the form of equations of processes in the system), but in the phase space of the system (in the form of phase trajectories). On the phase plane, the self-oscillatory regime corresponds to a limit cycle. Self-oscillatory modes are observed in non-linear systems, therefore, the study of these modes, the identification of the conditions for their occurrence, the study of the parameters of self-oscillations (amplitude and period) are important. For real systems, the determination of self-oscillations is a difficult problem. Criteria can be used to show that there are no closed phase trajectories in the phase portrait of the system, i.e., no self-oscillations. There are various criteria for the absence of closed phase trajectories, which give sufficient conditions for the impossibility of the occurrence of self-oscillations.

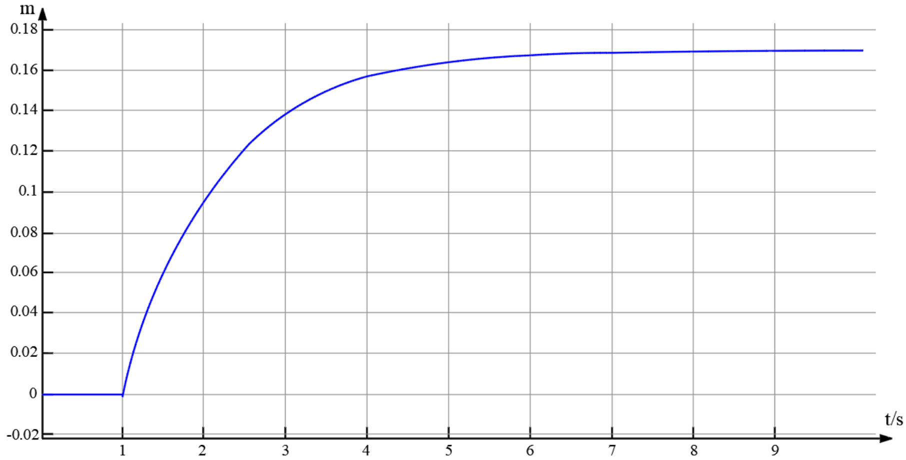

Figure 5. Curve of the transient process of the brushless motor on MATLAB/Simulink (X-axis - electromagnetic torque of the “FC–BM” system, Y-axis - time in seconds)

The program for determining self-oscillations and stability, shown in Figure 6, is compiled taking into account the standard function of the MATLAB odephas3 program, which provides the construction of a graph of the phase trajectory, the system of (1), in phase coordinates for a three-dimensional process [12]–[14] (4th line). The system of differential (1) in the program is presented in lines 11-13. The visualization of the phase trajectory of the “FC-BM” system is shown in Figure 7. The phase trajectory gives a qualitative assessment of all dynamic processes occurring in non-linear systems since they are not included in your time.

```
1  function PROGRAMMA
2  clc
3  y0 = [0;0;0];
4  options = odeset('OutputFcn', @odephas3);
5  [T,y] = ode113(@systm,[0 0.0077],y0,options);
6  grid on
7  function dy = systm(t,y)
8  dy = zeros(3,1); u=10;
9  a1=84.46; a2=1.38; a3=1.7; a4=181.8;
10 a5=444.44; a6=325.33; a7=9.33; a8=427.35;
11 dy(1)=a1*y(3)+a2*y(2)*y(3);
12 dy(2)=a3*y(1)*y(3)-a4*y(2);
13 dy(3)=a5*u-a6*y(1)-a7*y(1)*y(2)-a8*y(3);
14 end
15 end
```

Figure 6. Program for determining self-oscillation and stability

Figure 7 shows a phase trajectory, which shows the stability of the motion of a brushless motor without self-oscillations [15], [16]. However, the study of the stability of the closed system “FC-BM” by the method of phase trajectories is possible only when the number of differential equations does not exceed more than three equations. In this regard, we consider another approach to solving the problem of stability of the closed non-linear system “FC-BM” using the MATLAB/Simulink program with the number of differential equations $n > 3$. The equations of dynamics of the closed system “FC - BM” will be written in (4):

$$\begin{aligned} \frac{dx_1}{dt} &= a_1 x_2 + a_2 x_2 x_3 \\ \frac{dx_2}{dt} &= -a_3 x_4 - a_4 x_1 - a_5 x_1 x_3 - a_6 x_2, \\ \frac{dx_3}{dt} &= a_7 x_1 x_2 - a_8 x_3 \\ \frac{dx_4}{dt} &= a_9 (b_1 \cdot (u - b_2 x_1) + b_3 \cdot arctg(u - b_2 x_1)) - \frac{1}{T_p} x_4, \end{aligned} \tag{4}$$

where $a_1 = 84,46$; $a_2 = 1,38$; $a_3 = 444,44$; $a_4 = 325,33$; $a_2 = 1,38$; $a_6 = 427,35$; $a_7 = 1,7$; $a_8 = 181,8$; $a_9 = \frac{K_p}{T_p}$; $b_1 = 2$; $b_2 = 0,2$; $b_3 = 5$.

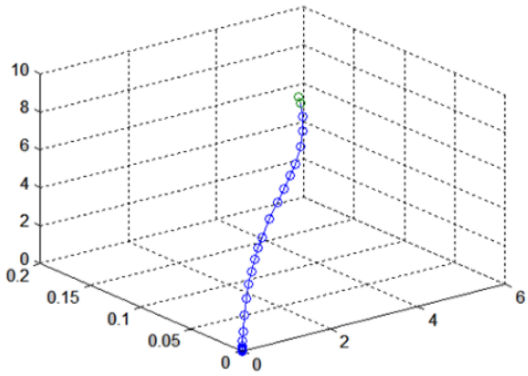

Figure 7. Phase trajectory of the closed system “FC-BM”

3. RESULTS AND DISCUSSION

The simplest method for studying non-linear systems is linearization. For the convenience of solving the stability problem for a closed non-linear “FC–BM” system in the MATLAB/Simulink environment, we use the method of harmonic linearization. The essence of the method is to replace a non-linear element of the system with an equivalent linear one, which converts harmonic oscillations in the same way as a non-linear element and is characterized by an equivalent complex gain.

Such a replacement makes it possible to study non-linear systems by frequency methods. In particular, using the frequency method, one can detect the presence of self-oscillations, investigate their stability and determine their amplitude and frequency, as well as solve the problems of correcting a non-linear system. Self-oscillations are undamped oscillations supported by an external source of energy, the supply of which is regulated by the oscillatory system itself. Self-oscillations are characterized by the following properties: i) they are not forced by any external periodic processes, but represent their own (free) movements of the system and ii) have an amplitude and frequency that do not depend on the initial conditions, but are determined solely by the parameters of the system.

They do not arise for any one set of values of the system parameters but are observed in some, usually quite wide, range of values of these parameters. That is, according to the harmonic linearization method, to determine the roots of the characteristic equation of the TF, we transform the system of (3) into symbolic equations, and in the brushless motor speed control system, we replace the function $y = arctg(x)$ two members of the Taylor series $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, -\infty < x < \infty$ [17] and function $y = abs(x)$ $by\, y = x^2/x$. In addition to this product of variables, in the system of (3) we replace $- x_2 \cdot x_3$ by $x_2 \cdot sign(x_3)$, $x_1 \cdot x_3$ by $x_1 \cdot sign(x_3)$ and $x_1 \cdot x_2$ by $x_2 \cdot sign(x_1)$ to linearize the product of variables and write in the program for determining self-oscillation and stability. Program for determining self-oscillation and stability is shown in Figure 8.

The calculation procedure is as follows:

a. Symbolic variables W$_i$ is introduced into the program for determining self-oscillations and stability (2nd line) and symbolic equations $f_i = 0$ (from the 4th to the 9th lines) according to the rules of the algorithmic language of the MATLAB/Simulink program.
b. On the 10th line of the program for determining self-oscillations and stability, the solve function in the MATLAB/Simulink environment calculates the TFs in symbolic form for each variable of the closed non-linear system "FC-BM".
c. Using the known parameters (lines 11 and 12) and the TFs of each link (lines 13 and 14), the process of forming the system TFs into a standard form is carried out. TFs are converted to standard form by the eval function (line 15).
d. The pole function calculates the roots of the characteristic equation of the TF of the closed non-linear system "FC-BM", the output variable is the angular velocity of the BM. The stability of the system is determined by the form of the roots of the characteristic equation.
e. The TF and the roots of the characteristic equation of the TF of the closed non-linear system "FC-BM" are shown in Figure 9.

```
1  function USTOI
2  syms w1 w2 w3 w4
3  clc
4  f1=sym('(1/w1)*x1-a1*x2-a2*(x2*sign(x3))');
5  f2=sym('a3*x1+w2*x2+a4*(x1*sign(x3))-a5*x4');
6  f3=sym('w3*x3-a6*(x2*sign(x1))');
7  fa=sym('a7*(((u-a8*x1)^2)/(u-a8*x1))');
8  fb=sym('a9*sign((u-a8*x1)-((u-a8*x1)^c)/c)');
9  f4=sym('w4*x4'-(fa+fb));
10 [x1,x2,x3,x4]=solve(f1,f2,f3,f4);
11 a1=84.46; a2=1.38; a3=325.33; a4=9.33;
12 a5=444.44;  a6=1.7; a7=2; a8=0.2; a9=5; c=3; u=10;
13 w1=tf([1],[1 0]); w2=tf([1 427.35],[0 1]);
14 w3=tf([1 181.8],[0 1]); w4=tf([1 1000],[0 1]);
15 R1=eval(x1); Wq=minreal(R1)
16 p=pole(Wq)
17 step(Wq*2,'k.-',0.5)
18 grid
```

Figure 8. Program for determining self-oscillation and stability

```
Transfer function:
                  9.538e005
--------------------------------------
s^3 + 1427 s^2 + 4.561e005 s + 2.874e007


p =

   1.0e+003 *

    -1.0000
    -0.3437
    -0.0836
```

Figure 9. The TFs and the roots of the characteristic equation of the TF of the closed non-linear system "FC-BM"

The closed system "FC-BM" is stable, since the roots of the characteristic equation p of the TF of the system turned out to have a negative real part [18]–[20]. However, the establishment of only one fact of the stability of the closed system "FC-BM" is not enough for the existence of normal operation of the system. In this regard, we will determine whether there are self-oscillations in the "FC-BM" system using the phase trajectory method. To do this, we transform the characteristic equation of the TF of the "FC-BM" system (Figure 8) into a normal system of differential equations, which can be written in (3):

$$\frac{dx_1}{dt} = x_2,$$

$$\frac{dx_2}{dt} = x_3, \qquad (5)$$

$$\frac{dx_3}{dt} = -a_1 u - a_2 x_3 - a_3 x_2 - a_4 x_1,$$

where x_i–phase coordinates of the closed system "FC-BM", u–disturbing influence, $a_1 = 9{,}538e05$; $a_2 = 1427$; $a_3 = 4{,}561e05$; $a_4 = 2{,}874e07$.

The program for calculating the phase trajectory according to the system of (4) is shown in Figure 10. The phase trajectory of the movement of the "FC-BM" system is shown in Figure 11. The phase trajectory of the closed non-linear system "FC-BM" shows the stability of the system and the absence of self-oscillations in the system [21]–[23].

The phase trajectory of movement shows the change in the controlled variable in the phase space. The phase trajectory in the phase space also gives a geometric representation of the dynamics of the process under study. However, there is no time coordinate. Time is displayed in an implicit form, so the phase trajectory does not give only the temporal characteristic of the system, but is a qualitative full dynamic characteristic [24], [25].

```
function PROGR
clc
x0 = [0;0;0];
options = odeset('OutputFcn', @odephas3);
[T,x] = ode113(@systm,[0 0.0075],x0,options);
grid on
function dx = systm(t,x)
dx = zeros(3,1); a1=9.538e5; a2=1427;
a3=4.561; a4=2.874e7; u=0.5;
dx(1)=x(2);
dx(2)=x(3);
dx(3)=a1*u-a2*x(3)-a3*x(2)-a4*x(1);
end
end
```

Figure 10. Program for calculating the phase trajectory of the "FC-BM" system

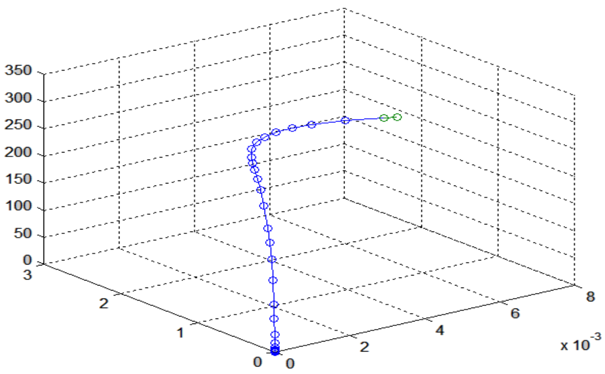

Figure 11. The phase trajectory of the movement of the "FC-BM" system

4. CONCLUSION

At present, in the study of complex dynamic processes observed in various branches of natural science, along with analytical methods, computer modeling is widely used. Systems of differential equations are widely used as mathematical models. The program for calculating the phase trajectory of the dynamics of a closed system "FC-BM" gives a complete picture of the stability of the system.

Non-linear differential equations of dynamics are transformed into a symbolic form. Thanks to this, the transfer function and the roots of the characteristic equation of the TF are found. The method of harmonic linearization is an approximate method for studying a non-linear system for the presence of self-oscillations.

The obtained transfer function of the block diagram of a brushless motor makes it possible, with the help of MATLAB/Simulink, to obtain transient and frequency characteristics and to study the quality of transients, which is especially important when designing a control system for a brushless motor. Modern high-speed computers effectively give a numerical solution to ordinary differential equations without requiring its solution in an analytical form. This allows some researchers to assert that the solution to the problem was obtained if it was possible to reduce it to the solution of an ordinary differential equation.

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