A study on the solution of interval linear fractional programming problem

Yamini Murugan, Nirmala Thamaraiselvan
Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur, India

Article Info

Article history:

Received Feb 9, 2023
Revised Jul 25, 2023
Accepted Sep 22, 2023

Keywords:

Comparative study
Interval arithmetic
Interval linear fractional programming problem
Interval linear programming
Variable transformation

ABSTRACT

Interval linear fractional programming problem (ILFPP) approaches uncertainties in real-world systems such as business, manufacturing, finance, and economics. In this study, we propose solving the interval linear fractional programming (ILFP) problem using interval arithmetic. Further, to construct the problem, a suitable variable transformation is used to form an equivalent ILP problem, and a new algorithm is depicted to obtain the optimal solution without converting the problem into its conventional form. This paper compares the range, solutions, and approaches of ILFP with fuzzy linear fractional programming (FLFP) in solving real-world optimization problems. The illustrated numerical examples show a better range of interval solutions on practical applications of ILFPs and uncertain parameters.

Corresponding Author:

Nirmala Thamaraiselvan
Department of Mathematics, SRM Institute of Science and Technology
Kattankulathur, Tamil Nadu 603203, India
Email: nirmalat@srmist.edu.in

1. INTRODUCTION

Linear fractional programming (LFP) problem is used worldwide in solving real-life problems. An LFP problem is noteworthy due to its crucial role in modern applications like production planning (inventory/sales, output/employee), financial and corporate planning (debt/equity ratio), and healthcare and hospital planning (cost/patient, nurse/patient ratio), to name a few. Many researchers have worked diligently on the linear fractional programming problem since it offers a more effective method than a linear programming problem. Experts commonly determine the potential values of several parameters involved in this model's objective function and constraints. The current process involves an experimental approach and a knowledgeable understanding of the parameters' nature to fix these parameters at a specific value. The parameters for real-world situations are less exact than those for fixed actual values since profit/cost, student/cost, and time/cost are all subject to vary for various reasons.

In real-world situations, data measurement and observational ambiguity are frequent occurrences. Particularly in the case of optimization problems, the parameters can be ambiguous, in which case they can be expressed as intervals. One such approach is interval linear fractional programming (ILFP), a specialized optimization methodology combining interval analysis and fractional programming to deal with problem-solving under uncertainty. Providing decision-makers with more reliable and robust solutions offers a practical approach in dealing inaccurate objective function and constraints data. ILFP is employed in various domains, including engineering, finance, economics, and operations research. It is helpful in situations involving uncertainty, such as production planning, portfolio optimization, resource allocation, and supply chain management.

This work aims to thoroughly understand ILFP, its formulation, and its practical applications.

In the literature, different methods for solving various models of the linear fractional programming problem are discussed. Moore *et al.* [1] analyzed the interval and its computational algorithms with various applications. In 1960, Hungarian mathematician B. Matros developed the area of LFP. Bajalinov [2] studied the various types, methods and applications of linear fractional programming problem. Charnes and Cooper [3] transformed the LFP problem into a LP problem. Swarup [4] enhanced the popular simplex method to solve linear fractional function programming problems without converting them to LP problems. Bitran and Magnanti [5] studied duality and sensitivity analysis of fractional problem. Singh [6] dealt with optimality conditions in fractional programming. Hladík [7] considered generalized ILFP problem for computing the range of optimal values. Effati and Pakdaman [8] dealt with an interval-valued objective function with linear fractional bounds by transforming IVLFP into an optimization problem. Mitlif [9] used the development Lagrange method to solve problems with constrained and unconstrained linear fractional programming with interval coefficients in the objective function. The objective functional functions' OVR was studied by [10] without converting them into LP problems. Majeed [11] applied interval values to solve problems in linear fractional bounded variable programming. Rahman *et al.* [12] studied the necessary optimality conditions of a constrained optimization problem with interval-valued objective function. Batamiz *et al.* [13] dealt with interval multi-objective linear programming (IMOLP). Omran *et al.* [14] studied three level fractional programming problem with rough coefficient in constraints. Borza *et al.* [15] depicted a parametric approach for solving FPP with interval co-efficients. Pokharna and Tripathi [16] derived necessary and sufficient conditions of IFPP and its application. Mustafa and Sulaiman [17] solved by converting LFPP with rough intervals into LPP with interval co-efficients. Das *et al.* [18] proposed a method to solve fuzzy linear fractional programming problems under non-negative fuzzy variables. Chinnadurai and Muthukumar [19] proposed a procedure to solve a fully fuzzy linear fractional programming problem using the upper and lower bounds. To obtain the pareto optimum solution, Stanojevic and Stanojevic [20] developed a technique for dealing with linear fractional programming problems with uncertain coefficients in the objective function. Borza and Rambely [21] discussed a method for solving linear fractional programming with fuzzy coefficients based on α-cuts and max-min. Ammar and Emsimir [22] solved triangular fuzzy rough integer linear programming (TFRILP) problems with α-level.

In section 2, some basic concepts for interval parameters, interval arithmetic and ranking functions are discussed. In section 3, we find the ideas for solving ILFPP and the conversion to ILPP with the fundamental theorems and depicts the algorithm for solving interval linear fractional programming problem (ILFPP). In section 4, we show the real-life applications of ILFPP and compare the results of ILFPP with FLFPP [18], and a graphical illustration is provided to compare and analyze the efficiency of the proposed method. In section 5, conclusion of the proposed method are deliberated.

2. PRELIMINARIES

2.1. Interval number

Let $\tilde{u} = [u_1, u_2]$=$\{x \in \mathbb{R}$: $u_1 \leq x \leq u_2$ and $u_1, u_2 \in\mathbb{R}\}$ represent an interval on the real number line $\mathbb{R}$. If $u_1 = u_2 = u$, then $\tilde{u} = [u, u]$ represents a real number or a degenerate interval. We associate intervals with ordered pairs $\langle m, w\rangle$ as follows: for $\tilde{u} = [u_1, u_2] \subseteq \mathbb{R}$, we define $m(\tilde{u}) = \left(\dfrac{u_1 + u_2}{2}\right)$ and $w(\tilde{u}) = \left(\dfrac{u_2 - u_1}{2}\right)$. Therefore, $\tilde{u} \to \langle m(\tilde{u}), w(\tilde{u})\rangle$ is uniquely determined. Conversely, if $\langle m(\tilde{u}), w(\tilde{u})\rangle$ is given, we can determine the interval as follows: the left endpoint of $\tilde{u}$ is $m(\tilde{u}) - w(\tilde{u})$, and the right endpoint is $m(\tilde{u}) + w(\tilde{u})$. Hence for the given $\langle m(\tilde{u}), w(\tilde{u})\rangle$, the interval is unique.

2.2. Interval vector

An interval vector, denoted as $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \ldots, \tilde{v}_n)^t$, represents a vector whose components are enclosed within closed intervals. We use $\mathbb{IR}^n$ to indicate the collection of all n-component interval vectors. We define $m(\tilde{v})$ as a vector whose elements consist of the respective midpoint of the components of $\tilde{v}$, precisely $m(\tilde{v}) = (m(\tilde{v}_1), m(\tilde{v}_2), \ldots, m(\tilde{v}_n))^t$. In addition, we define $w(\tilde{v}) = (w(\tilde{v}_1), w(\tilde{v}_2), \ldots, w(\tilde{v}_n))^t$ as the width of interval vector.

2.3. Interval arithmetic

Ma *et al.* [23] introduced a novel fuzzy arithmetic approach focusing on location indices and fuzziness index functions applied to interval numbers. In standard arithmetic, we typically employ position index numbers. However, in the context of lattice L, fuzziness index functions are designed to adhere to lattice laws, particularly the least upper bound and greatest lower bound operations within the lattice L. To clarify, for any elements x and y belonging to the lattice L, we define $x \vee y = \max\{x, y\}$ and $x \wedge y = \min\{x, y\}$. For any two intervals $\tilde{x}, \tilde{y} \in \mathbb{IR}$ and for $* \in \{+, -, \cdot, \div\}$, the arithmetic operations on IR [24] is defined as:

$$\tilde{x} * \tilde{y} = \langle m(\tilde{x}) * m(\tilde{y}),\ max\{w(\tilde{x}), w(\tilde{y})\}\rangle$$

In particular:

- Addition: $\tilde{x} + \tilde{y} = \langle m(\tilde{x}) + m(\tilde{y}),\ max\{w(\tilde{x}), w(\tilde{y})\}\rangle$
- Subtraction: $\tilde{x} - \tilde{y} = \langle m(\tilde{x}) - m(\tilde{y}),\ max\{w(\tilde{x}), w(\tilde{y})\}\rangle$
- Multiplication: $\tilde{x} \cdot \tilde{y} = \langle m(\tilde{x}) \cdot m(\tilde{y}),\ max\{w(\tilde{x}), w(\tilde{y})\}\rangle$
- Division: $\tilde{x} \div \tilde{y} = \langle m(\tilde{x}) \div m(\tilde{y}),\ max\{w(\tilde{x}), w(\tilde{y})\}\rangle$, provided, $m(\tilde{y}) \neq 0$

2.4. Ranking of interval numbers

Sengupta and Pal [25] suggested a easy and powerful index to compare any two intervals on $\mathbb{IR}$ through the satisfaction of decision-makers. Let $\preceq$ be an extended order relation between the interval numbers. For any two intervals $\tilde{u} = [u_1, u_2], \tilde{v} = [v_1, v_2] \in \mathbb{IR}$ then for $m(\tilde{u}) < m(\tilde{v})$, we construct a premise($\tilde{u} \preceq \tilde{v}$) which implies that $\tilde{u}$ is inferior to $\tilde{v}$ (or $\tilde{v}$ is superior to $\tilde{u}$). An acceptability function $A_{\preceq}$:$\mathbb{IR}\times\mathbb{IR}\rightarrow [0, \infty)$ is defined as:

$A_{\preceq}(\tilde{u}, \tilde{v})$= $A(\tilde{u} \preceq \tilde{v})$= $\dfrac{m(\tilde{v}) - m(\tilde{u})}{w(\tilde{v}) + w(\tilde{u})}$, where $w(\tilde{v}) + w(\tilde{u}) \neq 0$

$A_{\preceq}$ may be interpreted as the grade of acceptability of the first interval number to be inferior to the second interval number.

2.5. Feasible and optimal solutions of interval linear fractional programming problem

There are 2 points for this subsection, as follows:

a. Interval feasible solution: let $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^t$ denote an interval vector, where each $\tilde{x}_i$ is an inter-valued decision variable. If $\tilde{x}$ satisfies the system of constraints and non-negative restrictions of the ILFPP, it is referred to as an interval feasible solution.

b. Interval optimal solution: an interval $\tilde{x}^*$ is considered an optimal solution for the ILFP problem if no interval vector $\tilde{x} \in \tilde{F}$ (interval feasible set) exists such that $\tilde{\phi}(\tilde{x}^*) \preceq \tilde{\phi}(\tilde{x})$.

3. PROPOSED METHOD

3.1. Interval linear fractional programming problem

General form: consider linear fractional programming problem involving interval numbers as (1)-(3):

$$Maximize/Minimize \quad \tilde{\phi}(\tilde{x}) \approx \frac{\tilde{P}(\tilde{x})}{\tilde{Q}(\tilde{x})} = \frac{\sum_{j=1}^{n} \tilde{p}_j^t \tilde{x}_j + \tilde{p}_0}{\sum_{j=1}^{n} \tilde{q}_j^t \tilde{x}_j + \tilde{q}_0} \tag{1}$$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \preceq \tilde{b}_i \quad , i = 1, 2, ..., m \tag{2}$$

$$m(\tilde{x}_j) \geq 0 \quad , j = 1, 2, ..., n \tag{3}$$

where $\tilde{x}$ is an interval vector of decision variables, $\tilde{p}, \tilde{q} \in \mathbb{IR}^n$ and $\tilde{p}_0,\ \tilde{q}_0 \in \mathbb{IR}$ are interval vectors representing the range of possible values for the coefficient vector and constant term, respectively. $\tilde{A} \in \mathbb{IR}^{m\times n}$

are interval matrices and $\tilde{b} \in \mathbb{IR}^m$ are interval vectors. Considering that all operations and conditions are constrained to exclusively apply to the midpoint of the interval, while also considering the maximum width as defined in the arithmetic operations. We assume that ILFPP satisfies the following condition: $\tilde{q}(\tilde{x}) \succ \tilde{0}\ \forall\ \tilde{x} \in \tilde{F},$ where $\tilde{F}$ be the feasible set with interval numbers.

3.2. Conversion of interval linear fractional programming problem to interval linear programming problem

Now by using variable transformation, $\tilde{x} = \dfrac{\tilde{y}}{\tilde{y}_0}$ and $\tilde{y}_0 = \dfrac{1}{\tilde{q}^t\tilde{x} + \tilde{q}_0}$ in the problem (1)-(3) and assuming $\sum_{j=1}^{n} \tilde{q}^t\tilde{y} + \tilde{q}_0\tilde{y}_0 \approx \tilde{1}$, the problem transforms into ILPP:

$$Maximize\ \tilde{\phi}(\tilde{x}) = \frac{\sum_{j=1}^{n} \tilde{p}_j^t \frac{\tilde{y}_j}{\tilde{y}_0} + \tilde{p}_0}{\sum_{j=1}^{n} \tilde{q}_j^t \frac{\tilde{y}_j}{\tilde{y}_0} + \tilde{q}_0} = \frac{\sum_{j=1}^{n} \tilde{p}_j^t\tilde{y}_j + \tilde{p}_0\tilde{y}_0}{\sum_{j=1}^{n} \tilde{q}_j^t\tilde{y}_j + \tilde{q}_0\tilde{y}_0} \implies Maximize\ \ \tilde{\zeta}(\tilde{y}) = \sum_{j=1}^{n} \tilde{p}_j^t\tilde{y}_j + \tilde{p}_0\tilde{y}_0 \quad (4)$$

subject to

$$\sum_{j=1}^{n} \tilde{q}_j\tilde{y}_j + \tilde{q}_0\tilde{y}_0 \approx \tilde{1} \quad (5)$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{y}_j - \tilde{b}_i\tilde{y}_0 \preceq \tilde{0} \quad ,i = 1, 2, ..., m \quad (6)$$

$$m(\tilde{y}_j), m(\tilde{y}_0) \geq 0 \quad (7)$$

3.3. Main results

Theorem 3.1. *If $\tilde{x}$ is a interval feasible solution of ILFPP then $\tilde{y}$ is a interval feasible solution to ILPP and the objective functions are equivalent at these points (i.e.,) $\tilde{\phi}(\tilde{x}) = \tilde{\zeta}(\tilde{y})$*

Proof. Let us consider the feasibility of ILFP to show the feasibility of the solution of ILPP.

– To begin with, proving all the constraints of ILFP compatible with the corresponding linear analogue, we consider $\tilde{x}$ be the feasible solution of ILFP problem with $\tilde{x} = \dfrac{\tilde{y}}{\tilde{y}_0}$ and $\tilde{y}_0 = \dfrac{1}{\tilde{q}^t\tilde{x} + \tilde{q}_0}$

$$\tilde{A}\tilde{y} - \tilde{b}\tilde{y}_0 \ \approx\ \tilde{A}\tilde{y}_0\tilde{x} - \tilde{b}\tilde{y}_0 \ \approx\ \tilde{y}_0(\tilde{A}\tilde{x} - \tilde{b}) \approx \tilde{y}_0 * 0. \quad (8)$$

$$\tilde{q}^t\tilde{y} + \tilde{q}_0\tilde{y}_0 \ \approx\ \tilde{y}_0(\tilde{q}^t\tilde{x} + \tilde{q}_0) \ \approx\ \tilde{y}_0(\tilde{q}^t\tilde{x} + \tilde{q}_0) \approx \frac{\tilde{q}^t\tilde{x} + \tilde{q}_0}{\tilde{q}^t\tilde{x} + \tilde{q}_0} \approx \tilde{1} \quad (9)$$

Hence, this proves $(\tilde{y}, \tilde{y}_0)$ is the feasible solution of ILPP. In addition to the non-negative restrictions, $\tilde{y} \succeq \tilde{0}$ and $\tilde{y}_0 = \dfrac{1}{\tilde{Q}(\tilde{x})} \succeq \tilde{0}$

– To prove the objective functions are equivalent at these points, we consider,

$$\tilde{\zeta}(\tilde{y}, \tilde{y}_0) \ \approx\ \tilde{p}^t\tilde{y} + \tilde{p}_0\tilde{y}_0 \ \approx\ \tilde{y}_0(\tilde{p}^t\tilde{x} + \tilde{p}_0) \ \approx\ \frac{1}{\tilde{q}^t\tilde{x} + \tilde{q}_0}(\tilde{p}^t\tilde{x} + \tilde{p}_0) \ \approx \tilde{\phi}(\tilde{x}) \quad (10)$$

In (10) thus demonstrates that the objective functions at these places are equivalent; hence, if ILFP is feasible, ILPP is likewise possible.

Theorem 3.2. *If interval vector $\tilde{y} = (\tilde{y}_0, \tilde{y}_1, \cdots, \tilde{y}_n)^t$ is the interval feasible solution of the ILFP problem(1)-(3) then $\tilde{y}_0$, is an interval number that is positive.*

Proof. Let us suppose $\tilde{x}' = (\tilde{x}'_1, \tilde{x}'_2, \cdots, \tilde{x}'_n)^t$ and $\tilde{y}' = (\tilde{y}'_0, \tilde{y}'_1, \tilde{y}'_2, \cdots, \tilde{y}'_n)^t$ as the interval feasible solutions of ILFPP and ILPP respectively. Assuming $(\tilde{y}', \tilde{y}'_0) = (\tilde{0}, \tilde{y}')$ $i.e.,$ $\tilde{y}'_0 \approx \tilde{0}$ with the interval feasible solutions $\tilde{x}' \in \tilde{F}\ \ and\ \ \tilde{y}' \in \tilde{M}$ where $\tilde{F}, \tilde{M}$ are the interval feasible sets of ILFP and ILP problems respectively. Then:

$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}'_j \preceq \tilde{b}_i \quad i = 1, 2, ..., m \quad (11)$$

$$m(\tilde{x}'_j) \geq 0 \qquad j = 1, 2, ..., n \tag{12}$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{y}'_j \preceq \tilde{0} \quad i = 1, 2, ..., m \tag{13}$$

$$m(\tilde{y}'_j) \geq 0 \qquad j = 1, 2, ..., n \tag{14}$$

On multiplying constraint (13) with an arbitrary interval $\tilde{\mu}$ and adding it to constraint (11) of the system. Similarly with the non-negative restrictions (12) and (14) which gives:

$$\sum_{j=1}^{n} \tilde{a}_{ij}(\tilde{x}'_j + \tilde{\mu}\tilde{y}'_j) \preceq \tilde{b}_i \quad i = 1, 2, ..., m \tag{15}$$

$$m(\tilde{x}'_j + \tilde{\mu}\tilde{y}'_j) \geq 0 \qquad j = 1, 2, ..., n \tag{16}$$

Then, $m(\tilde{x}'_j + \tilde{\mu}\tilde{y}'_j)$ is in $\tilde{F}$ for $m(\tilde{\mu}) > 0$. But $m(\tilde{\mu})$ value may be required as large as possible then $\tilde{F}$ is unbounded which is a contradiction to our assumption on the feasible set $\tilde{F}$. Hence $\tilde{y}_0$ is a positive interval number.

Theorem 3.3. *If the interval optimal solution for the ILP problem is $\tilde{y}^*$, then the optimal solution for the ILFP problem is $\tilde{x}^*$ where $\tilde{x}^* = \frac{\tilde{y}^*}{\tilde{y}_0^*}$.*

Proof. Considering the interval vector $\tilde{y}^* = (\tilde{y}_0^*, \tilde{y}_1^*, \tilde{y}_2^*, \cdots, \tilde{y}_n^*)^t$ is the optimal solution of ILPP (17):

$$\tilde{\zeta}(\tilde{y}^*) \succeq \tilde{\zeta}(\tilde{y}) \ \forall \, \tilde{y} \in \tilde{M} \tag{17}$$

where $\tilde{M}$ denotes the interval feasible set of ILPP. On proving by contradiction, suppose that $\tilde{x}^*$ is not an interval optimal solution of ILFP problem. So there must exists another interval vector $\tilde{x}' \ni \tilde{\phi}(\tilde{x}') \succeq \tilde{\phi}(\tilde{x}^*)$ with $\tilde{x}^* = \frac{\tilde{y}^*}{\tilde{y}_0^*}$

$$\tilde{\phi}(\tilde{x}^*) = \frac{\sum \tilde{p}^t \tilde{x}^* + \tilde{p}_0}{\sum \tilde{q}^t \tilde{x}^* + \tilde{q}_0)} \approx \frac{\sum \tilde{p}^t \tilde{y}^* + \tilde{p}_0 \tilde{y}_0^*}{\sum \tilde{q}^t \tilde{y}^* + \tilde{q}_0 \tilde{y}_0^*} \tag{18}$$

Let us take $\tilde{Q}(\tilde{x}) \approx \tilde{1}$

$$\tilde{\phi}(\tilde{x}^*) \approx \tilde{\zeta}(\tilde{y}^*) \tag{19}$$

It means that, $\tilde{\phi}(\tilde{x}^*) \succeq \tilde{\zeta}(\tilde{y}^*)$. Since $\tilde{x}'$ is the interval feasible solution of ILFP problem which conveys $\tilde{y}'$ is the interval feasible solution of ILP problem.

$$\tilde{\zeta}(\tilde{y}') \succeq \tilde{\zeta}(\tilde{y}^*) \tag{20}$$

which contradicts our assumption $\tilde{y}^*$ is the interval optimal solution of maximization ILPP. This shows that $\tilde{x}^*$ is the interval optimal solution of ILFP problem. This completes the proof of the theorem.

3.4. Algorithm for solving interval linear fractional programming problem

ILFPP involves uncertainty in the objective function and constraint coefficients. The following outlines key steps for resolving such problems:

a. Step 1: convert a minimization-type problem to a maximization-type problem.

b. Step 2: by variable transformation of $\tilde{x} = \frac{\tilde{y}}{\tilde{y}_0}$, the ILFP problem is transformed into its equivalent ILP problem.

c. Step 3: the equivalent ILPP is, $Max \ \tilde{\phi}(\tilde{x}) \equiv Max \ \tilde{\zeta}(\tilde{y})$

d. Step 4: in similar way, constraints of the given ILFPP is transformed.

e. Step 5: the intervals of ILPP is converted in its mid-point and width form using the interval arithmetic.

f. Step 6: solve the transformed ILPP using simplex two-phase method involving interval numbers.

g. Step 7: find the values of $\tilde{y} = (\tilde{y}_0, \tilde{y}_1, \tilde{y}_2, \cdots, \tilde{y}_n)^t$.

h. Step 8: to obtain the required interval optimal solution, substitute the values of $\tilde{y}$ in the objective function.

4. RESULTS AND DISCUSSION

4.1. Example 1 [18]

TATA Hospital Jamshedpur, India, has two nutritional experiments (vitamin A and Calcium) with two products, milk (glass) and salad (500 mg) in Table 1, with profits of around 6 dollars and around 2 dollars per unit, respectively. However, the cost for each one unit of the above product is around 1 dollar and around 1 dollar, respectively. Consider that a fixed cost of around 2 dollars is added to the cost function. Determine the maximum profit of these two products. Here, the environmental coefficients such as profit (due to market situations), cost (due to market conditions), vitamin A and calcium (due to the presents of the suppliers) are imprecise numbers with interval numbers over the planning horizon due to incomplete information. For example, the profit of product A is [4,8] dollars. Similarly, the other parameters and variables are assumed to be interval numbers. Hence, the above problem can be formulated as the following ILFP problem:

Table 1. Nutrients

Nutrient	Milk (glass)	Salad (500 mg)	Minimum nutrient required
Vitamin A	1	1	7
Calcium	2	3	17

Solution: Let $\tilde{x}_1$ and $\tilde{x}_2$ be the amount of vitamin A and calcium produced in this problem, respectively. The preceding situation in real life can be expressed as:

$$Maximize \quad \tilde{\phi} = \frac{[4,8]\tilde{x}_1 + [1,3]\tilde{x}_2}{[0,2]\tilde{x}_1 + [0,2]\tilde{x}_2 + [1,3]}$$

subject to

$$[0,2]\tilde{x}_1 + [0,2]\tilde{x}_2 \preceq [3,11]$$
$$[1,3]\tilde{x}_1 + [2,4]\tilde{x}_2 \preceq [7,27]$$
$$\tilde{x}_1, \tilde{x}_2 \succeq \tilde{0}$$

Solving the given problem by our method and expressing the interval parameters in terms of the midpoint and width as $\tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a})\rangle$. It can be transformed into ILPP using the variable transformation discussed in subsection 3.2. The equivalent interval linear programming problem is:

$$Max \ \tilde{\zeta} = \langle 6,2\rangle\tilde{y}_1 + \langle 2,1\rangle\tilde{y}_2 + \tilde{0}\tilde{y}_0$$

subject to

$$\langle 1,1\rangle\tilde{y}_1 + \langle 1,1\rangle\tilde{y}_2 + \langle 2,1\rangle\tilde{y}_0 \approx \langle 1,1\rangle$$
$$\langle 1,1\rangle\tilde{y}_1 + \langle 1,1\rangle\tilde{y}_2 - \langle 7,4\rangle\tilde{y}_0 \preceq \tilde{0}$$
$$\langle 2,1\rangle\tilde{y}_1 + \langle 3,1\rangle\tilde{y}_2 - \langle 17,10\rangle\tilde{y}_0 \preceq \tilde{0}$$
$$\tilde{y}_1, \tilde{y}_2, \tilde{y}_0 \succeq \tilde{0}$$

Phase-1

		$\tilde{c_j}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{-1}$	
$B.V$	$\tilde{c_B}$	$\tilde{Y_B}$	$\tilde{y_1}$	$\tilde{y_2}$	$\tilde{y}_0 \downarrow$	$\tilde{s_1}$	$\tilde{s_2}$	$\tilde{A_1}$	θ
$\leftarrow \tilde{A}_1$	$\tilde{-1}$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 2,1\rangle$	$\tilde{0}$	$\tilde{0}$	$\tilde{1}$	$\langle 0.5,1\rangle \rightarrow$
$\tilde{s_1}$	$\tilde{0}$	$\tilde{0}$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle -7,4\rangle$	$\tilde{1}$	$\tilde{0}$	$\tilde{0}$	
$\tilde{s_2}$	$\tilde{0}$	$\tilde{0}$	$\langle 2,1\rangle$	$\langle 3,1\rangle$	$\langle -17,10\rangle$	$\tilde{0}$	$\tilde{1}$	$\tilde{0}$	
		$\tilde{\zeta_j}$	$\langle -1,1\rangle$	$\langle -1,1\rangle$	$\langle -2,1\rangle$	$\tilde{0}$	$\tilde{0}$	$\tilde{-1}$	
		$\tilde{\zeta_j} - \tilde{c_j}$	$\langle -1,1\rangle$	$\langle -1,1\rangle$	$\langle -2,1\rangle \uparrow$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	

		$\tilde{c_j}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
$B.V$	$\tilde{c_B}$	$\tilde{Y_B}$	$\tilde{y_1}$	$\tilde{y_2}$	$\tilde{y}_0$	$\tilde{s_1}$	$\tilde{s_2}$	θ
$\tilde{y}_0$	$\tilde{0}$	$\langle 0.5,1\rangle$	$\langle 0.5,1\rangle$	$\langle 0.5,1\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	
$\tilde{s_1}$	$\tilde{0}$	$\langle 3.5,1\rangle$	$\langle 4.5,1\rangle$	$\langle 4.5,1\rangle$	$\langle 0,4\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	
$\tilde{s_2}$	$\tilde{0}$	$\langle 8.5,1\rangle$	$\langle 10.5,1\rangle$	$\langle 11.5,1\rangle$	$\langle 0,10\rangle$	$\langle 0,1\rangle$	$\langle 1,1\rangle$	
		$\tilde{\zeta_j}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
		$\tilde{\zeta_j} - \tilde{c_j}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	

Since $\tilde{\zeta_j} - \tilde{c_j} \succeq 0$ for all j, hence we go to phase-2

Phase-2

		$\tilde{c_j}$	$\langle 6,2\rangle$	$\langle 2,1\rangle$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
$B.V$	$\tilde{c_B}$	$\tilde{Y_B}$	$\tilde{y_1}\downarrow$	$\tilde{y_2}$	$\tilde{y}_0$	$\tilde{s_1}$	$\tilde{s_2}$	θ
$\tilde{y}_0$	$\tilde{0}$	$\langle 0.5,1\rangle$	$\langle 0.5,1\rangle$	$\langle 0.5,1\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 1,1\rangle$
$\leftarrow\tilde{s_1}$	$\tilde{0}$	$\langle 3.5,1\rangle$	$\langle 4.5,1\rangle$	$\langle 4.5,1\rangle$	$\langle 0,4\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	$\langle 0.78,1\rangle\rightarrow$
$\tilde{s_2}$	$\tilde{0}$	$\langle 8.5,1\rangle$	$\langle 10.5,1\rangle$	$\langle 11.5,1\rangle$	$\langle 0,10\rangle$	$\langle 0,1\rangle$	$\langle 1,1\rangle$	$\langle 0.81,1\rangle$
		$\tilde{\zeta_j}$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	
		$\tilde{\zeta_j}-\tilde{c_j}$	$\langle -6,2\rangle\uparrow$	$\langle -2,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	

		$\tilde{c_j}$	$\langle 6,2\rangle$	$\langle 2,1\rangle$	$\tilde{0}$	$\tilde{0}$	$\tilde{0}$	
$B.V$	$\tilde{c_B}$	$\tilde{Y_B}$	$\tilde{y_1}\downarrow$	$\tilde{y_2}$	$\tilde{y}_0$	$\tilde{s_1}$	$\tilde{s_2}$	θ
$\tilde{y}_0$	$\tilde{0}$	$\langle 0.111,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 1,4\rangle$	$\langle -0.11,1\rangle$	$\langle 0,1\rangle$	
$\tilde{y_1}$	$\langle 6,2\rangle$	$\langle 0.778,1\rangle$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 0,4\rangle$	$\langle 0.222,1\rangle$	$\langle 0,1\rangle$	
$\tilde{s_2}$	$\tilde{0}$	$\langle 0.331,1\rangle$	$\langle 0,1\rangle$	$\langle 1,1\rangle$	$\langle 0,10\rangle$	$\langle -2.31,1\rangle$	$\langle 1,1\rangle$	
		$\tilde{\zeta_j}$	$\langle 6,2\rangle$	$\langle 6,2\rangle$	$\langle 0,4\rangle$	$\langle 1.332,2\rangle$	$\langle 0,2\rangle$	
		$\tilde{\zeta_j}-\tilde{c_j}$	$\langle 0,2\rangle$	$\langle 4,2\rangle$	$\langle 0,4\rangle$	$\langle 1.332,2\rangle$	$\langle 0,2\rangle$	

By the proposed method, the obtained interval optimal solution is $\tilde{y}_1 = \langle 0.778, 1\rangle$, $\tilde{y}_2 = \langle 0, 0\rangle$, $\tilde{y}_0 = \langle 0.111, 1\rangle$. The profit obtained on the amount of calcium and vitamin A produced is $Max\ \tilde{\zeta} = \langle 4.667, 2\rangle \implies \tilde{\zeta} = [2.67, 6.67]$

Comparative study: the numerical example illustrates the application of ILFPP in an actual-world scenario. Figure 1 and Table 2 compares the range of the solution between ILFPP and FLPP $\tilde{\zeta} = [2.67, 6.67]$. Therefore, it may be inferred that the interval solutions determined by ILFPP have a shorter range than those obtained by FLFPP, implying more precision and less ambiguity. The outcome is better and more reliably estimated within the provided range of $\tilde{\zeta} = [2.67, 6.67]$

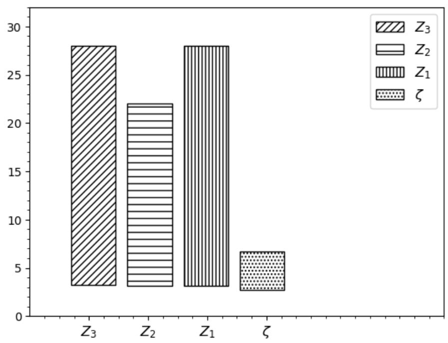

Figure 1. Comparative graph on the interval range of example 1

Table 2. Optimal solution ranges: proposed method vs. existing methods

Das *et al.* [18] method	Chinnadurai and Muthukumar [19] method	Stanojevic and Stanojevic [20] method	Proposed method
Z_1= [3.14,28]	Z_2=[3.14,22]	Z_3=[3.2,28]	ζ= [2.67,6.67]

4.2. Example 2 [18]

Dali Company is Taiwan's leading producer of soft drinks and low-temperature foods. Dali is planning to develop the South-East Asian market and broaden the visibility of Dali products in the Chinese market. In particular, following the entry of Taiwan into the World Trade Organization, Dali plans to seek strategic alliances with prominent international companies and to introduce an international blend to meet any future goals. In the domestic soft drinks market, Dali supplies tea beverages from four distribution centers in Taichung, Chiayi, Kaohsiung, and Taipei. At the same time, production is based at three plants in Changhua, Touliu and Hsinchu. According to the preliminary environmental information, Table 3 summarizes the potential supply (thousand dozen bottles) available from the given three plants. The forecast demand (thousand dozen bottles) from the four distribution centers is shown in Table 4. Table 5 and Table 6 summarizes the profit of the company gained by each route and the unit shipping cost for each route for the upcoming season, respectively. The environmental coefficient and related parameters generally are imprecise numbers with interval possibility distributions over the planning horizon due to incomplete or unobtainable information. The management of Dali is initiating a study to maximize the profit as much as possible.

Table 3. Supply of the plants

Source	Changhua	Touliu	Hsinchu
Supply	[7.2,8.8]	[12,16]	[10.2,13.8]

Table 4. Demand from the destinations

Destination	Taichung	Chiayi	Kaohsiung	Taipei
Demand	[6.2,7.8]	[8.9,11.1]	[6.5,9.5]	[7.8,10.2]

Table 5. Profits for the company

Source	Destination			
	Taichung	Chiayi	Kaohsiung	Taipei
Changhua	$[8,10.8]	$[20.4,24]	$[8,10.6]	$[18.8,22]
Touliu	$[14,16]	$[18.2,22]	$[10,13]	$[6,8.8]
Hsinchu	$[18.4,21]	$[9.6,13]	$[7.8,10.8]	$[14,16]

Table 6. Shipping costs

Source	Destination			
	Taichung	Chiayi	Kaohsiung	Taipei
Changhua	$[1.5,2.5]	$[4,6]	$[1.3,2.5]	$[3,5]
Touliu	$[2.5,4]	$[2,4]	$[2.3,4]	$[1.5,2.5]
Hsinchu	$[3,5]	$[2,4]	$[1.5,2.7]	$[2,4]

Solution: let $\tilde{x}_{11}, \tilde{x}_{12}, \tilde{x}_{13}, \tilde{x}_{14}, \tilde{x}_{21}, \tilde{x}_{22}, \tilde{x}_{23}, \tilde{x}_{24}, \tilde{x}_{31}, \tilde{x}_{32}, \tilde{x}_{33}, \tilde{x}_{34}$ are the tea beverages and the real-world problem can be formulated as:

$$Max\ \tilde{\phi} = \frac{\begin{array}{c}[8,10.8]\tilde{x}_{11} + [20.4,24]\tilde{x}_{12} + [8,10.6]\tilde{x}_{13} + [18.8,22]\tilde{x}_{14} + [14,16]\tilde{x}_{21} + [18.2,22]\tilde{x}_{22}\\ + [10,13]\tilde{x}_{23} + [6,8.8]\tilde{x}_{24} + [18.4,21]\tilde{x}_{31} + [9.6,13]\tilde{x}_{32} + [7.8,10.8]\tilde{x}_{33} + [14,16]\tilde{x}_{34}\end{array}}{\begin{array}{c}[1.5,2.5]\tilde{x}_{11} + [4,6]\tilde{x}_{12} + [1.3,2.5]\tilde{x}_{13} + [3,5]\tilde{x}_{14} + [2.5,4]\tilde{x}_{21} + [2,4]\tilde{x}_{22}\\ + [2.3,4]\tilde{x}_{23} + [1.5,2.5]\tilde{x}_{24} + [3,5]\tilde{x}_{31} + [2,4]\tilde{x}_{32} + [1.5,2.7]\tilde{x}_{33} + [2,4]\tilde{x}_{34}\end{array}}$$

$$\begin{gathered}
\tilde{x}_{11} + \tilde{x}_{12} + \tilde{x}_{13} + \tilde{x}_{14} \preceq [7.2,8.8]\\
\tilde{x}_{21} + \tilde{x}_{22} + \tilde{x}_{23} + \tilde{x}_{24} \preceq [12,16]\\
\tilde{x}_{31} + \tilde{x}_{32} + \tilde{x}_{33} + \tilde{x}_{34} \preceq [10.2,13.8]\\
\tilde{x}_{11} + \tilde{x}_{21} + \tilde{x}_{31} \preceq [6.2,7.8]\\
\tilde{x}_{12} + \tilde{x}_{22} + \tilde{x}_{32} \preceq [8.9,11.1]\\
\tilde{x}_{13} + \tilde{x}_{23} + \tilde{x}_{33} \preceq [6.5,9.5]\\
\tilde{x}_{14} + \tilde{x}_{24} + \tilde{x}_{34} \preceq [7.8,10.2]\\
\tilde{x}_{ij} \succeq 0 \text{ where } i = 1,2,3\ ;\ j = 1,2,3,4.
\end{gathered}$$

With the results proven and the algorithm described in section 4, the ILFPP can be solved by the proposed method, resulting in the interval optimal solution obtained as,
$\tilde{y}_{11}, \tilde{y}_{12}, \tilde{y}_{13}, \tilde{y}_{14}, \tilde{y}_{21}, \tilde{y}_{23}, \tilde{y}_{24}, \tilde{y}_{31}, \tilde{y}_{32}, \tilde{y}_{33}, \tilde{y}_{34} = \tilde{0}, \tilde{y}_{22} = \langle 0.333, 1\rangle, \tilde{y}_0 = \langle 0.333, 1\rangle$ and the profit obtained is $Max\ \zeta = \langle 6.69, 1.9\rangle \implies \zeta = [4.79, 8.59]$
Comparative study: the numerical illustration exhibits the use of ILFPP in a practical manner. Figure 2 and Table 7 compare the range of the solution between ILFPP and FLPP $\tilde{\zeta} = [4.79, 8.59]$. It depicts that the interval solutions found via ILFPP have a shorter range than those derived from FLFPP, indicating more precision and reducing ambiguity in the profit.

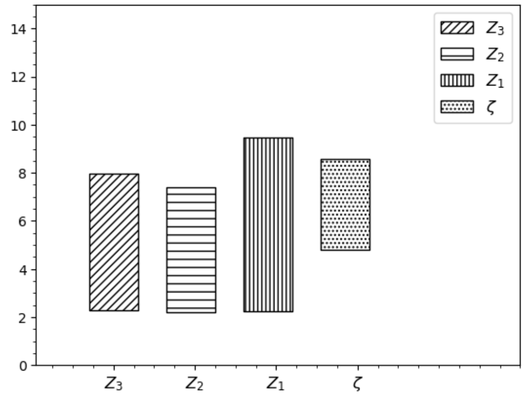

Figure 2. Comparative graph on the interval range of example 2

Table 7. Optimal solution ranges: proposed method vs. existing methods

Das *et al.* [18] method	Chinnadurai & Muthukumar [19] method	Stanojevic & Stanojevic [20] method	Proposed method
Z_1=[2.26,9.48]	Z_2=[2.2,7.39]	Z_3=[2.3,7.96]	ζ=[4.79,8.59]

4.3. Example 3 [18]

Let's examine the given ILFP problem:

$$Max\ \tilde{\phi} = \frac{[3,7]\tilde{x}_1 + [2,4]\tilde{x}_2}{[4,6]\tilde{x}_1 + [1,3]\tilde{x}_2 + [0,2]}$$

subject to

$$[2,4]\tilde{x}_1 + [3,7]\tilde{x}_2 \preceq [11,19]$$
$$[4,6]\tilde{x}_1 + [3,7]\tilde{x}_2 \preceq [8,12]$$
$$\tilde{x}_1, \tilde{x}_2 \succeq 0$$

Solution: using the proposed algorithm described in subsection 3.4 the ILFPP can be solved as follows:

$$Max\ \tilde{\zeta} = \frac{\langle 5,2\rangle\tilde{x}_1 + \langle 3,1\rangle\tilde{x}_2}{\langle 5,1\rangle\tilde{x}_1 + \langle 2,1\rangle\tilde{x}_2 + \langle 1,1\rangle}$$

subject to

$$\langle 3,1\rangle\tilde{x}_1 + \langle 5,2\rangle\tilde{x}_2 \preceq \langle 15,4\rangle$$
$$\langle 5,1\rangle\tilde{x}_1 + \langle 5,2\rangle\tilde{x}_2 \preceq \langle 10,2\rangle$$
$$\tilde{x}_1, \tilde{x}_2 \succeq 0$$

The interval optimal solution obtained is
$\tilde{y}_1 = 0,\ \tilde{y}_2 = \langle 0.4, 1\rangle,\ \tilde{y}_0 = \langle 0.2, 1\rangle\ and\ Max\ \tilde{\zeta} = \langle 1.2, 1\rangle \implies \tilde{\zeta} = [0.2, 2.2]$
Comparative study: compared to FLFPP, the interval solutions in Figure 3 and Table 8 derived using ILFPP show an interval version with a better mid-point, as per the study's findings, within the provided range of solution $\tilde{\zeta} = [0.2, 2.2]$.

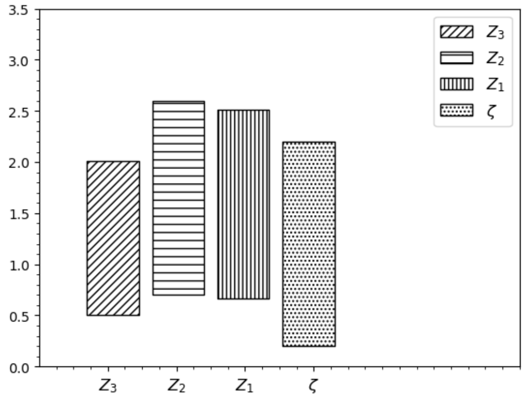

Figure 3. Comparative graph on the interval range of example 3

Table 8. Optimal solution ranges: proposed method vs. existing methods

Das *et al.* [18] method	Chinnadurai and Muthukumar [19] method	Stanojevic and Stanojevic [20] method	Proposed method
Z_1=[0.67,2.51]	Z_2=[0.7,2.6]	Z_3=[0.5,2.01]	ζ=[0.2,2.2]

4.4. Result analysis

The real-world problems from the cited papers [18]–[20] with fuzzy triangular numbers are re–formulated and bounded as interval numbers with the upper and lower limits. Additionally, from the perspective of intervals, we solved the system using the proposed method. This approach compares the optimal solutions of our process with that of the cited papers. Since the minimal-range interval solutions enhance robustness against ambiguous data, our method provides effective results with fewer computational difficulties and reduces the range of the interval bounds. Graphical representations of the results give a better understanding of the range of the interval solution.

5. CONCLUSION

This paper proposes an interval version of the algorithm to obtain an optimal interval solution. This method transforms the interval linear fractional programming problem into interval linear programming problem for facile computations. The motive of the technique is to reduce the vagueness in uncertain conditions to some extent in the interval version. The graphical representation shows the range of the interval, and the numerical example of a real-life application illustrates the efficiency of the proposed method.

REFERENCES

[1] R. E. Moore, R. B. Kearfott, and M. J. Cloud *Introduction to Interval Analysis*, Society for Industrial and Applied Mathematics, 2009, doi: 10.1137/1.9780898717716.

[2] E. Bajalinov, *Linear-fractional programming: theory, methods, applications and software*, New York: Springer New York, 2003.

[3] A. Charnes and W. W. Cooper, "Programming with linear fractional functionals," *Naval Research Logistics Quarterly*, vol. 9, no. 3–4, pp. 181–186, Sep. 1962, doi: 10.1002/nav.3800090303.

[4] K. Swarup, "Linear Fractional Functionals Programming," *Operations Research*, vol. 13, no. 6, pp. 1029–1036, Dec. 1965, doi: 10.1287/opre.13.6.1029.

[5] G. R. Bitran and T. L. Magnanti, "Duality and Sensitivity Analysis for Fractional Programs," *Operations Research*, vol. 24, no. 4, pp. 675–699, Aug. 1976, doi: 10.1287/opre.24.4.675.

[6] C. Singh, "Optimality conditions in fractional programming," *Journal of Optimization Theory and Applications*, vol. 33, no. 2, pp. 287–294, Feb. 1981, doi: 10.1007/BF00935552.

[7] M. Hladík, "Generalized linear fractional programming under interval uncertainty," *European Journal of Operational Research*, vol. 205, no. 1, pp. 42–46, Aug. 2010, doi: 10.1016/j.ejor.2010.01.018.

[8] S. Effati and M. Pakdaman, "Solving the Interval-Valued Linear Fractional Programming Problem," *American Journal of Computational Mathematics*, vol. 2, no. 1, pp. 51–55, 2012, doi: 10.4236/ajcm.2012.21006.

[9] R. J. Mitlif, "Development Lagrange Method for Solving Linear Fractional Programming Problems with Intervals Coefficients in the Objective Function," *Al-Mustansiriyah Journal of Science*, vol. 27, no. 1, pp. 88–90, 2016.

[10] F. S. P. S. Abad, M. Allahdadi, and H. M. Nehi, "Interval linear fractional programming: optimal value range of the objective function," *Computational and Applied Mathematics*, vol. 39, no. 4, pp. 1–17, 2020, doi: 10.1007/s40314-020-01308-2.

[11] A. S. Majeed, "A proposed Technique for Solving Interval-Valued Linear Fractional Bounded Variable Programming Problem," *Academic Science Journal*, vol. 14, no. 1, pp. 237–251, 2018.

[12] M. S. Rahman, A. A. Shaikh, and A. K. Bhunia, "Necessary and sufficient optimality conditions for non-linear unconstrained and constrained optimization problem with interval valued objective function," *Computers and Industrial Engineering*, vol. 147, p. 106634, Sep. 2020, doi: 10.1016/j.cie.2020.106634.

[13] A. Batamiz, M. Allahdadi, and M. Hladík, "Obtaining Efficient Solutions of Interval Multi-objective Linear Programming Problems," *International Journal of Fuzzy Systems*, vol. 22, no. 3, pp. 873–890, 2020, doi: 10.1007/s40815-020-00800-5.

[14] M. Omran, O. Emam, and A. Mahmoud, "On Solving Three Level Fractional Programming Problem with Rough Coefficient in Constraints," *British Journal of Mathematics and Computer Science*, vol. 12, no. 6, pp. 1–13, 2016, doi: 10.9734/bjmcs/2016/21932.

[15] M. Borza, A. S. Rambely, and M. Saraj, "Parametric approach for an absolute value linear fractional programming with interval coefficients in the objective function," *AIP Conference Proceedings*, vol. 1602, pp. 415–421, 2014, doi: 10.1063/1.4882519.

[16] N. Pokharna and I. P. Tripathi, "Interval fractional programming: Optimality and its application to portfolio optimization," *Research Square*, 2023, doi: 10.21203/rs.3.rs-1383605/v1.

[17] R. B. Mustafa and N. A. Sulaiman, "A New Approach to Solving Linear Fractional Programming Problem with Rough Interval Coefficients in the Objective Function," *Ibn AL- Haitham Journal For Pure and Applied Sciences*, vol. 35, no. 2, pp. 70–83, 2022, doi: 10.30526/35.2.2809.

[18] S. Das, S. A. Edalatpanah, and T. Mandal, "Application of linear fractional programming problem with fuzzy nature in industry sector," *Filomat*, vol. 34, no. 15, pp. 5073–5084, 2020, doi: 10.2298/FIL2015073D.

[19] V. Chinnadurai and S. Muthukumar, "Solving the linear fractional programming problem in a fuzzy environment: Numerical approach," *Applied Mathematical Modelling*, vol. 40, no. 11–12, pp. 6148–6164, Jun. 2016, doi: 10.1016/j.apm.2016.01.044.

[20] B. Stanojević and M. Stanojević, "Solving Method for Linear Fractional Optimization Problem with Fuzzy Coefficients in the Objective Function," *International Journal of Computers Communications and Control*, vol. 8, no. 1, pp. 146–152, Nov. 2012, doi: 10.15837/ijccc.2013.1.178.

[21] M. Borza and A. S. Rambely, "An approach based on α-cuts and max-min technique to linear fractional programming with fuzzy coefficients," *Iranian Journal of Fuzzy Systems*, vol. 19, no. 1, pp. 153–169, 2022, doi: 10.22111/ijfs.2022.6558.

[22] E. S. Ammar and A. Emsimir, "A mathematical model for solving fuzzy integer linear programming problems with fully rough intervals," *Granular Computing*, vol. 6, no. 3, pp. 567–578, 2021, doi: 10.1007/s41066-020-00216-4.

[23] M. Ma, M. Friedman, and A. Kandel, "A new fuzzy arithmetic," *Fuzzy Sets and Systems*, vol. 108, no. 1, pp. 83–90, 1999, doi: 10.1016/S0165-0114(97)00310-2.

[24] S. Ganesan and G. Kandasamy, "Using Interval Parameters for Latest Start Time and Mission Floats Operation Networks," *Mathematical Modelling of Engineering Problems*, vol. 10, no. 2, pp. 687–694, 2023, doi: 10.18280/mmep.100240.

[25] A. Sengupta and T. K. Pal, "On comparing interval numbers," *European Journal of Operational Research*, vol. 127, no. 1, pp. 28–43, 2000, doi: 10.1016/S0377-2217(99)00319-7.

BIOGRAPHIES OF AUTHORS

Yamini Murugan received an M.Sc. degree in Mathematics (2020) from SRM Institute of Science and Technology, Kattankulathur, India. She is currently doing her research in the Department of Mathematics, College of Engineering and Technology SRM Institute of Science and Technology, India. Her researches of interest are in the area of interval optimization and its application. She can be contacted at email: ym7201@srmist.edu.in.

Nirmala Thamaraiselvan is currently working as an Assistant Professor in the Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur, Chennai, India. She has been awarded Ph.D. degree from SRM University. She has 11 years of experience in teaching. Her research interests include interval optimization and its application. She can be contacted at email: nirmalat@srmist.edu.in.