A new look on CSI imperfection in downlink NOMA systems

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ABSTRACT
Observing that cooperative scheme benefits to non-orthogonal multiple access (NOMA) systems, we focus on system performance analysis of downlink. However, spectrum efficiency is still high priority to be addressed in existing systems and hence this paper presents full-duplex enabling in NOMA systems. Other challenge needs be considered related to channel state information (CSI). In particular, we derive closed-form expressions of outage probability for such NOMA systems under the presence of CSI imperfection. Furthermore, to fully exploit practical environment, we provide system model associated with Nakagami-m fading. The Monte-Carlo simulations are conducted to verify the exactness of considered systems.

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1. INTRODUCTION
Due to advantages of superior massive connectivity and spectral efficiency, non-orthogonal multiple access (NOMA) has received considerable consideration as the prominent approach for future wireless networks [1], [2]. As promising feature of NOMA, the same radio resources with different power factors based on their channel conditions share for signals from the plurality of users. At the receiving end, the user with poor channel conditions is able to decode its own message as considering other user’s messages as interference. In constrast, the users with better channel conditions need successive interference cancellation (SIC) to aliviate another users’ messages and then they decode their own signal [3], [4]. By allocating different levels of power to users, NOMA relying on power domain to permit multiple users to achieve the same non-orthogonal resources [5]-[8]. The users’ messages would be detached with the help of SIC. In such, improving connectivity in IoT needs implementation of NOMA technique [9]. In [10] the authors targeted to security-required and the delay-sensitive users on the same non-orthogonal resource to satisfy requirements in IoT employing NOMA. They proved that the proposed system relying on NOMA paradigm is better than the benchmark OMA scheme. The throughput and energy efficiency are studied in the NOMA scenarios to perform a cellular massive IoT in [11]. further improving network connectivity, NOMA can be applied with cognitive radio and millimeter wave networks [12], [13]. In the manner of cooperative scheme, NOMA exhibits some advantages in term of outage probability and further enhance network connectivity [14]-[20].

Basically, the channel Rayleigh fading is conducted in systems introduced in the above-mentioned studies. In general, the dense obstacles and scattering are in the channels between the base station (BS) and the user which follow Rayleigh fading. Since the propagation encounters fewer obstacles and less scattering,
the Nakagami-m distribution is suitable to model the BS-relay links compared with Rayleigh fading. A key factor, channel fading makes a crucial influence on the performance of NOMA systems. In [21], the authors have studied the NOMA system over Nakagami-m fading to highlight outage performance, but the perfect channel estimation is assumed. Actually, the performance is further degraded by imperfect channel estimation. Therefore, it is very challenging to provide an accurate analysis of the NOMA system under joint impacts of imperfect channel state information (CSI) and Nakagami-m fading.

Although the existing works about NOMA have assumed perfect CSI or the order of the instantaneous channel gain at the transmitter side [22]-[25]. However, this assumption is not suitable in some communication scenarios, such as underwater acoustic (UWA) [26] and high-speed railway (HSR) [27] systems, due to the rapidly changing channel and the large feedback delay. Motivated by such analysis, we consider NOMA system in imperfect CSI case.

2. SYSTEM MODEL

In this paper, we assume the base station(S), two user NOMA($U_i$, $i \in \{1, 2\}$) shown in Figure 1. These nodes are equipped single antenna. To improve bandwidth usage efficiency, the user 1 ($U_1$) works in full-duplex (FD) mode. Channels in link S-$U_1$, S-$U_2$, $U_1$-$U_2$ are denoted by $h_1$, $h_2$ and $h_3$, respectively. Self-interference channel existing at user $U_1$ is denoted by $g_1$. These channels are followed Nakagami-m channel model.

![Figure 1. System model](image)

The error channel estimation coefficients is modeled by [28]

$$h_j = \tilde{h}_j + e_j,$$

(1)

where $\tilde{h}_j (j \in \{1, 2, 3\})$ denotes the estimated channel coefficient and $e_j$ denotes the estimated channel error which can be approximated as a Gaussian random variable with $CN(0, \sigma_j^2)$

In the block time $t$, the base station S transmits the information $a_1 x_1(t) + a_2 x_2(t)$ to $U_i$. It is noted that $x_1$ and $x_2$ are the information of $U_1$ and $U_2$, respectively. To proceed NOMA, $a_1$ and $a_2$ are the power allocation coefficients where $a_1^2 + a_2^2 = 1$ and $a_1^2 > a_2^2$. Thus, the received signal at $U_1$ can be computed by

$$y_{U_1}(t) = \sqrt{P_S} (a_1 x_1(t) + a_2 x_2(t)) (\tilde{h}_1 + e_1) + \sqrt{P_I} g_1 x_2 (t - \tau) + n_1$$

$$= \sqrt{P_S} \tilde{h}_1 (a_1 x_1(t) + a_2 x_2(t)) + \sqrt{P_S} e_1 (a_1 x_1(t) + a_2 x_2(t)) + \sqrt{P_I} g_1 x_2 (t - \tau) + n_1$$

(2)

where $P_S$ and $P_I$ are the power transmission of $S$ and $U_i$, respectively. $n_1$ is the additive white Gaussian noise (AWGN) with $CN(0, N_0)$, $g_1$ is the residual interference coefficient and $\tau$ denotes signal processing delay.
Then, the signal to interference plus noise ratio (SINR) to detect $x_2$ at $U_1$ is given by

$$\Gamma_{1,2} = \frac{P_Sa_2^2 |\tilde{h}_1|^2}{P_Sa_2^2 |\tilde{h}_1|^2 + P_S\sigma_2^2 + P_1 |g_1|^2 + N_0}$$

$$= \frac{\rho a_2^2 |\tilde{h}_1|^2}{\rho a_2^2 |\tilde{h}_1|^2 + \rho \sigma_2^2 + \rho |g_1|^2 + 1}$$

(3)

where $\rho = \frac{P_S}{N_0} = \frac{P_0}{N_0}$ is the signal-to-noise ratio (SNR) and the SINR at $U_1$ to detect the own signal $x_1$ is given as

$$\Gamma_1 = \frac{\rho a_2^2 |\tilde{h}_1|^2}{\rho \sigma_2^2 + \rho |g_1|^2 + 1}$$

(4)

Now, the base station $S$ transmits signals to $U_2$ through two links (direct and relay links). The received signal at $U_2$ in direct link is expressed as

$$y_{U_2}(t) = \sqrt{P_S} (a_1x_1(t) + a_2x_2(t)) (\tilde{h}_2 + e_2) + n_2$$

$$= \sqrt{P_S}h_2 (a_1x_1(t) + a_2x_2(t)) \sqrt{P_S} \sigma_2 (a_1x_1(t) + a_2x_2(t)) + n_2$$

(5)

effective noise

It is noted that the signals received at $U_2$ from $U_1$ node could be formulated as

$$y_{U_1\rightarrow U_2}(t) = \sqrt{P_1} (\tilde{h}_3 + e_3) x_2(t - \tau) + n_3$$

(6)

where $n_2$ and $n_3$ are the AWGN with $CN(0, N_0)$.

Then, the SINR to detect $x_2$ of $U_2$ is given as

$$\Gamma_2 = \frac{\rho a_2^2 |\tilde{h}_2|^2}{\rho a_2^2 |\tilde{h}_2|^2 + \rho \sigma_2^2 + 1}$$

(7)

From (6), the SINR to detect $x_2$ at $U_2$ is formulated by

$$\Gamma_{2,1} = \frac{\rho |\tilde{h}_3|^2}{\rho \sigma_3^2 + 1}$$

(8)

### 3. PERFORMANCE ANALYSIS

#### 3.1. Channel mode

The probability density function (PDF) of their gains follow gamma distributions is formulated as

$$f_{|h_j|^2}(x) = \frac{x^{m_j-1}e^{-\frac{x}{\Omega_j}}^m}{\Gamma(m_j)} \left(\frac{m_j}{\Omega_j}\right)^{m_j}$$

(9)

where $m_j$ and $\Omega_j$ are the fading severity factor and mean, respectively.

#### 3.2. Outage probability of $U_1$

The outage probability of $U_1$ can be expressed as [29]

$$OP_{U_1} = 1 - \Pr(\Gamma_{1,2} > \gamma_{th2}, \Gamma_1 > \gamma_{th1})$$

(10)
where $\gamma_{thi} = 2^{R_i} - 1$ and $R_i$ is the target rate. With the help of (3) and (4), it can be written as

$$\mathcal{O}P_{U_1} = 1 - \Pr \left( \frac{\rho a_1^2 |h_1|^2}{\rho a_2^2 |h_1|^2 + \rho g_1^2 + 1} > \gamma_2, \frac{\rho a_2^2 |h_1|^2}{\rho a_2^2 |h_1|^2 + \rho g_1^2 + 1} > \gamma_1 \right)$$

$$= 1 - \Pr \left( |h_1|^2 > \frac{\phi (\rho a_1^2 + \rho g_1^2 + 1)}{\rho} \right)$$

(11)

where $\phi = \max \left( \frac{\gamma_2^2}{\gamma_1^2}, \frac{\gamma_1^2}{\gamma_2^2} \right)$. Then, (11) is calculated as

$$\mathcal{O}P_{U_1} = 1 - \int_0^\infty f_{|g_1|^2}(x) \int_x^\infty f_{|h_1|^2}(y) dy dx$$

(12)

Putting (9) into (12), we have

$$\mathcal{O}P_{U_1} = 1 - \frac{1}{\Gamma(m_{g_1}) \Gamma(m_{h_1})} \frac{m_{g_1}}{\Omega_{g_1}} \frac{m_{h_1}}{\Omega_{h_1}}$$

$$\times \int_0^\infty x^{m_{g_1} - 1} e^{-\frac{m_{g_1}}{\Omega_{g_1}} x} \int_0^{\phi (\rho a_1^2 + \rho g_1^2 + 1)} \frac{y^{m_{h_1} - 1} e^{-\frac{y}{\Omega_{h_1}}}}{\rho} dy dx$$

(13)

Based on [30, Eq. 3.351.2], (13) can be computed as

$$\mathcal{O}P_{U_1} = 1 - \frac{1}{\Gamma(m_{g_1})} \frac{m_{g_1}}{\Omega_{g_1}} \frac{m_{h_1}}{\Omega_{h_1}}$$

$$\times \int_0^\infty x^{m_{g_1} - 1} e^{-\frac{m_{g_1}}{\Omega_{g_1}} x} \sum_{k=0}^{m_{h_1} - 1} \frac{k!}{\phi^{m_{h_1}}} \left( \frac{m_{g_1}}{\phi} \frac{(\sigma_1^2 + 1/\rho)}{k! \Gamma(m_{g_1})} \right)^k$$

(14)

By applying [30, Eq. 1.111], (14) can be further simplified as

$$\mathcal{O}P_{U_1} = 1 - \sum_{k=0}^{m_{h_1} - 1} \sum_{n=0}^{m_{g_1} - 1} \frac{k!}{\phi^{m_{h_1}}} \left( \frac{m_{g_1}}{\phi} \frac{(\sigma_1^2 + 1/\rho)}{k! \Gamma(m_{g_1})} \right)^k$$

$$\times \int_0^\infty x^{m_{g_1} + n - 1} e^{-\frac{m_{g_1}}{\Omega_{g_1}} + \frac{m_{g_1}}{\Omega_{h_1}} - \phi^{m_{h_1}}} (\frac{m_{g_1}}{\phi})^{m_{g_1}} dx$$

(15)

Finally, using [30, Eq. 3.351.3] the outage probability in closed-form for user $U_1$ can be obtained as

$$\mathcal{O}P_{U_1} = 1 - e^{-\frac{m_{h_1}}{\Omega_{h_1}} (\sigma_1^2 + 1/\rho)} \sum_{k=0}^{m_{h_1} - 1} \sum_{n=0}^{m_{g_1} - 1} \frac{k!}{\phi^{m_{h_1}}} \left( \frac{m_{g_1}}{\phi} \frac{(\sigma_1^2 + 1/\rho)}{k! \Gamma(m_{g_1})} \right)^k$$

$$\times \frac{\Gamma(m_{g_1} + n)}{\Gamma(m_{g_1}) k! \Gamma(m_{h_1})} \frac{(\omega_m \gamma_1)^k (m_{g_1})^{m_{g_1}}}{\Gamma(\Omega_{g_1})^{k-m_{g_1}-n} (m_{g_1} \Omega_{h_1} + \omega_m \gamma_1 \Omega_{g_1})^{m_{g_1}+n}}$$

(16)

### 3.3 Outage probability of $U_2$

In this case, it is assumed that the far user $U_2$ exploits the selection combine (SC) approach to process two paths achieved from the S and $U_1$. Thus, the outage probability of $U_2$ is given as [29]

$$\mathcal{O}P_{U_2} = \Pr \left( \max (\Gamma_2, \min (\Gamma_{1,2}, \Gamma_{2,1}) < \gamma_2) \right)$$

$$= \underbrace{\Pr (\Gamma_2 < \gamma_2) \Pr (\min (\Gamma_{1,2}, \Gamma_{2,1}) < \gamma_2)}_{A_2}$$

(17)
Then, putting (7) into the first term of (17), we have

\[
A_1 = 1 - \Pr (\Gamma_2 > \gamma_2) \\
= 1 - \Pr \left( |\tilde{h}_2|^2 > \frac{\gamma_2 (\rho \sigma_2^2 + 1)}{\rho (a_1^2 - \gamma_2 a_2^2)} \right) \\
= 1 - \int_{\frac{\gamma_2 (\rho \sigma_2^2 + 1)}{\rho (a_1^2 - \gamma_2 a_2^2)}}^{\infty} f_{|\tilde{h}_2|^2}(x) dx
\] (18)

Similarly, \( A_1 \) is obtained as

\[
A_1 = 1 - e^{-\frac{\gamma_2 m_{h_2} (\rho \sigma_2^2 + 1)}{\rho h_{\gamma_2} (a_1^2 - \gamma_2 a_2^2)}} \sum_{q=0}^{\infty} \frac{1}{q!} \left( \frac{\gamma_2 m_{h_2} (\rho \sigma_2^2 + 1)}{\rho h_{\gamma_2} (a_1^2 - \gamma_2 a_2^2)} \right)^q
\] (19)

Furthermore, the second term of (17) is expressed as

\[
A_2 = \Pr (\min (\Gamma_{1,2}, \Gamma_{2,1}) < \gamma_2) \\
= 1 - \Pr (\Gamma_{1,2} > \gamma_2) \Pr (\Gamma_{2,1} > \gamma_2)
\] (20)

Then, we can write \( \Pr (\Gamma_{1,2} > \gamma_2) \) as

\[
\Pr (\Gamma_{1,2} > \gamma_2) = \Pr \left( |\tilde{h}_1|^2 > \frac{\gamma_2 (\rho \sigma_2^2 + \rho |g_1|^2 + 1)}{(a_1^2 - \gamma_2 a_2^2) \rho} \right) \\
= \int_{\frac{\gamma_2 (\rho \sigma_2^2 + |g_1|^2 + 1)}{(a_1^2 - \gamma_2 a_2^2) \rho}}^{\infty} f_{|g_1|^2}(x) \int_{\frac{\gamma_2 (\rho \sigma_2^2 + |g_1|^2 + 1)}{(a_1^2 - \gamma_2 a_2^2) \rho}}^{\infty} f_{|\tilde{h}_1|^2}(y) dy dx
\] (21)

Similar in above, \( \Pr (\Gamma_{1,2} > \gamma_2) \) can be obtained as

\[
\Pr (\Gamma_{1,2} > \gamma_2) = 1 - e^{-\frac{\gamma_2 m_{h_1} (\rho \sigma_2^2 + |g_1|^2)}{(a_1^2 - \gamma_2 a_2^2) \rho h_{\gamma_2}} \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \frac{k!}{n!} \frac{(m_{g_1} + n)! (\sigma_2^2 + 1/\rho)^{k-n}}{\Gamma (m_{g_1} + n) \Omega_{g_1, \gamma_2}^{\gamma_2 m_{h_1} \Omega_{g_1, \gamma_2}}} \left( \frac{\gamma_2 m_{h_1} \Omega_{g_1}}{a_1^2 - \gamma_2 a_2^2} \right)^{m_{g_1} - n}
\] (22)

Next, \( \Pr (\Gamma_{2,1} > \gamma_2) \) can be formulated as

\[
\Pr (\Gamma_{2,1} > \gamma_2) = \Pr \left( |\tilde{h}_3|^2 > \frac{\gamma_2 (\rho \sigma_3^2 + 1)}{\rho} \right) = \int_{\frac{\gamma_2 (\rho \sigma_3^2 + 1)}{\rho}}^{\infty} f_{|\tilde{h}_3|^2}(x) dx
\] (23)

Similar, after some manipulations, we have

\[
\Pr (\Gamma_{2,1} > \gamma_2) = e^{-\frac{\gamma_2 m_{h_3} (\rho \sigma_3^2 + 1)}{m_{h_3}} \sum_{p=0}^{\infty} \frac{1}{p!} \left( \frac{\gamma_2 m_{h_3} (\rho \sigma_3^2 + 1)}{\rho \Omega_{h_3}} \right)^p}
\] (24)
Putting (22), (23) and (26) into (20) and then with help the result and (19). Finally, the outage probability of $U_2$ is formulated by

$$
\text{OP}_{U_2} = \left(1 - e^{-\frac{\gamma_2 m_{h_2} (\sigma_2^2 + 1)}{\rho \Omega_{h_2} (a_1^2 - \gamma_2 a_2^2)}} \right) \left(1 - \frac{\gamma_2 m_{h_2} (\rho \sigma_2^2 + 1)}{\rho \Omega_{h_2}} \right) \sum_{n=0}^{m_{h_2} - 1} \frac{1}{n!} \left(\frac{\gamma_2 m_{h_2} (\rho \sigma_2^2 + 1)}{\rho \Omega_{h_2}} \right)^n \\
\times \left(1 - e^{-\frac{\gamma_2 m_{h_2} (\sigma_2^2 + 1)}{\rho \Omega_{h_2} (a_1^2 - \gamma_2 a_2^2)}} \right) \sum_{n=0}^{m_{h_2} - 1} \frac{1}{n!} \left(\frac{\gamma_2 m_{h_2} (\rho \sigma_2^2 + 1)}{\rho \Omega_{h_2}} \right)^n \\
\times e^{-\frac{\gamma_2 m_{h_2} (\sigma_2^2 + 1/\rho)}{\Omega_{h_1}}} \sum_{k=0}^{m_{h_1} - 1} \sum_{n=0}^{k} \frac{k!}{n!} \Gamma(m_g + n)! (\sigma_1^2 + 1/\rho)^{k-n} \\
\times \frac{1}{\Omega_{h_1}^{m_{h_1}}} \left(\frac{\gamma_2 m_{h_2} (\sigma_2^2)}{a_1^2 - \gamma_2 a_2^2} \right) \left(\frac{\gamma_2 m_{h_2} (\Omega_{g_1} + \gamma_2 m_{h_2} \Omega_{g_2})}{(a_1^2 - \gamma_2 a_2^2)} \right)^{-m_{g_1} - n}
$$

(25)

3.4. Throughput

Based on achievable outage probability, we can compute throughput as below

$$
\mathcal{T}P = (1 - \text{OP}_{U_1}) R_1 + (1 - \text{OP}_{U_2}) R_2
$$

(26)

4. NUMBER RESULT

In this section, we perform simulations to check the corrections of derived formulas. We set $a_1^2 = 0.8$, $a_2^2 = 0.2$, $R_1 = R_2 = 1$, $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 0.01$, $m = m_{g_1} = m_{h_1} = m_{h_2} = m_{h_3} = 2$, $\Omega_{h_1} = 2$, $\Omega_{h_2} = \Omega_{h_3} = 1$ and $\Omega_{g_1} = 0.05$.

Figure 2 shows outage performance of user $U_1$ versus the transmit SNR. We can see FD case exhibits its superiority compared with HD case. At high SNR region, SINR would be better and hence such outage probability can be improved at high SNR. Similar trend can be seen in Figure 3 for case of user $U_2$. We change value of fading severity parameter $m$ and a gap exists when comparing these cases of $m$. Similar performance is presented in Figure 4. Figure 5 for user $U_1, U_2$ respectively for two cases of noise term. Finally, throughput meets highest as SNR is greater than 20 dB shown in Figure 6. The reason is that throughput depends on outage probability.

![Figure 2](attachment:image2.png)

**Figure 2.** The outage probability of $U_1$ versus SNR varying $m$ with $\sigma^2 = 0.01$. 

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5. CONCLUSION

This paper has studied cooperative NOMA scheme under impact of imperfect CSI. Based on the nature of cooperation, the near user benefits from full-duplex design to serve far user, which improve performance of far user. This results in an enhancement of the downlink NOMA over Nakagami-$m$ fading. We analyze the system outage probability and throughput under different variations of the key system parameters. Finally, simulation results validate our considered system.

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