Total power deficit estimation for isolated power system network using $H_\infty$ norm method

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ABSTRACT
An isolated electrical network with an independent local distributed generator is very sensitive towards the contingencies between load demand and supply. Although the network system has less complexity in term of structure, its stability condition is crucial due to its stand-alone operating condition. Hence, to monitor the stability of the network, total power deficit is the only variable that can be estimated. This study proposed to utilize the $H_\infty$ filtering problem to design an estimator that can estimate the total power deficiency. For verification, the design estimator was tested under two cases which are nominal case and parameter uncertainty case. In nominal case, the system parameters are assumed to be known while in parameter uncertainty case the system has contaminated with uncertain parameter where its admissible value is laying in a given polytope. With complying the linear matrix inequality (LMI) constraint, the main objective function of the $H_\infty$ norm was guaranteed with minimum upper bound. The results show that the dynamical behaviors of designed estimator successfully trace the actual power deficit signal at steady state with minimal estimation error.

Keywords: $H_\infty$ filtering, Input state estimation, Isolated power system, Power deficiency, Robust estimation

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1. INTRODUCTION
A small electrical network operating in island condition is very sensitive towards any disturbance due to the mixed energy demand and sources. The stability condition is achieved when the dynamical behavior in the generation side is able to sustain for any level of demand and ensured the dynamical frequency network within the permissible level. The unbalance power condition may lead to the severe frequency deviation problem [1]–[5]. The investigations on the influence of frequency load control on the islandic power system network fed by mixed generation sources are reported in [6]–[10]. The unbalance power estimation is highly required before performing the network safety actions to avoid the damage of local generator and total load black-out. The effect of power system imbalance towards the renewable energy penetration and power electronics application was investigated as in [11]–[14]. These studies show that the mixed energy and fast growing of electrical technology and strategy may influence the power flow quality and system stability. Hence, monitoring the network condition is crucial due to the unpredictable uncertain contingencies. One of the power system parameters which reflects the power system dynamic is the total power deficiency.
Total power deficiency is defined as the magnitude of the imbalance between load demand and generated supplied. This magnitude is considered as variable as the state is basically reflect in the network frequency dynamic. The basic to identify the power deficit of the power system network with distributed generator dependance is referred to the general swing equation. This approach has commonly been used by the load shedding scheme to determine the size of load to be shed upon the disturbance through the rate of changed of frequency initial slope as reported in [15]–[20]. All of these works are assumed that the system structure has the well-known parameters aspecially the equivalent system inertia which may deviated due to the variation of network capacity. Thus, the power deficit determination might be not accurate.

Inertia constant is the parameter which directly influence the frequency dynamics during the transient before the steady-state. The investigation of frequency dynamic and system weight response was reported in [21]–[25]. These studies proof that the system inertia is the important parameter to be accounted when investigating the caused of poor stability condition. Hence, the design of filter using theoretical approach is another solution to enable the power deficit estimation to become more reliable in any conditions. This paper utilized a generator model operating in island mode which interconnected with loads. An optimal $H_\infty$ filtering problem was chosen as an approach to design an estimator for this network model. The design procedure is referred from the work done by Geromel and Oliveira [26], on linear time-invariant continuous time system. Then the designed estimator was verified for nominal and robust estimation during uncertain inertia constant parameter case. There are two contributions that has been highlighted in this paper which are first, the new approach to estimate the total power deficit for isolated power system network using $H_\infty$ norm. Second is an approach to estimate the input vector by introducing an additional block function denoted by $h$ augmented with the isolated power system network model.

2. METHOD

The framework of isolated power system network is identical to hydropower system that consists of hydro governor, transient droop compensation, hydro turbine and load which can be described by (1)-(3). The input for this model is the electrical load demand $P_L$ while the state variables are the frequency $\omega$, mechanical power $P_m$ and the governor power $P_{gV}$. The output of this model is the frequency deviation $\frac{d\omega}{dt}$. Noted that, all the variables are in per unit system. Plus, the model does not consider an automatic generation control (AGC) so that the dynamical behaviour of the frequency becomes non-conservative.

\[
\frac{d\omega}{dt} = \frac{1}{2H} (P_m - P_L) \quad (1)
\]

\[
\frac{dP_m}{dt} = \frac{1}{\tau_r} (P_{gV} - P_m) \quad (2)
\]

\[
\frac{dP_{gV}}{dt} = \frac{1}{\tau_{gV}} (P_{ref} - \frac{\omega}{R} - P_{gV}) \quad (3)
\]

The total power deficiency defines the total power deference between load demand power and mechanical power of the generator. In this research, the network dynamical frequency response was observed and then augmented with the estimator system to estimate the total power deficit. Consider the isolated power system network that is represented in linear-time invariant, continuous time system:

\[
\dot{x}(t) = Ax(t) + B\omega_d(t)
\]

\[
y(t) = C_2x(t) + D\omega_d(t)
\]

\[
z(t) = C_1x(t)
\]

Where $x \in \mathbb{R}^n$ is the state variable, $\omega_d \in \mathbb{R}^m$ is the disturbance input; $y \in \mathbb{R}^r$ is measured output and $z \in \mathbb{R}^s$ is the vector to be estimated. The matrices $A$ and $B$ were obtained through (1)-(3). $y$ is the output state denoting the frequency dynamic while $z$ is the output state denoting the expected power deficit to be estimated. Noted that the power deficit is originally not the state variable. It is related to the electrical changes in demand that can be reached from the input state. To make the linear estimator solution become feasible, the power deficit related to the input state must be observable. Hence, it must be converted into the state variable. Figure 1 shows the block diagram to formulate the estimator design problem. The low pass filter denoted as $h$ was introduced to convert the power deficit input state to become as a variable state and

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augmented with the generator model. The purpose of this approach is to facilitate the design process and to ensure the proper estimator transfer function.

The full state-space realizations with augmented of system ‘h’ can be written as (5):

\[
\begin{align*}
\dot{x} &= A_n x + B_n \omega_d \\
y &= C_n x + D_n \\
P_d &= C_g x
\end{align*}
\]

with

\[
A_n = 
\begin{bmatrix}
0 & 1/2H & 0 & 0 \\
0 & -1/\tau_T & 1/\tau_T & 0 \\
-1/RT_{gv} & 0 & -1/T_{gv} & 0 \\
0 & 0 & 0 & -1/T_h
\end{bmatrix};
\]

\[
B_n = 
\begin{bmatrix}
-1/2H \\
0 \\
0 \\
B_h
\end{bmatrix};
\]

\[
C_n = [1 \ 0 \ 0 \ 0]
\]

\[
C_g = [0 \ 0 \ 0 \ C_h]
\]

Noted that \(T_h\) is the time constant of an additional low pass filter while the other symbol parameters are already mentioned. The problem to be dealt with is to design an estimator to estimate \(\hat{P}_d\) of \(P_d\) which given by \(\hat{P}_d = \mathcal{F} \cdot y\), where \(\mathcal{F}\) belongs to a linear estimator with minimum state space realization in the form:

\[
\begin{align*}
\dot{\hat{x}} &= A_f \hat{x} + B_f (C_n x + D_n) \\
&= A_f \hat{x} + B_f C_n x + B_f D_n \\
\hat{P}_d &= C_f \hat{x}
\end{align*}
\]

The matrix \(A_f \in \mathbb{R}^{n_f \times n_f}, B_f \in \mathbb{R}^{n_f \times n_r}, C_f \in \mathbb{R}^{s \times n_f}\) are to be determined. Connecting (5) to (7), it corresponds to the overall augmented system that consist of network model \(x\) and estimator model \(\hat{x}\).

\[
\begin{bmatrix}
\dot{x} \\
\dot{\hat{x}}
\end{bmatrix} = 
\begin{bmatrix}
A_n & 0 \\
B_n & C_n
\end{bmatrix}
\begin{bmatrix}
x \\
\hat{x}
\end{bmatrix} + 
\begin{bmatrix}
B_n \\
B_f D_n
\end{bmatrix} \omega_d
\]

The transfer function from noise input \(\omega_d\) to the estimation error \(e\) can be written as \(T_{\omega}(s) := \hat{C}(sI - \hat{A})^{-1} \hat{B} = GE - h\) and the matrices \(\hat{A}, \hat{B}, \hat{C}\) of compatible dimensions are given by:

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Total power deficit estimation for isolated power system network using H∞ ... (Mohd Saifuzzam Jamri)
The goal for this formulation is to solve the following design problem: 

\[ H_\infty \] estimation problem: to find a guaranteed estimation performance index \( \gamma \) such that 

\[ \|GE - h\|^2_\infty \leq \gamma \] 

produce a feasible upper bound \( \gamma \) over the estimator state-space realization in (7). This \( H_\infty \) norm can be solved accordance to the lyapunov equation solution in schur compliment form as shown in.

\[
\begin{align*}
\min & \quad \gamma \\
\text{Subject to} & \quad \bar{P} > 0; \quad \left[ \begin{array}{cc}
\bar{A} & \bar{B} \\
\bar{B} & 0
\end{array} \right] < 0
\end{align*}
\]

Where: \( \bar{P} \) is positive definite matrix

2.1. Nominal case

In nominal case, the isolated power system model parameters are assumed precisely known. The upper bound of the \( H_\infty \) norm of \( T(s) = GE - h \) provided in (10) deserves some comments. For precisely known parameters, the objective function \( \|GE - h\|^2_\infty \leq \gamma \) yields the very small norm below the small optimal \( \gamma > 0 \) (i.e., exactly equal to zero) and does not introduce any convertism. In this case, the overall system matrix structure can be converted into the error system through the decomposition approach. From (9), the overall augmented system matrix can be decomposed by pre-post multiply with the invertible transformation matrix \( T \) and \( T^{-1} \). Choosing \( T = \left[ \begin{array}{cc}
I & 0 \\
0 & I
\end{array} \right] \), \( A_f = A_n - B_f C_n \), and \( C_f = C_g \), the new matrix \( \bar{A}_n, \bar{B}_n \) and \( \bar{C}_n \) is becomes:

\[
\begin{align*}
\bar{A}_n &= T \left[ \begin{array}{cc}
A_n & 0 \\
B_f C_n & A_n - B_f C_n
\end{array} \right] T^{-1} = \left[ \begin{array}{cc}
A_n & 0 \\
0 & A_n - B_f C_n
\end{array} \right] \\
\bar{B}_n &= T \left[ \begin{array}{c}
B_n \\
B_f D_n
\end{array} \right] = \left[ \begin{array}{c}
B_n \\
B_n + B_f D_n
\end{array} \right] \\
\bar{C}_n &= [-C_g \ C_g] T^{-1} = [0 \ C_g]
\end{align*}
\]

Now the overall augmented system matrix has been decomposed and become two separate systems which are augmented generator system and error system. By only taking out the error system (the second row) and substitute into (10), the Schur compliment matrix can write as (12):

\[
\begin{align*}
\bar{P} > 0; \quad & \left[ \begin{array}{cc}
(A_n - B_f C_n) \bar{P} + \bar{P} (A_n - B_f C_n) & \bar{B}_n + \bar{B}_n D_n \\
-\gamma I & \bar{P} C_g'
\end{array} \right] < 0
\end{align*}
\]

If this matrix is feasible, then the objective function stated in (10) will be satisfied. However, this matrix is hard to solve due to the multiplication of the multiple variables. To this end, the matrix needs to convert into linear matrix inequality (LMI) form. If this conversion is accomplished, the solution will become feasible.

By partitioned \( \bar{P} \) as \( \bar{P} := \left[ \begin{array}{cc}
\bar{P}^{-1} & 0 \\
0 & I
\end{array} \right] \) and pre and post multiply this matrix to (12). Then the matrix will become LMI as (13):

\[
\bar{A} := \left[ \begin{array}{cc}
A_n & 0 \\
B_f C_n & A_f
\end{array} \right] \\
\bar{B} := \left[ \begin{array}{c}
B_n \\
B_f D_n
\end{array} \right] \\
\bar{C} := [-C_g \ C_f]
\]
\[
\begin{bmatrix}
Y & 0 \\
0 & I
\end{bmatrix} > 0 \quad \Rightarrow \quad \begin{bmatrix}
YA_n + QC_n + A_n'Y + C_n'Q' & YB_n - QD_n & C_g' \\
YB_n - QD_n & -I & 0 \\
C_g' & 0 & -\gamma I
\end{bmatrix} < 0 \quad (13)
\]

The new matrix variables are given by positive definite matrix \( Y = \bar{P}^{-1} \) and matrix \( Q \). Hence, the estimator design problem is equivalent to the programming problem expressed in terms of LMI (13). The estimator matrix is defined by \( A_f = A_n - B_fC_n; B_f = -Y^{-1}Q; C_f = C_g \).

### 2.2. Parameter uncertainty case

In this case, the isolated power system model has contaminated with the parameter uncertainty. The uncertain parameter that has been considered in this case is the inertia constant \( H \). This is the only parameter that significantly affect the slope of dynamical frequency and can be clearly seen in (1). Note that the parameter uncertainty is confined to a given polytope satisfying \( M = \sum_{i=1}^{p} M_i \) for some \( \lambda_i > 0 \) such that \( \lambda_1 + \cdots + \lambda_p = 1 \) and \( \lambda := \begin{bmatrix} A_n & B_n \\ C_n & D_n \end{bmatrix} \). Only \( A_n \) and \( B_n \) matrix of the augmented system is assumed to be unknown.

The guaranteed estimation performance is accordance to the objective function such that \( \sup \| GE - h \|_{\infty}^2 \leq \gamma \) and produced a feasible estimator which minimize the \( \gamma \) over the estimator state-space realization in (7). The matrix in (10) can be written as (14):

\[
\bar{P} > 0 \quad \Rightarrow \quad \begin{bmatrix}
\bar{A}_i\bar{P} + \bar{P}\bar{A}_i' & \bar{B}_i & \bar{P}\bar{C}_i' \\
\bar{B}_i & -I & 0 \\
\bar{P}\bar{C}_i & 0 & -\gamma I
\end{bmatrix} < 0; i = 1, \ldots, N \quad (14)
\]

Partitioning the matrix \( \bar{P} := \begin{bmatrix} X & \bar{Y} \\ \bar{U} & \bar{V} \end{bmatrix} \) satisfying the inequality (14) and multiply to the left by \( j' := \text{diag}(j', 1, I) \) and to the right by \( j := \begin{bmatrix} X^{-1} & Y \\ 0 & V \end{bmatrix} \). Introducing the new variable as explained in [24], furthermore multiplying the constrain \( \bar{P} > 0 \) to the left by \( j' \) and to the right by \( j \), the above inequalities is equivalent to:

\[
\begin{bmatrix}
Z & Z \\
Z & \gamma
\end{bmatrix} > 0; \quad \Rightarrow \quad \begin{bmatrix}
ZA_{n_i} + A_{n_i}'Z & ZA_{n_i} + A_{n_i}'Y + C_{n_i}'F' + Q' & ZB_{n_i} + CF_{n_i} + A_{n_i}'Y + C_{n_i}'F' & ZB_{n_i} + FD_{n_i} + C_g' \\
ZB_{n_i} + FD_{n_i} + C_g' & -I & 0 \\
C_g' & -\gamma I
\end{bmatrix} < 0 \quad (15)
\]

where \( i = 1, \ldots, N \)

Hence, the estimator design problem is equivalent to the following programming problem expressed in terms of LMI on the variable positive definite matrix \( Z = Z' \) and \( Y = Y' \) and matrix \( Q, G \) and \( F \). The filter matrix is defined by \( A_f = -Y^{-1}Q(I - Y^{-1}Z)^{-1}; B_f = -Y^{-1}F; C_f = G(I - Y^{-1}Z)^{-1} \).

### 3. RESULTS AND DISCUSSION

In nominal case, the inertia constant parameter value is assumed known with \( H = 5 \) and an additional of transfer function \( h = 1/0.001s + 1 \). The estimation performance index \( \gamma \) such that \( \|GE - h\|_{\infty}^2 \leq \gamma \) was solved with resulted upper bound \( \gamma = 8.0067e - 6 \). The sudden load demand changed at time 3 seconds has triggered the turbine system and made the network frequency drop and deviated to the new operating condition. Figure 2 shows the estimated total power deficit in time response while Table 1 shows the tabulated data related to the estimation error performance at steady-state response through the integral absolute error (IAE), integral square error (ISE) and root means square error (RMSE). The result shows that the estimator able to estimate the total power deficit accurately with small estimation error.
Figure 2. Estimated total power deficit using $H_\infty$ norm with 0.2 per unit sudden load demand changed

Table 1. The estimation error performance using $H_\infty$ norm method under nominal case

<table>
<thead>
<tr>
<th>Inertia constant value</th>
<th>IAE</th>
<th>ISE</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0017</td>
<td>2.75E-06</td>
<td>2.13E-04</td>
</tr>
</tbody>
</table>

For the parameter uncertainty case, the inertia constant parameter value $H$ is assumed known but with uncertain minimum and maximum values which range 4.5 to 5.5 and an additional of transfer function $h = 1/0.001s + 1$. Under the 0.2 per unit of sudden load demand changed at time 3 seconds, the robust $H_\infty$ estimation was guaranteed accordance to the objective function $\sup ||GE - h||_\infty \leq \gamma$ such that the resulted the upper bound $\gamma = 0.0459$. Figure 3 shows the estimated total power deficit in time response when the system contaminated with uncertain inertia constant value.

Figure 3. Estimated total power deficit using $H_\infty$ norm with 0.2 per unit sudden load demand changed and uncertain inertia constant values

The response shows that the $H_\infty$ estimator is fragile when the inertia constant parameter changed. There is offset between the estimated and the reference at the transient and steady-state response. Table 2 shows the error performance at steady-state response through the IAE, ISE and RMSE. The results proof that the error yields the similar performance for each inertia constant value but has about 4.5% of different between estimated and the reference total power deficit value.

Table 2. The estimation error performance at steady state using $H_\infty$ norm method under robust case

<table>
<thead>
<tr>
<th>Inertia constant value</th>
<th>IAE</th>
<th>ISE</th>
<th>RMSE</th>
<th>Estimated total power deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>0.4412</td>
<td>0.0039</td>
<td>0.0088</td>
<td>0.1911</td>
</tr>
<tr>
<td>5</td>
<td>0.4425</td>
<td>0.0039</td>
<td>0.0088</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>0.4443</td>
<td>0.0039</td>
<td>0.0089</td>
<td></td>
</tr>
</tbody>
</table>
4. CONCLUSION

The total power deficit for isolated power system network was successfully estimated by using the $H_{\infty}$ norm method. The model was simulated via MATLAB with Simulink toolbox. The simulation result is used to validate the effectiveness of designed estimator and the error performance analysis was focused on steady-state response for two cases which are nominal and parameter uncertainty case. The objective function was guaranteed with subject to the LMI constrain to obtain the minimum upper bound of the $H_{\infty}$ norm worst magnitude in singular value. The results show that the $H_{\infty}$ norm estimator has an ability to estimate the total power deficit with very small estimation error in nominal case but not in parameter uncertainty case when the network has perturbed by uncertain inertia constant parameter.

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