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Recent developments in vehicle routing problem under time uncertainty: a comprehensive review

Akhmetbek Yernar, Cemil Turan
Department of Computer Science, Faculty of Engineering and Natural Sciences, SDU University, Kaskelen, Kazakhstan

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ABSTRACT

This review paper examines recent advancements in vehicle routing optimization under time uncertainty, focusing on the vehicle routing problem (VRP). It systematically analyzes research papers to identify strategies for optimizing routes despite temporal uncertainties, covering key areas such as optimization algorithms, uncertainty modeling techniques, and simulation methods. The study investigates dynamic dispatching models, reliability considerations, and multi-objective optimization approaches. By synthesizing existing literature, this paper presents the current state of research in vehicle routing under time uncertainty and suggests potential future research directions. Our findings indicate that integrating robust optimization techniques with advanced simulation methods could significantly enhance decision-making processes in uncertain environments. Additionally, the paper highlights the role of machine learning and artificial intelligence in developing adaptive algorithms that respond to dynamic changes in real-time. As the need for efficient logistics solutions grows, this comprehensive review underscores the importance of addressing uncertainties in vehicle routing to improve operational efficiency, reduce costs, and enhance customer satisfaction.

Corresponding Author:

Cemil Turan
Department of Computer Science, Faculty of Engineering and Natural Sciences, SDU University
Kaskelen, Almaty, Kazakhstan
Email: cemil.turan@sdu.edu.kz

1. INTRODUCTION

In transportation logistics, the vehicle routing problem (VRP) is a basic task that optimizes vehicle routing to serve a group of geographically dispersed consumers. In practical operations, vehicles and drivers encounter a myriad of uncertainties that can significantly impact the planning and execution of routes. Factors such as uncertain customer demands, variable travel times due to traffic congestion or weather conditions, and unpredictable service times pose formidable challenges to route optimization [1]. Neglecting these uncertainties can lead to route failures, service delays, and diminished customer satisfaction, ultimately affecting the profitability of logistics operations. As highlighted by Yang *et al.* [2], uncertainties in travel times and service requirements are inherent in real-world scenarios, making it imperative to address these uncertainties in VRP.

The optimization of vehicle routing under time uncertainties presents a complex challenge in transportation logistics, where dynamic travel times and service requirements introduce significant planning complexities. Uncertainties from factors like traffic congestion and unpredictable delays can disrupt planned routes, leading to suboptimal solutions and increased costs. Additionally, research mentions the trade-off between cost and customer service in stochastic VRPs, highlighting the need for robust optimization strategies to balance conflicting objectives [3]. Neglecting time uncertainties can result in route failures, delays, and reduced customer satisfaction, underscoring the importance of addressing uncertainties to enhance operational efficiency and service quality [2].

VRPs under time uncertainties arise from the intricate nature of transportation logistics. The dynamic landscape of travel times and service demands introduces a layer of complexity that challenges traditional routing strategies. Factors like traffic congestion [4], unexpected delays, and fluctuating customer requirements can disrupt planned routes, leading to inefficiencies and increased operational costs. By exploring this domain, researchers aim to develop robust optimization models that can effectively navigate these uncertainties and enhance the resilience and reliability of routing solutions. This review seeks to consolidate existing knowledge, identify research gaps, and pave the way for innovative approaches to address the evolving challenges in VRPs under time uncertainties.

The objectives of this review encompass a comprehensive exploration of the challenges and opportunities in VRPs under time uncertainties, drawing upon the contributions of various researchers in the field. The upcoming paper will thoroughly explore how robust optimization and metaheuristic algorithms can improve vehicle routing. It will explain how research methodology of the work, categorize different routing problems, discuss various solution methods, and summarize the main findings.

2. METHOD

Figure 1 depicting the distribution of published papers over the years reveals notable trends in the evolution of research on vehicle routing problems with time uncertainty (VRPTU). Initially, the field saw modest activity, with the first papers emerging around 2005-2006. However, since approximately 2016, there has been a discernible surge in the popularity and scholarly attention directed toward VRPTU, as evidenced by the increasing number of published papers. This upward trend suggests a growing recognition of the significance and complexity of VRPTU challenges within the academic and research communities, prompting intensified investigations and advancements in solution methodologies and approaches.

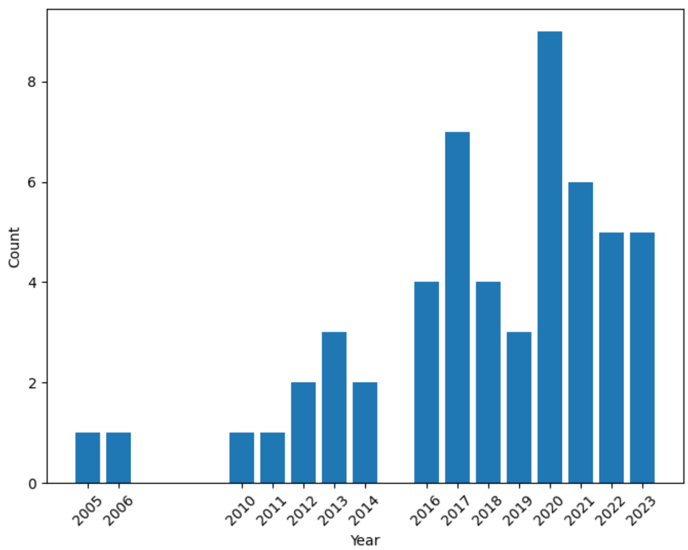

Figure 1. Published article count distributed by year

2.1. Article collection

In conducting this review, a systematic approach was followed to gather relevant literature. Initially, a thorough search was performed using specific keywords such as ”vehicle,” ”routing,” ”problem,” ”time,” and ”uncertainty” in reputable databases like Scopus. This search yielded a substantial number of articles.

Subsequently, each article underwent scrutiny to ensure it directly addressed the focal point of our study: the VRP under time uncertainty. Articles that only touched on related aspects without a substantial focus on VRP and temporal uncertainties were excluded.

Furthermore, a detailed examination of the selected articles was conducted to determine if they discussed stochastic or dynamic elements inherent in VRP. Review and survey papers, though informative, were omitted to prioritize primary research contributions and empirical studies. This rigorous method identified 54 highly relevant research articles that significantly contribute to understanding recent advancements in optimizing vehicle routes amidst time uncertainties.

2.2. Article distribution and sorting

This review incorporates a structured taxonomy to categorize and analyze recent developments in VRPs under time uncertainty. Several dimensions have been introduced to offer a comprehensive framework for understanding the complexities of dynamic routing problems. The selected categories for analysis are as follows:

a. Uncertainty: uncertainty refers to the unpredictability or variability of temporal factors in VRPs. This uncertainty encompasses three primary components: service time, travel time, and demand uncertainties.
b. Nature: identifies sources of uncertainty (e.g., vehicle speeds and traffic congestion) influencing decision-making in routing optimization.
c. Solution methods: differentiates between algorithmic approaches (e.g., exact, heuristic, metaheuristic, and reinforcement learning) used in routing optimization.
d. Objective function: guides solution approaches by outlining optimization goals such as minimizing vehicle count or travel costs.
e. Fleet size: categorizes fleets as single or multiple vehicles, considering their impact on dynamic or stochastic problem structures.
f. Benchmark: describes benchmark problems used for evaluation (e.g., Solomon benchmarks), aiding in algorithm performance assessment.
g. Time constraints: encompasses constraints like time windows (soft, hard, or deadline-based) and service times, crucial for dynamic problem-solving.

3. TAXONOMY

This chapter delves into the multifaceted nature of uncertainties encountered in VRP under time uncertainty. Through a systematic categorization, this chapter aims to provide a structured framework for understanding and addressing the diverse challenges posed by dynamic and stochastic elements in routing optimization. Table 1 (in Appendix) [1]-[55] presents a comprehensive overview of vehicle routing optimization under time uncertainty, encapsulating key aspects such as solution methods, fleet configurations (single vs multi), types of uncertainties, nature of uncertainties (e.g., traffic conditions, demand variations), time constraints (e.g., hard or soft time windows), additional considerations or features, and benchmarking criteria. The ”Authors” column lists the researchers associated with the studies discussed, while the ”#,” or numerical identifier, likely denotes the sequence of entries. Each row in the table represents a unique study or paper, showcasing the diverse methodologies and approaches used to address the complexities of VRPs amidst time uncertainty. Each subsection of the taxonomy delineates specific aspects of uncertainty, drawing insights from existing literature and highlighting key considerations essential for designing robust routing strategies. This taxonomy establishes the foundation for thorough analysis and solution development in the field of VRP in the face of time uncertainty by clarifying the nature of uncertainty in multiple dimensions.

3.1. Uncertainty

Uncertainty in the VRP refers to the unpredictability or variability in factors influencing route planning and optimization. This uncertainty can manifest in various forms, such as fluctuating travel times due to traffic conditions, unpredictable service durations at customer locations, uncertain demand patterns, or unexpected disruptions like vehicle breakdowns. In VRP-TU problems, uncertainties are classified into three categories: travel uncertainty due to unpredictable travel times, service uncertainty arising from uncertain service durations, and demand uncertainty stemming from fluctuations in customer demand.

3.1.1. Service time

Service times present a challenging optimization scenario in logistics and transportation management. In this variant of VRP, the exact duration of service at customer locations is uncertain, leading to dynamic scheduling complexities. For instance, Goel *et al.* [51] addressed this challenge by proposing a robust VRP model that incorporates probabilistic service time distributions and utilizes a tabu search (TS) algorithm to resolve the issue efficiently. Similarly, Režnar *et al.* [6] developed a stochastic VRP formulation considering uncertain service times, employing an improved differential evolution (IDE) algorithm to determine route feasibility under varying service time scenarios. Furthermore, Wu *et al.* [17] tackled vehicle disruption uncertainty using the TS algorithm, addressing challenges such as traffic accidents that affect vehicle operations. These studies exemplify the diverse methodologies and techniques employed to tackle the VRP with uncertainty constraints on service times, emphasizing the importance of robust and flexible routing solutions in dynamic operational environments.

3.1.2. Travel time

Travel time poses a significant challenge in logistics optimization due to its uncertainty, influenced by factors like traffic variations, weather conditions, and unforeseen events affecting route efficiency. For example, Hammouti *et al.* [5] proposed a proactive strategy considering vehicle travel time uncertainty, integrating modified clustering search-based genetic algorithm (MCSGA) approaches for route robustness evaluation [56]. Lai *et al.* [19] addressed travel time uncertainty in VRP using a hybrid genetic algorithm (GA) and neighbor search algorithm for route optimization. Additionally, Mańdziuk and Świechowsk [38] developed a VRP model considering dynamic travel time uncertainties caused by traffic congestion, enhancing route planning adaptability through simulation-based optimization techniques. These studies showcase diverse strategies for managing uncertainty in travel time within VRP, highlighting the necessity of adaptive routing solutions in dynamic operational contexts.

3.1.3. Demand

Demand introduces complexities in logistics planning due to uncertainty in customer requests for goods or services. This uncertainty can result from fluctuating market demands, seasonal changes, or unexpected shifts in customer preferences. Yang *et al.* [2] tackled uncertain customer demands, travel times, and service times by optimizing the trade-off between total travel time and driver consistency, proposing a hybrid algorithm for efficient problem-solving. Amiri *et al.* [55] presented a robust mathematical model for the electric vehicle routing problem (EVRP) with heavy-duty battery electric trucks, addressing energy consumption uncertainty and optimizing transportation costs while maximizing customer satisfaction using metaheuristic algorithms. Rajabi-Bahaabadi *et al.* [54] addressed the VRP with soft time windows and correlated stochastic arc travel times through a stochastic programming model and a hybrid algorithm combining TS with an ant system for solution. These studies demonstrate various approaches to handling demand uncertainty in logistics, emphasizing optimization and efficiency in route planning.

3.2. Nature

This uncertainty stems from various sources, such as traffic jams, unpredictable road conditions, unexpected vehicle disruptions, and other real-time events that impact travel times and service schedules. For instance, Zhang *et al.* [4] identified the impact of traffic congestion on time-constrained VRP, proposing adaptive routing strategies using real-time traffic data to mitigate delays and optimize route efficiency. Similarly, Feng *et al.* [14] studied the effects of road conditions and weather-related disruptions on VRP, developing robust optimization models to handle uncertain time constraints effectively. Wu *et al.* [17] investigated the implications of vehicle disruptions and unexpected events on route planning in VRP, introducing dynamic scheduling algorithms to adjust routes in response to real-time disruptions. These studies underscore the critical need for adaptive and responsive routing solutions in managing uncertainty related to time constraints in VRP, highlighting the importance of real-time data integration and dynamic optimization algorithms.

3.3. Fleet size

The size of the fleet plays a critical role in logistics optimization. For single-fleet scenarios, where a singular set of vehicles is deployed, Chen *et al.* [35] highlighted the challenge of managing uncertain time constraints efficiently, emphasizing the need for adaptive routing strategies and real-time data integration to optimize route schedules and minimize delays. Conversely, in multi-fleet environments involving multiple types of vehicles with varying capacities and capabilities, Çimen and Soysa [47] addressed the complexities of coordinating diverse fleets under uncertain time constraints, proposing hybrid optimization algorithms that balance workload distribution, fleet utilization, and route efficiency to enhance overall operational performance. These studies underscore the importance of fleet size considerations in navigating uncertainty in time-constrained VRP, highlighting the nuanced strategies required for effective route planning and fleet management.

3.4. Benchmark

Utilizing benchmark datasets like the Solomon dataset, which provides standardized instances for evaluating routing algorithms, researchers focus on optimizing routes under uncertain time constraints. For instance, Goel *et al.* [51] addressed uncertain time constraints in VRP using the Solomon dataset, developing adaptive routing strategies to minimize delays and improve route efficiency. Conversely, using real-life city examples as benchmark data introduces dynamic real-world factors such as traffic patterns and road conditions into VRP models, enhancing their applicability. Feng *et al.* [14] tackled uncertain time constraints in VRP using real-life city data, especially considering cities of England, proposing data-driven optimization algorithms to optimize routes based on real-time conditions adaptively. Additionally, researchers may generate random instances for benchmarking, allowing for controlled experimentation and evaluation of VRP solutions under varying degrees of uncertainty as [9], [15], [29], [30]. These studies underscore the importance of benchmark data selection, whether from standardized datasets like Solomon, real-life city scenarios, or randomly generated instances, in developing robust and applicable solutions for time-constrained VRP scenarios.

3.5. Time constraints

The categorization of time windows as soft, hard, deadlines, or non-existent (no time windows) has a significant impact on route optimization solutions for solving the VRP problem with uncertain time constraints. Figure 2 illustrates the distribution of different time window constraints in VRPs, showing a predominance of ”hard” constraints, followed by ”soft” and ”yes” constraints, with ”no”, ”deadlines”, and unspecified constraints being less common. Soft time windows allow for flexibility, permitting slight deviations from scheduled delivery times with potential penalties for delays. On the other hand, hard time windows impose strict constraints, mandating deliveries within specific time frames without exceptions. For instance, [34], [40], [46], [50], [57] addressed VRP with soft time windows, optimizing routes while accommodating delivery variations within acceptable limits. Conversely, [1], [39], [45], [53] focused on VRP with hard time windows, ensuring deliveries occur within predefined time frames. Additionally, [14], [22], [38] explored VRP without time windows, optimizing routes without any specified time constraints, allowing for maximum flexibility in delivery schedules. These studies underscore the importance of tailoring route optimization approaches to different time window variants or the absence thereof, based on operational needs and constraints. Deadline constraints, characterized by lower bound time windows, impose a minimum time limit for deliveries, balancing delivery punctuality and operational flexibility [7], [16], [21].

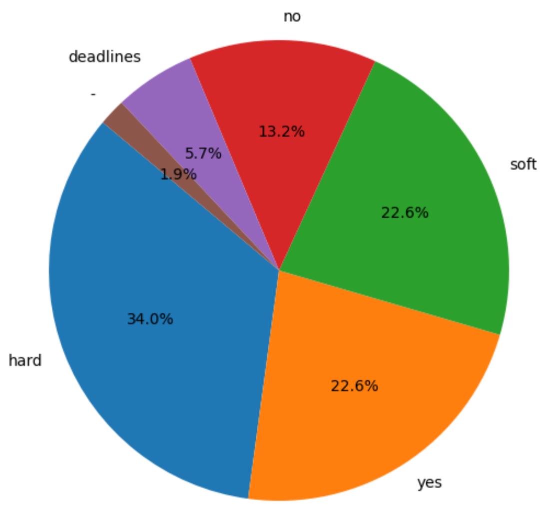

Figure 2. Time window constraint distributions

4. SOLUTION METHODS

In addressing the complexities of VRPTU, various solution methods have been developed to optimize routing decisions under dynamic and uncertain conditions. Figure 3 provides a distribution overview of these solution methods, showcasing their prevalence and utilization in the field. Machine learning-based methods, comprising 15% of the solutions, leverage data-driven techniques such as reinforcement learning and graph attention networks to enhance routing decisions. Mathematical programming methods, accounting for 22.6% of solutions, utilize optimization models like mixed integer linear programming (MILP) and Exact Branch-and-Price-and-Cut Algorithms for rigorous optimization. Heuristic and metaheuristic methods, dominating with 49.1% representation, employ strategies like GA, TS, and ant colony optimization (ACO) for efficient approximate solutions. Optimization and search techniques, constituting 7.5% of solutions, utilize algorithms such as large neighborhood search (LNS) and branch-price-and-cut for systematic exploration of solution spaces. Additionally, 5.7% of solutions fall under undefined methods, possibly representing hybrid or novel approaches tailored to VRPTU challenges. These solution methods collectively contribute to advancing robust and adaptable routing strategies in dynamic operational environments.

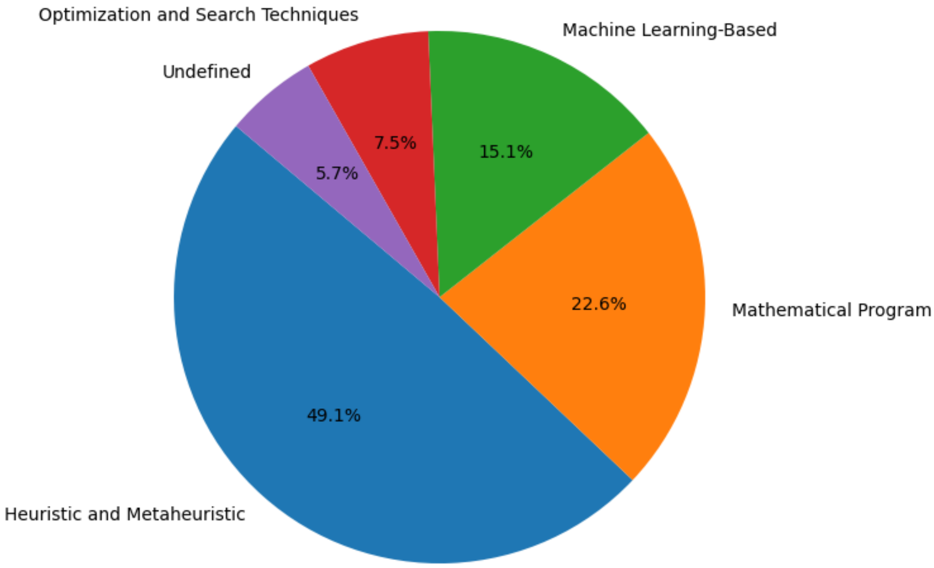

Figure 3. Solution methods distribution

4.1. Mathematical programming-based methods

Mathematical programming-based methods in vehicle routing involve formulating problems as mathematical optimization models to find optimal or near-optimal solutions. For instance, MILP models the problem with integer constraints, and [5] demonstrated its effectiveness in VRP. The exact branch-and-price-and-cut algorithm combines branch-and-bound techniques with column generation, as applied by [25], [26], [33] in solving large-scale VRP instances. Additionally, Dantzig-Wolfe decomposition utilizes dynamic programming and exact methods to decompose the problem into subproblems, addressing time-dependent routing decisions; Lee *et al.* [16] utilized this approach for vehicle routing with time windows and dynamic travel times. These methods offer rigorous and systematic approaches for handling complex VRPs efficiently.

4.2. Heuristic and metaheuristic methods

Heuristic and metaheuristic methods in vehicle routing refer to algorithmic approaches prioritizing computational efficiency and approximated solutions. For instance, GA mimic natural selection to improve solutions iteratively; [15], [32] demonstrated GA's application in VRP. TS algorithm explores neighboring solutions while avoiding revisiting previously visited states, as highlighted by [17], [20]. These methods, including hybrid algorithms [58], ACO [43], [59], and greedy [53] and insertion algorithms [20], offer versatile and effective approaches for addressing various constraints and complexities in VRP.

4.3. Machine learning-based methods

Machine learning-based methods in vehicle routing utilize data-driven techniques to enhance routing decisions and optimize solution quality. For instance, the learnable genetic algorithm combining neighbor search integrates GA with neighbor search strategies to learn optimal solutions through iterative refinement; Lai *et al.* [19] applied this approach to vehicle routing with promising results. Additionally, the reinforcement learning approach integrated with a graph attention network employs reinforcement learning techniques with graph attention networks to learn optimal routing policies; Zhang *et al.* [29] demonstrated the effectiveness of this approach in optimizing vehicle routes. Furthermore, the adaptive variable neighborhood search (AVNS) dynamically adjusts search strategies based on solution quality, optimizing routes iteratively; [36], [40], [55] showcased AVNS's effectiveness in improving solution quality for VRPs. These machine learning-based methods offer adaptive and intelligent solutions for addressing the complexities of VRP.

4.4. Optimization and search techniques

Optimization and Search Techniques in vehicle routing encompass various strategies to efficiently explore solution spaces and improve routing decisions. For example, the MCSGA integrates clustering and GA to optimize routes; Hammouti *et al.* [5] demonstrated its effectiveness in solving VRPs. Additionally, the two-stage stochastic mixed-integer programming model optimizes routes by considering stochastic elements and integer programming techniques; Yu *et al.* [28] applied this model to address uncertainties in vehicle routing scenarios. Moreover, the hybrid simulated annealing (SA) and TS algorithm combine SA's global search capabilities with TS's local search strategies to explore solution spaces and refine routes efficiently; Shi *et al.* [52] utilized this hybrid approach for effective VRP. These optimization and search techniques offer diverse and effective strategies for tackling complex vehicle routing challenges.

5. CONCLUSION

This comprehensive review paper provides valuable insights into the challenges and advancements in optimizing vehicle routing under time uncertainty. By analyzing a wide range of research papers focused on the VRP, the review highlights the importance of addressing uncertainties such as dynamic travel times, service requirements, and fleet size considerations in route optimization. The study emphasizes the need for robust optimization strategies, adaptive routing solutions, and real-time data integration to effectively manage uncertainties and enhance operational efficiency in logistics and transportation management. Looking forward, future research could focus on developing more sophisticated algorithms that combine machine learning techniques with traditional optimization methods to predict and respond to uncertainties dynamically. Additionally, exploring the integration of advanced technologies, such as the internet of things (IoT) and blockchain, could improve data reliability and security in real-time routing decisions. Another promising direction involves multi-modal routing, where combining different transportation modes could enhance flexibility and reduce costs under uncertain conditions.

APPENDIX

Table 1. Taxonomy of problems

#	Authors	Single vs multi	Solution method	Uncertainty	Nature	Time windows	Extra	Benchmark
1	Hammouti *et al.* [5]	multi	MIP and MCSGA	travel	travel time uncertain	exists	electric vehicle	Solomon
2	Režnar *et al.* [6]	multi	ALNS algorithm	service	dynamic speed	exists	Considering each street speeds different	generated
3	Adulyasak and Jaillet [7]	multi	RBC	travel	stochastic	deadlines	there is must visit node and node with deadline	not provided
4	Anderluh *et al.* [8]	multi	GRASP	travel	road accidents	doesn't exist	2 echelon, van and bike	city Vienna FCD
5	Fallah *et al.* [9]	multi	IDE	service	stochastic service time	exists	-	generated

Table 1. Taxonomy of problems *(continued)*

#	Authors	Single vs multi	Solution method	Uncertainty	Nature	Time windows	Extra	Benchmark
6	Ning and Su [10]	multi	GA	travel	dynamic travel time (jams and so on)	exists	-	not provided
7	Allahyari *et al.* [11]	multi	GRASP and ILS	travel	dynamic speed	hard	uncertain demand	Solomon
8	Liu and Qu [12]	multi	A-routing	travel	road accidents	doesn't exist		two large cities
9	Yuliza *et al.* [13]	multi	Insertion algorithm	travel	dynamic speed	exists	-	City Sako District, Palembang
10	Yang *et al.* [2]	multi	LNS and SA	demand, travel, service	traffic accidents	exists	-	not provided
11	Feng *et al.* [14]	multi	CPLEX and Annealing algorithm	travel	dynamic speed	doesn't exist	fuel consumption	England city
12	Haghani and Jung [15]	multi	GA	travel	dynamic speed	soft	-	random generated
13	Lee *et al.* [16]	multi	Dantzig-Wolfe decomposition	travel	traffic accidents	deadlines	-	not provided
14	Wu *et al.* (disruption) [17]	multi	TS	service	vehicle disruption	hard	-	Solomon
15	Groß *et al.* [18]	multi	Metaheuristics	travel	interval time for streets, stochastic travel time	no	2 echelon, truck 1 and 2	City
16	Lai *et al.* [19]	multi	GA and neighbor search	travel	traffic congestion	hard	communication network problem solving	not provided
17	Li and Chung [20]	multi	TS and insertion algorithm	travel, demand	stochastic	no	demand also uncertain	not provided
18	Jaillet *et al.* [21]	multi	Relative indexing solutions	travel, service, customer	stochastic	soft, deadlines	uncertain demands, trying all objectives	randomly generated
19	Tordecilla *et al.* [22]	multi	semi-heuristic with MSC and FIS	travel	stochastic	no	-	Benchmark of Chao *et al.* [23]
20	Lu *et al.* [24]	multi	MILP	travel	stochastic travel time	hard	two-echelon, train capacity uncertain	Euro-China Expressway
21	Vega *et al.* [25]	multi	Branch-price-and-cut algorithm	service, travel	stochastic travel time	hard	two-echelon, vehicle and deliverymen	Solomon
22	Wang *et al.* [26]	multi	Branch-price-and-cut algorithm	demand, travel time	stochastic, generated	can come early, but not late	-	not provided
23	Yuan *et al.* [27]	multi	Quantum evolution	travel	stochastic travel time	soft	-	Solomon
24	Yu *et al.* [28]	multi	MIP, re-optimization, heuristic approaches	travel, service, customer	stochastic	exists	-	Augerat *et al.* (1998)

Table 1. Taxonomy of problems *(continued)*

#	Authors	Single vs Multi	Solution method	Uncertainty	Nature	Time windows	Extra	Benchmark
25	Zhang *et al.* [29]	multi	RL	travel	dynamic congestions	soft	multi-depot	generated
26	Agra *et al.* [30]	multi	adjustable robust optimization (exact method)	travel	weather conditions	hard	maritime	generated
27	Grzegorz *et al.* [31]	multi	not provided	travel	traffic jams, unstable speeds	hard	periodic vehicle routing problem (PVRP)	not provided
28	Ando and Taniguchi [32]	multi	GA	travel	road conditions	not provided	-	South Osaka area
29	Yin *et al.* [33]	multi	branch-and-price-and-cut algorithm	demand, travel time	stochastic	deadlines	two-echelon	not provided
30	El-Sherbeny [34]	single and multi	Heuristic method	service, travel, demand	fuzzy	soft	-	not provided
31	Chen *et al.* [35]	single	bi-objective reliable path-finding algorithm	travel	traffic accidents, bad weather, and other sources of variability in travel times	exists	electric vehicle	(RTIS) in Hong Kong
32	Braaten *et al.* [36]	multi	ALNS	travel	traffic congestion, varying speeds, unexpected delays	hard	-	not provided
33	Nucci [37]	single	Artificial Immune Heuristic	travel time, service	traffic congestion, varying speeds, unexpected delays	hard	-	Italy
34	Mańdziuk and Świechowsk [38]	multi	UCT method	travel	traffic jams	no	-	not provided
35	Nasri *et al.* [39]	multi-	Adaptive large neighborhood search (ALNS) algorithm	service, travel	traffic congestion, varying speeds, unexpected delays	hard	-	Solomon
36	Zhang *et al.* [40]	multi	ALNS	service time of battery and travel time	traffic congestion, varying speeds, unexpected delays	soft	electric vehicle	generated
37	Chu *et al.* [41]	multi	two-stage heuristic approach	travel times	stochastic	exists	-	not provided
38	Wu and Hifi [42]	multi	Best-first strategy (BFS) heuristic	travel time	traffic congestion, varying speeds, unexpected delays	hard	-	not provided
39	Abous-leiman *et al.* [43]	single and multi	ACO	traffic information	traffic congestion, varying speeds, unexpected delays		-	not provided
40	Rouky *et al.* [1]	multi	ACO	travel times and service times	uncertain travel times and service times	hard	-	generated
41	Hu *et al.* [44]	multi	AVNS	demand, travel time	traffic congestion and fluctuations in customer demand	hard	-	Solomon

Table 1. Taxonomy of problems *(continued)*

#	Authors	Single vs Multi	Solution method	Uncertainty	Nature	Time windows	Extra	Benchmark
42	Zhu *et al.* [45]	multi	multi-objective mixed integer programming	arrival time uncertain	uncertainty of flight arrival times	hard	aircraft refuelers	not provided
43	Nguyen *et al.* [46]	multi	Satisficing measure approach (SMA)	exists	distributions for travel times on edges,	soft		not provided
44	Çimen and Soysal [47]	multi	ADP based heuristic algorithm	travel time	stochastic vehicle speed	soft	-	not provided
45	Rabbani *et al.* [48]	multi	meta-heuristic algorithm, specifically SA	traffic time variability	accidents, weather conditions, and traffic congestion.	exists	-	not provided
46	Allahviranloo *et al.* [49]	multi	PGA	travel time	stochastic	hard	-	generated
47	Sungur *et al.* [50]	multi	MADS	service time and customers	stochastic	soft	-	urban area with known customer locations
48	Goel *et al.* [51]	multi	ACO	travel time and demand, service time	traffic, unpredictable customer demands, and uncertain service times correlated to demand	hard	-	Solomon
49	Shi *et al.* [52]	multi	hybrid, SA and TS	service times, travel times, and synchronized visits	traffic jams, uncertain service times, uncertain travel times, and synchronized visits	exists	-	not provided
50	Huang and Blazquez [53]	multi	hybrid, the Greedy algorithm, Insertion algorithm, ACO	customer demand uncertainty and time-dependent link travel time uncertainty	traffic conditions, accidents, and weather conditions	hard	-	environment for the city of Taipei, Taiwan.
51	Zhang *et al.* [4]	multi	Branch-and-Price (B&P)	uncertain travel time and uncertain service time	traffic congestion, varying vehicle speeds	exists	-	not provided
52	Zhang *et al.* [3]	multi	TS heuristic algorithm	travel and service times	roadway capacity variations, traffic demand fluctuations	soft	-	Solomon
53	Rajabi-Bahaabadi *et al.* [54]	multi	ACO	travel time	traffic congestion, varying vehicle speeds	soft	-	Seattle, Washington, San Diego
54	Amiri *et al.* [55]	multi	ALNS meta-heuristic algorithm	customer locations, service time, charging stations, fuel consumption	traffic congestion, varying vehicle speeds	soft	electric vehicles	not provided

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BIOGRAPHIES OF AUTHORS

Akhmetbek Yernar received his bachelor's degree in information systems from SDU University, Faculty of Engineering and Natural Sciences, between 2018 and 2022. Currently, he is pursuing her master's degree in computer science at the same institution, with an expected completion date in 2024. Alongside his studies, he has taken on the role of an educator at SDU within the Faculty of Engineering and Natural Sciences. His commitment to teaching parallels his academic endeavors, providing students with valuable insights from the intersection of theory and practical application. His research interests lie in education, machine learning, and data science. These domains offer vast potential for innovative teaching methodologies and enhancing learning experiences through technological integration. He can be contacted at email: yernar.akhmetbek@sdu.edu.kz.

Cemil Turan is an Associate Professor and Principal Research Fellow at SDU University in Almaty, Kazakhstan. He holds a Ph.D. in electrical and computer engineering from Mevlana Rumi University in Turkey, where he specialized in sparse adaptive filtering techniques for acoustic echo cancellation. With extensive experience in teaching and research, he has contributed significantly to signal processing, computer vision, and artificial intelligence. His research interests include adaptive filters, sparse system identification, and machine learning applications in computer vision. He has published numerous papers in international journals and conferences. He can be contacted at email: cemil.turan@sdu.edu.kz.