Robust sliding mode observer based-simultaneous state and actuator fault estimation for a class of switching systems

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ABSTRACT

This paper investigates the state and actuator fault reconstruction problem in a class of switched linear systems subjected to unknown external disturbances according to average dwell time (ADT) technique. First, a robust switched sliding mode observer (SMO) is developed to simultaneously reconstruct the states of the switched system and the actuator faults. A novel and less conservative sufficient stability conditions are then established using the multiple quadratic Lyapunov function technique and the ADT approach. These conditions are formulated as linear matrix inequalities (LMI) to facilitate the design of the SMO. The observer gains matrices are obtained throughout the resolution of LMI using convex optimization techniques. Next, actuator faults are estimated by utilizing the concept of equivalent output injection, achieved through an analysis of the state error dynamics during the sliding motion. Finally, simulation results are considered to illustrate the applicability and efficiency of the developed method. It showcases the rapid and accurate convergence of the estimated system states and actuator fault to the real variables.

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1. INTRODUCTION

In recent years, there has been significant interest in the control and synthesis of switched systems, driven by their extensive applications in practical engineering systems. Switched systems are an important class of hybrid dynamical systems. They are governed by a finite number of linear subsystems and a controlled or autonomous logical switching sequence which determines the switching between these subsystems and indicates the active mode [1]–[4].

Fault diagnosis (FD) techniques are of great importance in ensuring the reliability and safe operation of practical engineering systems. In literature, FD methods are divided into two types based on a priori process knowledge including model-based FD methods and knowledge-based FD methods. Many FD methods are observer based. This method employs an analytical redundancy scheme, comparing measured signals with their estimates derived from a mathematical model. The resulting filtered disparity serves as a fault indication signal, commonly known as a residual signal. In fault-free conditions, the residual signal should approach zero, whereas the presence of faults typically results in a non-zero residual [5]. For fault detection and estimation problem, many observer-based methods have been established in the literature, such as adaptive observer method [6], [7], unknown input observer (UIO) method [8], fuzzy observer method [9], [10], descriptor observer method [11], [12], learning method based on neural network [13], interval observers [14] and sliding mode observer (SMO) method [15], [16].

The last decades have seen an increase in interest of application the sliding mode techniques for fault detection and estimation. However, in the literature, SMO are applied to several class of linear system [17], descriptor systems [18], linear parameter varying descriptor systems [19] and nonlinear systems [20]. SMO for switched systems have garnered significant attention over the past few decades, with numerous research papers dedicated to this subject [21]-[24]. For example, Islem *et al.* [21] have deal with the problem of states and actuator fault reconstruction applicable for a class of switched linear systems subject to exogenous disturbances. Yin *et al.* [22], two observer approaches have been proposed for switched systems to tackle state and fault estimation problems. The initial approach utilizes a linear descriptor reduced-order observer to accurately estimate both the system state and sensor faults simultaneously. The second approach employs a descriptor SMO technique to reconstruct sensor and actuator faults. Li *et al.* [23], the authors address the problem of sensor fault detection and estimation for buck-boost converter. By emploing switched Lyapunov function, an SMO is formulated for the switched error systems. This design ensures that the error systems achieve asymptotic stability by mean of H_∞ performance. Ali *et al.* [24], a robust SMO-based fault detection method for a boost converter is introduced. This approach incorporates the concept of average dwell time (ADT) along with H_∞ performance index considerations to mitigate the impact of disturbances on the residual signal. Additionally, the stability of switched systems is addressed using a Lyapunov functional technique.

The current study focuses on the problem of fault detection and estimation for continuous-time switched systems subject to unknown external disturbances and actuator faults. Sufficient stability conditions for the estimation error dynamics are provided in terms of linear matrix inequalities by using the multiple Lyapunov function and ADT approach. The advantage of employing these approaches is that they result in less conservative conditions compared to those using a common Lyapunov function which usually leads to a conservative result. The objective is to find an admissible switching signal that will ensure the convergence and stability of the designed SMO. The main contributions of the present manuscript are stated in the following:
- The state estimation approach is developed based on robust switched SMO for a class of switching systems with external disturbances and actuator faults.
- By utilizing the multiple Lyapunov function technique and H_∞ method combined with the ADT concept, a switching signal is constructed and sufficient conditions are derived to ensure the globally uniformly asymptotically stable (GUAS) behavior of the error reconstruction dynamics, while also reducing the conservativeness of the designed SMO.
- The developed switched SMO, incorporating feedforward signals, can simultaneously estimate actuator faults.
- The proposed estimation algorithm offers the advantage of quickly and accurately estimating faults, while also being suitable for reconstructing various types of faults.

The remainder of the paper is organized as follows: section 2 introduces the general model and preliminaries. In section 3, a robust switched SMO is designed to estimate the states of the original switched system and reconstruct actuator faults. Section 4 presents a numerical simulation to demonstrate the effectiveness and applicability of the theoretical results. Finally, section 5 provides the conclusion.

2. GENERAL MODEL AND PRELIMINARIES

Consider the dynamic continuous-time switched systems in the presence of actuator faults and unknown external disturbance modeled by:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + F_{\sigma(t)}f_a(t) + Rd(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^m$ are the state vector, the input vector and the output measurement vectors, respectively. $f_a(t) \in \mathbb{R}^q$ represents the actuator fault and $d(t) \in \mathbb{R}^l$ denotes the unknown external disturbances. The switching signal $\sigma(t) : \mathbb{R}^+ \to \Theta = \{1, 2, ..., N\}$ is a piecewise constant function and continuous from the right everywhere and N denotes the number of subsystems. When $\sigma(t) = i$, the i-th subsystem is activated for some $i \in \Theta$. In this case, the system (1) can be rewritten in the form (2):

$$\begin{cases}\dot{x}(t) = A_i x(t) + B_i u(t) + F_i f_a(t) + Rd(t)\\ y(t) = Cx(t)\end{cases} \tag{2}$$

Before starting the design of the switched SMO, the following assumptions are required:
Assumption 1.

$$rank(CF_i) = rank(F_i) = q$$

Assumption 2. The number of actuator faults and disturbances are assumed to satisfy the condition:

$$q + l \leqslant m$$

Assumption 3. We assume that $\|f_a(t)\| \leqslant \alpha_1$ holds for α_1 is a non-negative real number.
Assumption 4. All pair (A_i, C) are observable $\forall i \in \Theta$.

$$rank\begin{bmatrix} C \\ CA_i \\ \vdots \\ CA_i^{n-1}\end{bmatrix} = n$$

Initially, we introduce a definition and a lemma that establish the basis for defining the ADT concept and for presenting corresponding stability results associated with this concept.
Definition 1 [25]. Consider the time interval (τ_1, τ_2) of the switched system, and let $N_{\sigma(t)}(\tau_1, \tau_2)$ denotes switching numbers of $\sigma(t)$ during the interval (τ_1, τ_2). If the following condition holds:

$$N_{\sigma(t)}(\tau_1, \tau_2) \leqslant N_0 + \frac{\tau_2 - \tau_1}{\tau_a} \tag{3}$$

where $N_0 \geq 0$, then the constant $\tau_a > 0$ is called the ADT.
Lemma 1 [26]. Suppose that exist C^1 functions $V_{\sigma(t)} : \mathbb{R}^n \to \mathbb{R}$, two class K_∞ functions κ_1 and κ_2 and two constants $\alpha > 0$, $\mu > 1$ such that we have

$$\kappa_1(\|x(t)\|) \leq V_{\sigma(t)}(x(t)) \leq \kappa_2(\|x(t)\|) \tag{4}$$

$$\dot{V}_{\sigma(t)}(x(t)) \leqslant -\alpha V_{\sigma(t)}(x(t)) \tag{5}$$

and $\forall(\sigma(t_l) = i, \sigma(t_l{}^-) = j) \in N \times N, i \neq j$,

$$V_i(x(t)) \leqslant \mu V_j(x(t)) \tag{6}$$

Then the switched linear system $\dot{x}(t) = f_{\sigma(t)}(x(t))$ is GUAS for any switching signal with ADT:

$$\tau_a \geqslant \tau_a^* = \frac{\ln \mu}{\alpha} \tag{7}$$

3. MAIN RESULTS

In this part, we present a solution to the fault diagnosis problem based on SMO. Firstly, the model of a fault diagnosis SMO is given. Next, we employ the Lyapunov-Krasovskii functional method to ensure the estimation error stability and derive LMIs to establish sufficient conditions. The observer gain matrices are then computed by solving a set of LMI constraints. Finally, we describe the method for reconstructing actuator faults based on the concept of equivalent output injection.

3.1. Sliding mode observer design

The designed SMO for the switched system model (2) is described as (3):

$$\begin{cases} \dot{z}(t) = N_i z(t) + G_i u(t) + L_i y(t) - \Upsilon_i \nu(t) \\ \hat{x}(t) = z(t) + T_2 y(t) \\ \hat{y}(t) = C\hat{x}(t) \\ e_y(t) = y(t) - \hat{y}(t) \end{cases} \tag{8}$$

where $z(t) \in \mathbb{R}^n$, $\hat{x}(t) \in \mathbb{R}^n$, $\hat{y}(t) \in \mathbb{R}^m$ and $\nu(t)$ are respectively the observer state vector, the estimated state vector, the estimated output vector and the discontinuous switched component to induce a sliding motion [27]. $e_y(t)$ is the observer residual signal which is the difference between the measured output $y(t)$ and the estimated output $\hat{y}(t)$. The gain matrix $\Upsilon_i \in \mathbb{R}^{n\times m}$ is the feedforward injection map. $N_i \in \mathbb{R}^{n\times n}$, $G_i \in \mathbb{R}^{n\times p}$, $L_i \in \mathbb{R}^{n\times m}$and $T_2 \in \mathbb{R}^{n\times m}$ are the observer gains to be determined.

Defining the state error vector $e_x(t) = x(t) - \hat{x}(t)$, then from (8) we have:

$$e_x(t) = x(t) - z(t) - T_2 y(t) = (I_n - T_2 C)x(t) - z(t) \tag{9}$$

Since that for $rank \begin{bmatrix} I_n \\ C \end{bmatrix} = n$, there exists nonsingular matrices $T_1 \in \mathbb{R}^{n\times n}$ and $T_2 \in \mathbb{R}^{n\times m}$ such that:

$$T_1 + T_2 C = I_n \tag{10}$$

Then, the state error vector (9) is described by (11):

$$e_x(t) = T_1 x(t) - z(t) \tag{11}$$

Consequently, the dynamic of the state estimation error is governed by (12):

$$\dot{e}_x(t) = T_1 \dot{x}(t) - \dot{z}(t) \tag{12}$$

By substituting the expressions for $\dot{x}(t)$ and $\dot{z}(t)$ derived from (2) and (8) respectively, (12) becomes after some calculations:

$$\begin{aligned} \dot{e}_x(t) = {} & N_i e_x(t) + T_1 R d(t) + T_1 F_i f_a(t) + \Upsilon_i \nu(t) + (T_1 B_i - G_i)u(t) \\ & + (T_1 A_i + N_i T_2 C - L_i C - N_i)x(t) \end{aligned} \tag{13}$$

If we choose the matrices

$$G_i = T_1 B_i \tag{14}$$

$$E_i = N_i T_2 - L_i \tag{15}$$

$$N_i = T_1 A_i + E_i C \tag{16}$$

$$T_1 R = 0 \tag{17}$$

Then, it is easy to show that the error dynamics (13) can be rearranged as (18):

$$\dot{e}_x(t) = N_i e_x(t) + T_1 F_i f_a(t) + \Upsilon_i \nu(t) \tag{18}$$

To simultaneously determine the matrices T_1 and T_2 from equations (10) and (17), one can introduce the following augmented matrix:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} I_n & R \\ C & 0 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \tag{19}$$

Under Assumption 3, [28], a particular solution of (19) using the generalized inverse matrix is computed by:

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} I_n & R \\ C & 0 \end{bmatrix}^+ \tag{20}$$

According to (18), the dynamics of output estimation error $e_y(t)$ can be given by:

$$\dot{e}_y(t) = [CN_i e_x(t) + CT_1 F_i f_a(t) + C\Upsilon_i \nu(t)] \tag{21}$$

An ideal sliding mode occurs under conditions where $e_y(t) = 0$, $\dot{e}_y(t) = 0$. In (8) $\nu(t)$ is the external feed-forward compensation signal but in (21) it acts as an input signal. From (21), a virtual equivalent feedforward signal [18] may be described for each $C\Upsilon_i \neq 0$ as (22):

$$\nu(t) = -(C\Upsilon_i)^{-1} [CN_i e_x(t) + CT_1 F_i f_a(t)] \tag{22}$$

Therefore, the discontinuous output error injection term $\nu(t)$ has the following structure:

$$\nu(t) = \rho \operatorname{sgn}(e_y(t)) = \rho \frac{e_y(t)}{\|e_y(t)\|}, e_y(t) \neq 0 \tag{23}$$

where ρ is a positive scalar with:

$$\rho \geqslant \frac{\|T_1\| \, \|F_i\|}{\|\Upsilon_i\|} \alpha_1 \tag{24}$$

To obtain the gain matrices and the feedforward compensation signal $\nu(t)$ to ensure asymptotic stability of the SMO (8), the following theorem is provided for designing the proposed SMO.
Theorem 1. For given constants scalars $\alpha > 0$, $\mu > 1$, β, ϵ and $\gamma > 0$, if there exist matrices $P_i = {P_i}^T > 0$ and matrices $W_i = P_i E_i$ satisfying the inequalities $\forall (i,j) \in \Theta \times \Theta, i \neq j$:

$$P_i \leqslant \mu P_j \tag{25}$$

$$\begin{bmatrix} \Pi_i & P_i(T_1 F_i) & P_i \Upsilon_i \\ * & -\frac{1}{\varepsilon} I_p & 0 \\ * & * & -I_m \end{bmatrix} \leqslant 0 \tag{26}$$

where

$$\Pi_i = (T_1 A_i)^T P_i + P_i(T_1 A_i) + \alpha P_i + \beta I_n + C^T {W_i}^T + W_i C + \gamma^2 I_n \tag{27}$$

Then, for the switching signal $\sigma(t)$ with ADT constraint (7), there exists a SMO (8) for the switched system (2) such that the state estimation error (18) is GUAS.
Proof. In this part, we demonstrate the convergence property of the state estimations errors. Consider the following multiple Lyapunov function:

$$V_i(e_x(t)) = e_x^T(t) P_i e_x(t) \tag{28}$$

where $P_i = {P_i}^T > 0$. Based on (18), the derivative of $V_i(e_x(t))$ is obtained as (29):

$$\dot{V}_i(e_x(t)) = e_x^T(t) \left[N_i^T P_i + P_i N_i\right] e_x(t) + 2e_x^T(t) P_i T_1 F_i f_a(t) + 2e_x^T(t) P_i \Upsilon_i \nu(t) \tag{29}$$

If $\|f_a(t)\| \leqslant \alpha_1$, then:

$$\dot{V}_i(e_x(t)) \leq e_x^T(t) \left[N_i^T P_i + P_i N_i\right] e_x(t) + 2\alpha_1 \left\|e_x^T(t) P_i T_1 F_i\right\| + 2e_x^T(t) P_i \Upsilon_i \nu(t) \tag{30}$$

For any positive scalar β, we have the inequality:

$$2\alpha_1 \left\|e_x^T(t) P_i (T_1 F_i)\right\| \leqslant \beta^{-1} {\alpha_1}^2 \left\|e_x^T(t) P_i (T_1 F_i)\right\|^2 + \beta \tag{31}$$

Hence,

$$\begin{aligned} \dot{V}_i(e_x(t)) \leq{} & e_x^T(t) \left[N_i^T P_i + P_i N_i\right] e_x(t) + \beta^{-1} {\alpha_1}^2 \left\|e_x^T(t) P_i (T_1 F_i)\right\|^2 + \beta \\ & + e_x^T(t) P_i \Upsilon_i \nu(t) + \nu^T(t) \Upsilon_i^T P_i e_x(t) \end{aligned} \tag{32}$$

By taking $\varepsilon = \beta^{-1}{\alpha_1}^2$ in (32) implies:

$$\begin{aligned}\dot{V}_i(e_x(t)) \leq e_x^T(t)\left[N_i^T P_i + P_i N_i\right] e_x(t) + \varepsilon\left\|e_x^T(t) P_i (T_1 F_i)\right\|^2 + \beta \\ + e_x^T(t) P_i \Upsilon_i \nu(t) + \nu^T(t)\Upsilon_i^T P_i e_x(t)\end{aligned} \tag{33}$$

For $\|\nu(t)\| \leqslant \gamma \|e_x(t)\|$, the state estimation error converges asymptotically to zero and the gain from $\nu(t)$ to $e_x(t)$ is bounded by γ if:

$$\dot{V}_i(e_x(t)) + \alpha V_i(e_x(t)) - \nu^T(t)\nu(t) + \gamma^2 e_x^T(t) e_x(t) \leqslant 0 \tag{34}$$

Then,

$$\begin{aligned}&e_x^T(t)\left[N_i^T P_i + P_i N_i\right] e_x(t) + \alpha e_x^T(t) P_i e_x(t) + \varepsilon\left\|e_x^T(t) P_i (T_1 F_i)\right\|^2 + \beta \\ &+ e_x^T(t) P_i \Upsilon_i \nu(t) + \nu^T(t)\Upsilon_i^T P_i e_x(t) + \gamma^2 e_x^T(t) e_x(t) - \nu^T(t)\nu(t) \leq 0\end{aligned} \tag{35}$$

The inequality (35) can be reformulated as (36):

$$\begin{bmatrix} e_x^T(t) & \nu^T(t) \end{bmatrix}^T \Xi_i \begin{bmatrix} e_x(t) \\ \nu(t) \end{bmatrix} \leqslant 0 \tag{36}$$

where

$$\Xi_i = \begin{bmatrix} \tilde{\Delta}_i & P_i \Upsilon_i \\ \Upsilon_i^T P_i & -I_m \end{bmatrix}$$

with

$$\tilde{\Delta}_i = N_i^T P_i + P_i N_i + \alpha P_i + \beta I_n + \varepsilon P_i (T_1 F_i)(T_1 F_i)^T P_i + \gamma^2 I_n \tag{37}$$

Then, the inequality (36) is satisfied if the condition holds $\forall i \in \Theta$

$$\Xi_i \leqslant 0 \tag{38}$$

Changing N_i by their parameter (16) and define $W_i = P_i E_i$ with the application of Schur complement, the inequality (38) becomes:

$$\begin{bmatrix} \Pi_i & P_i(T_1 F_i) & P_i \Upsilon_i \\ * & -\frac{1}{\varepsilon} I_p & 0 \\ * & * & -I_m \end{bmatrix} \leqslant 0 \tag{39}$$

where

$$\Pi_i = (T_1 A_i)^T P_i + P_i (T_1 A_i) + \alpha P_i + \beta I_n + C^T {W_i}^T + W_i C + \gamma^2 I_n$$

This implies that $e_x(t)$ converges to zero based on Lyapunov stability theory in the presence of fault and signal $\nu(t)$. By selecting $\Upsilon_i = P_i^{-1} C^T$ [20], the linearity of inequality (39) with respect to P_i and W_i allows for a numerical solution within the LMI framework. This concludes the proof.

3.2. Actuator fault reconstruction approach

In this section, assuming the well-designed SMO gains specified in (7), we propose an efficient approach for the actuator fault reconstruction procedure. Relying on the results of Theorem 1, one obtains that $e_y(t) = \dot{e}_y(t) = 0$ as $t \to \infty$ then (22) becomes:

$$C T_1 F_i f_a(t) + C \Upsilon_i \nu(t) \tag{40}$$

Consequently,

$$C T_1 F_i f_a(t) = -C \Upsilon_i \nu(t) \tag{41}$$

Suppose that the discontinuous term $\nu(t)$ is replaced by a continuous approximation $\nu_\delta(t)$ using an alternative approach, as described in [27], and by employing (22) and (23):

$$\nu_\delta(t) = \alpha_1 \frac{\|T_1\|\,\|F_i\|}{\|\Upsilon_i\|} \frac{e_y(t)}{\|e_y(t)\| + \delta} \tag{42}$$

where δ is a small positive scalar selected to mitigate the chattering effect in the sliding motion. It can be shown that the discontinuous component $\nu(t)$ can be approximated to any desired degree of accuracy by (42) for a sufficiently small choice of δ. Since $rank(CT_1F_i) = q$ it follows from (41) that:

$$\hat{f}_a(t) \approx -(CT_1F_i)^+ C\Upsilon_i \nu_\delta(t) \tag{43}$$

Thus, the actuator fault can be approximated by this way.
Remark 1. While Essabre *et al.* [29] focused on analyzing constant actuator faults, our paper addresses the issue of time-varying actuator faults.
Remark 2. Du *et al.* [30] introduced unknown input observers (UIO) for switched linear systems. However, this technique is primarily designed to detect and estimate faults where the q-order derivative of the fault is zero. In contrast, our work addresses various types of time-varying actuator fault signals.
Remark 3. Johnson *et al.* [31] employ the common Lyapunov functional approach to assess the stability of the SMO design for continuous-time switched systems . Although widely utilized, this method has certain limitations, particularly in finding a common Lyapunov function that is applicable to all subsystems. In contrast, our approach in this study guarantees the global stability of the SMO by integrating the multiple Lyapunov functional approach with the ADT concept. This integration enables us to achieve less conservative outcomes compared to the utilization of a common Lyapunov function.

4. AN ILLUSTRATIVE EXAMPLE

In this section, we provide a simulated example to demonstrate the performance of the robust sliding mode observer using a numerical example. We examine a continuous-time switched system of the form (2), as proposed in [32] with $N = 2$ subsystems. The studied system is defined by:
Subsystem 1:

$$A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 2.5 & -1 & 0.5 \\ 0 & 1 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & -3 \\ 1 & 2.5 \\ 0 & 1 \end{bmatrix}, F_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, R = \begin{bmatrix} 0.7 \\ 0.8 \\ 0.5 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & -3 \\ 1 & 2.5 \\ 0 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, R = \begin{bmatrix} 0.7 \\ 0.8 \\ 0.5 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The LMIs are solved using the MATLAB Yalmip toolbox. The parameters $\beta = 0.5$ and $\gamma = 0.1$ are chosen to solve the optimization problem in Theorem 1. The designed SMO gains are obtained as:
For $i = 1$

$$N_1 = 10^6 \times \begin{bmatrix} 0.6954 & -1.0702 & 0.6954 \\ 0.5961 & -0.9173 & 0.5961 \\ -1.0928 & 1.6817 & -1.0928 \end{bmatrix}, G_1 = \begin{bmatrix} -0.6667 & -2.4083 \\ 0.5 & 1.15 \\ 0.3333 & 1.6417 \end{bmatrix}$$

$$L_1 = 10^5 \times \begin{bmatrix} -3.0676 & 4.6013 \\ -2.6293 & 3.9440 \\ 4.8204 & -7.2307 \end{bmatrix}, \Upsilon_1 = \begin{bmatrix} -0.0012 & 0.0018 \\ -0.0011 & 0.0016 \\ 0.0019 & -0.0029 \end{bmatrix}$$

For $i = 2$

$$N_2 = 10^6 \times \begin{bmatrix} 0.3892 & -0.7571 & 0.3892 \\ 0.3336 & -0.6489 & 0.3336 \\ -0.6117 & 1.1897 & -0.6117 \end{bmatrix}, G_2 = \begin{bmatrix} -0.7108 & -0.2492 \\ 0.1300 & -0.0850 \\ 0.6242 & 0.3058 \end{bmatrix}$$

$$L_2 = 10^5 \times \begin{bmatrix} -2.0331 & 3.0497 \\ -1.7427 & 2.6140 \\ 3.1949 & -4.7924 \end{bmatrix}, \Upsilon_2 = \begin{bmatrix} -0.0007 & 0.0011 \\ -0.0006 & 0.0009 \\ 0.0011 & -0.0017 \end{bmatrix}$$

For given $\mu = 2$ and $\alpha = 0.6$, one has $\tau_a^* = \frac{\ln \mu}{\alpha} = 1.1552$. A switching signal $\sigma(t)$ with an ADT of $\tau_a = 1.5 \geqslant 1.1552$ is generated and depicted in Figure 1.

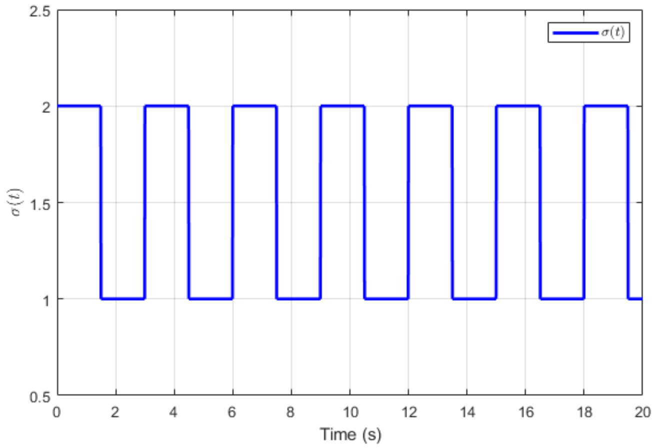

Figure 1. Switching signal $\sigma(t)$

In the simulation, the control input $u(t)$ is defined as follows:

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} 2\sin(t + 1.8) \\ 1.5\cos(2t + 3.6) \end{bmatrix}$$

To demonstrate the robustness of the SMO-based fault diagnosis design, we assume the external disturbance $d(t)$ to be Gaussian noise with a zero mean and a variance of 10^{-2}. In this example, we investigate the effectiveness of the proposed method by considering two cases of actuator faults. The first case involves actuator faults characterized by a constant value:

$$f_1(t) = \begin{cases} 0, & 0s \leqslant t < 5s \\ 1, & 5s \leqslant t < 14s \\ -1, & 14s \leqslant t \leqslant 20s \end{cases}$$

In this case, Figure 2 shows the constant actuator fault $f_1(t)$ and its estimated $\hat{f}_1(t)$.

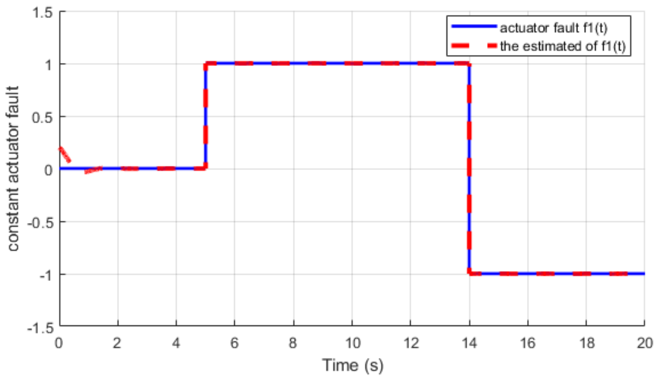

Figure 2. Constant fault $f_1(t)$ and its estimation $\hat{f}_1(t)$

If the actuator fault is a time-varying function as:

$$f_2(t) = \begin{cases} 0, & 0s \leqslant t < 4s \\ \cos(1.6t + 2.25), & 4s \leqslant t \leqslant 20s \end{cases}$$

In this case, the actuator fault $f_2(t)$ and its estimated $\hat{f}_2(t)$ are described in Figure 3.

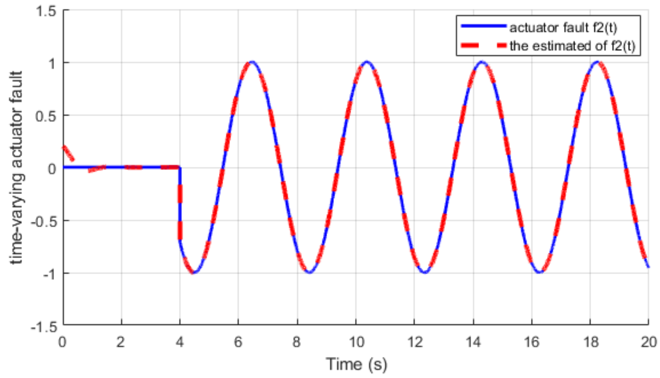

Figure 3. Time-varying fault $f_2(t)$ and its estimation $\hat{f}_2(t)$

Based on the outcomes of the simulation, it can be inferred that the method developed in this study has a good performance in estimating actuator faults, both constant and time-varying. Figures 4 to 6 showcase the system states alongside their corresponding estimates, affirming the effectiveness and precision of the designed SMO in estimating the state vector accurately. Despite the existence of unknown external disturbances, the simulation demonstrates the simultaneous achievement of accurate actuator fault and system state estimation by the developed SMO.

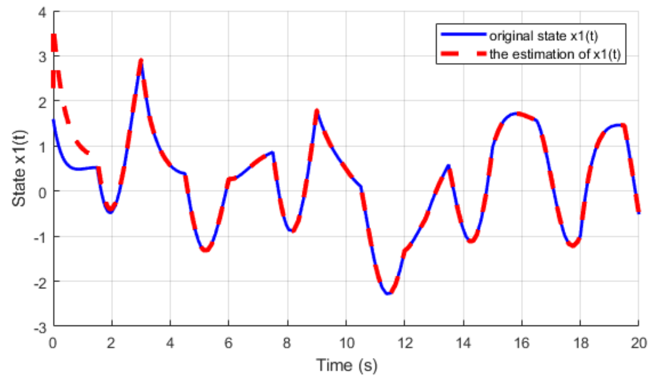

Figure 4. The state $x_1(t)$ and its estimate $\hat{x}_1(t)$

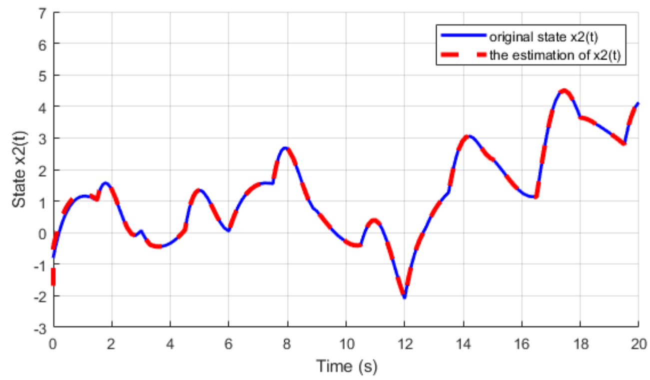

Figure 5. The state $x_2(t)$ and its estimate $\hat{x}_2(t)$

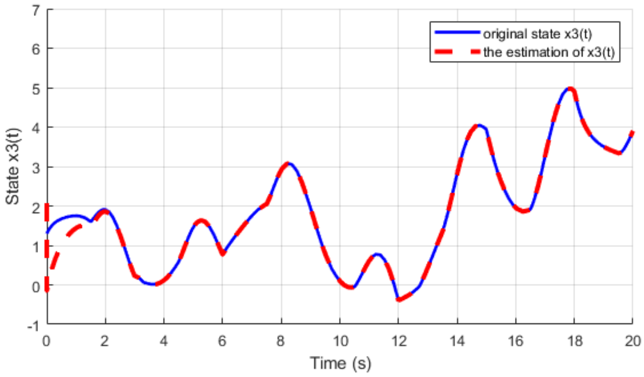

Figure 6. The state $x_3(t)$ and its estimate $\hat{x}_3(t)$

5. CONCLUSION

In this article, the problem of actuator fault reconstruction for a class of continuous-time switched systems subject to external disturbance according to ADT concept has been addressed. Employing a robust switched SMO effectively nullified the detrimental impact of disturbances on state estimation, achieving perfect reconstruction of the actuator faults. Moreover, through the utilization of the LMI method, the necessary observer matrices were synthesized efficiently. Finally, the efficiency of the designed method is verified through a numerical example. The study demonstrated that the developed methodology allows for simultaneous estimation of states and actuator faults with good performance. An open problem lies in extending these findings to construct fault tolerant controllers. For instance, the authors aim to integrate the method with fault tolerant control techniques. Leveraging the fault estimation obtained by the SMO, a reconfigurable fault tolerant control scheme can be devised to uphold satisfactory performance and stabilize the system in the event of a fault.

REFERENCES

[1] R. Goebel, R. G. Sanfelice, and A. R. Teel, "Hybrid dynamical systems," in *IEEE Control Systems Magazine*, vol. 29, no. 2, pp. 28-93, Apr. 2009, doi: 10.1109/MCS.2008.931718.

[2] Y. Eddoukali, A. Benzaouia, and M. Ouladsine, "Integrated fault detection and control design for continuous-time switched systems under asynchronous switching," *ISA transactions*, vol. 84, pp. 12-19, Oct. 2018, doi: 10.1016/j.isatra.2018.10.009.

[3] J. Li, X. Guo, C. Chen, and Q. Su, "Robust fault diagnosis for switched systems based on sliding mode observer," *Applied Mathematics and Computation*, vol. 341, pp. 193-203, Jan. 2019, doi: 10.1016/j.amc.2018.08.036.

[4] Z. Sun and S. S. Ge, "Analysis and synthesis of switched linear control systems," *Automatica*, vol. 41, no. 2, pp. 181-195, Feb. 2005, doi: 10.1016/j.automatica.2004.09.015.

[5] J. Chen and R. J. Patton, "Robust model-based fault diagnosis for dynamic systems," *Springer Science and Business Media*, vol. 3, Nov. 2012, doi: 10.1007/978-1-4615-5149-2.

[6] B. Jiang, M. Staroswiecki, and V. Cocquempot, "Fault accommodation for nonlinear dynamic systems," *IEEE Transactions on automatic Control*, vol. 51, no 9, pp. 1578-1583, Oct. 2006, doi: 10.1109/TAC.2006.878732.

[7] M. Essabre, I. Hmaiddouch, A. El Assoudi, and E. H. El Yaagoubi, "Design of unknown input observer for discrete-time Takagi Sugeno implicit systems with unmeasurable premise variables," *Bulletin of Electrical Engineering and Informatics*, vol. 12, no. 1, pp. 59-68, Feb 2023, doi: 10.11591/eei.v12i1.4107.

[8] F. J. Bejarano and A. Pisano, "Switched observers for switched linear systems with unknown inputs," *IEEE Transactions on Automatic Control*, vol. 56, no 3, pp. 681-686, Apr. 2011, doi: 10.1109/TAC.2010.2095990.

[9] K. Zhang, B. Jiang, P. Shi, and J. Xu, "Analysis and Design of Robust H_∞ Fault Estimation Observer With Finite-Frequency Specifications for Discrete-Time Fuzzy Systems," *IEEE Transactions on Cybernetics*, vol. 45, no 7, pp. 1225-1235, Sep. 2014, doi: 10.1109/TCYB.2014.2347697.

[10] K. Ouarid, M. Essabre, A. El Assoudi, and E. H. El Yaagoubi, "State and fault estimation based on fuzzy observer for a class of Takagi-Sugeno singular models," *Indonesian Journal of Electrical Engineering and Computer Science (IJEECS)*, vol. 25, no. 1, pp. 172–182, Jan. 2022, doi: 10.11591/ijeecs.v25.i1.pp172-182.

[11] J. Zhang, A. K. Swain, and S. K. Nguang, "Robust H_∞ adaptive descriptor observer design for fault estimation of uncertain nonlinear systems," *Journal of the Franklin Institute*, vol. 351, no 11, pp. 5162-5181, Nov. 2014, doi: 10.1016/j.jfranklin.2014.08.019.

[12] M. Elouni, H. Hamdi, B. Rabaoui, N. B. Braiek, and M. Rodrigues,"Robust sensor faults observer design for continuous-time switched systems using descriptor approach," *2023 20th International Multi-Conference on Systems, Signals & Devices (SSD)*, Mahdia, Tunisia, 2023, pp. 825-831, doi: 10.1109/SSD58187.2023.10411272.

[13] A. T. Vemuri, and M. M. Polycarpou, "A methodology for fault diagnosis in robotic systems using neural networks," *Robotica*, vol. 22, no. 4, pp. 419-438, Jul. 2004, doi: 10.1017/S0263574703005204.

[14] S. Ifqir, D. Ichalal, N. A. Oufroukh, and S. Mammar, “Robust interval observer for switched systems with unknown inputs: Application to vehicle dynamics estimation,” *European Journal of Control*, vol. 44, pp. 3-14, Sep. 2018, doi: 10.1016/j.ejcon.2018.09.018.

[15] X. G. Yan and C. Edwards, “Robust decentralized actuator fault detection and estimation for large-scale systems using a sliding mode observer,” *International Journal of control*, vol. 81, no. 4, pp. 591-606, Apr. 2008, doi: 10.1080/00207170701536056.

[16] M. Taouil, A. El Ougli, B. Tidhaf, and H. Zrouri, “Sensor fault reconstruction for wind turbine benchmark model using a modified sliding mode observer,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 13, no. 5, pp. 5066-5075, Oct 2023, doi: 10.11591/ijece.v13i5.pp5066-5075.

[17] C. P. Tan and C. Edwards, “An LMI approach for designing sliding mode observers,” *International Journal of Control*, vol. 74, no. 16, pp. 1559-1568, Feb. 2001, doi: 10.1109/CDC.2000.914193.

[18] T. K. Yeu, H. S. Kim, and S. Kawaji, “Fault detection, isolation and reconstruction for descriptor systems,” *Asian Journal of Control*, vol. 7, no. 4, pp. 356-367, Dec. 2005, doi: 10.1111/j.1934-6093.2005.tb00398.x.

[19] H. Hamdi, M. Rodrigues, C. Mechmeche, and N. Benhadj Braiek, “Fault diagnosis based on sliding mode observer for LPV descriptor systems,” *Asian Journal of Control*, vol. 21, no. 1, pp. 89-98, Jan. 2019, doi: 10.1002/asjc.2022.

[20] R. Shahnazi and Q. Zhao, “ Adaptive fuzzy descriptor sliding mode observer-based sensor fault estimation for uncertain nonlinear systems,” *Asian Journal of Control*, vol. 18, no. 4, pp. 1478-1488, Nov. 2015, doi: 10.1002/asjc.1249.

[21] L. Islem, T. Asma, and Z. Nadia, “Hybrid sliding mode observer for uncertain linear switching system with actuator faults,” *Transactions of the Institute of Measurement and Control*, vol. 41, no 15, pp. 4301-4310, Jan. 2019, doi: 10.1177/0142331219856198.

[22] S. Yin, H. Gao, J. Qiu, and O. Kaynak, “Descriptor reduced-order sliding mode observers design for switched systems with sensor and actuator faults,” *Automatica*, vol. 76, pp. 282-292, Feb. 2017, doi: 10.1016/j.automatica.2016.10.025.

[23] J. Li, K. Pan, and Q. Su, “Sensor fault detection and estimation for switched power electronics systems based on sliding mode observer,” *Applied Mathematics and Computation*, vol. 353, pp. 282-294, Jul. 2019, doi: 10.1016/j.amc.2019.02.010.

[24] S. Ali, M. T. Raza, G. Abbas, N. Ullah, S. Al Otaibi, and H. Luo, “Sliding Mode Observer-Based Fault Detection in Continuous Time Linear Switched Systems,” *Energies*, vol. 15, no. 3, pp. 1090, Feb. 2022, doi: 10.3390/en15031090.

[25] P. Gahinet and P. Apkarian, “An linear matrix inequality approach to H_∞ control, ” *International journal of robust and nonlinear control* , vol. 4, no. 4, pp. 421-448, Jul. 1994, doi: 10.1002/rnc.4590040403.

[26] L. Zhou, D. W. Ho, and G. Zhai, “Stability analysis of switched linear singular systems,” *Automatica*, vol. 49, no. 5, pp. 1481-1487, May 2013, doi: 10.1016/j.automatica.2013.02.002.

[27] C. Edwards, S. K. Spurgeon, and R. J. Patton, “Sliding mode observers for fault detection and isolation,” *Automatica*, vol. 36, no. 4, pp. 541-553, Apr. 2000, doi: 10.1016/S0005-1098(99)00177-6.

[28] M. Tabatabaeipour and T. Bak, “Robust observer-based fault estimation and accommodation of discrete time piecewise linear systems,” *Journal of the Franklin Institute*, vol. 351, no. 1, pp. 277-295, Jan. 2013, doi: 10.1016/j.jfranklin.2013.08.021.

[29] M. Essabre, I. Hmaiddouch, A. El Assoudi, and E. H. El Yaagoubi, “Unknown input observer for Takagi-Sugeno implicit models with unmeasurable premise variables,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 13, no. 5, pp. 5035–5046, Oct. 2023, doi: 0.11591/ijece.v13i5.pp5035-5046.

[30] D. Du, V. Cocquempot, and B. Jiang, “Robust fault estimation observer design for switched systems with unknown input,” *Applied Mathematics and Computation*, vol. 348, pp. 70-83, May 2019, doi: 10.1016/j.amc.2018.11.034.

[31] S. C. Johnson, A. Chakrabarty, J. Hu, S. H. Żak, and R. A. DeCarlo, “Dual-mode robust fault estimation for switched linear systems with state jumps,” *Nonlinear Analysis: Hybrid Systems*, vol. 27, pp. 125-140, Feb. 2018, doi: 10.1016/j.nahs.2017.08.009.

[32] J. Yang, Y. Chen, F. Zhu, and F. Wang, ”Simultaneous state and output disturbance estimations for a class of switched linear systems with unknown inputs,” *International Journal of Systems Science*, vol. 48, no. 1, pp. 22-33, Feb. 2016, doi: 10.1080/00207721.2016.1144227.

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