Realization of fractional order lowpass filter using different approximation techniques

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ABSTRACT

Many research groups are starting to pay serious attention to the problem of fractional-order circuits. In this paper, a new approach to designing fractional order low-pass filter (FOLPF) is presented. Finding a rational approximation of the fractional Laplace operator $s^\alpha$ is a crucial step in the design of fractional order filters. A comparative study of the most widely used approximation techniques named continued fraction expansion (CFE) method and Biquadratic Approximation (RE) method is performed. Then the transfer function of the proposed FOLPF is calculated. Using operational amplifier, the proposed filter is synthesized. The proposed circuit is simulated using Texas instruments TINA software. The results obtained outperform the existing methods.

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depth analysis of the many developments that have been made in the realm of realising the fractance device. A variety of different comparable circuit representations of fractional order elements are given by [29], [30]. Narayan et al. [31] present a realization of a fractional order bandpass filter by employing a reconfigurable device. The paper is organised as follows: different rational approximation methods are discussed in section 2, section 3 deals with the FOLPF, section 4 discusses the implementation of the filter, the finalized circuit and the corresponding results are presented in section 5, and finally conclusions are drawn in section 6.

2. RATIONAL APPROXIMATION METHODS
Given that the fractional order system is in fact an infinite order system, it is preferable to approximate lengths with finite values. Both the Laplace and the \( z \) domains can be used to make an approximation of the fractional order operator known as \( s^\alpha \). Approximations that fall within the s-domain umbrella include the oustaloup approach, the mastudas method, and the least square method. Each approach has a number of advantages and disadvantages. The continued fraction expansion (CFE) method and Reyad Elkhazali (RE) approximation are both taken into consideration in this article as potential methods of approximation. Because of the complexity of the hardware, only approximation up to the second order is taken into consideration. It can be written as (1) [4], which is the second order approximation of \( s^\alpha \).

\[
s^\alpha = \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0}
\]  

where \( a_0, a_1, a_2 \) are the filter coefficients.

2.1. Continued fraction expansion method
The CFE of \((1 + x)^\alpha\) is used to characterise this technique [4], [32].

\[
(1 + x)^\alpha = \frac{1}{1 - \alpha x} \frac{1 + (1+\alpha)x}{2+\alpha x} \frac{1 + (2+\alpha)x}{3+\alpha x} \frac{1 + (3+\alpha)x}{4+\alpha x} \frac{1 + (4+\alpha)x}{5+\alpha x} \ldots
\]  

(2)

when \( x = s - 1 \) is substituted, the above CFE converges down the negative real axis from \( s = -\infty \) to \( s = 0 \) in the finite complex \( s \)-plane. Considering the first four terms of the expansion a second order approximation will be derived with the coefficients as (3):

\[
a_0 = \alpha^2 + 3\alpha + 2
\]
\[
a_1 = 8 - 2\alpha^2
\]
\[
a_2 = \alpha^2 - 3\alpha + 2
\]  

(3)

2.2. Biquadratic Approximation method
Khazali et al. [14] presented this approximation in 2019. This method is designated as RE to designate the name of the scientist. A biquadratic approximation approach can be used to estimate a fractance operator. The rational approximation will take the shape of a cascade connection of biquadratic transfer functions as (4):

\[
\left(\frac{\omega_i}{\omega_g}\right)^\alpha = \prod_{i=1}^{n} H_i \left( \frac{\omega_i}{\omega_g} \right) = \prod_{i=1}^{n} \frac{N_i \left( \frac{\omega_i}{\omega_g} \right)}{D_i \left( \frac{\omega_i}{\omega_g} \right)}
\]  

(4)

where

\[
\omega_i = \text{Center Frequency}
\]
\[
\omega_g = \text{Geometric Mean}
\]  

(5)

The remaining center frequencies may be computed using a recursive formula using \( \omega_1 \) as in (6):

\[
\omega_i = \omega_2^{2(i-1)} \omega_1, \quad i = 2, 3, 4, \ldots, n.
\]  

(6)

\( \omega_2 \) is calculated by solving in (7):

\[
a_0 a_2 \eta^4 + a_1 (a_2 - a_0) \gamma^3 + (a_1^2 - a_0^2 - a_2^2) \eta^2 + a_1 (a_2 - a_0) \gamma + a_0 a_2 \eta = 0
\]  

(7)

where \( \eta = \tan \left( \frac{\alpha \pi}{4} \right) \). The expression for the quadratic will be (8):
The values of the coefficients $a_0$, $a_1$, $a_2$ are calculated in (9):

$$a_0 = \alpha \alpha + 2 \alpha + 1$$

$$a_2 = \alpha \alpha - 2 \alpha + 1$$

$$a_1 = (a_2 - a_0) \tan \left( \frac{(2+\alpha)\pi}{4} \right)$$

After calculating the coefficients, the rational approximation for $s^\alpha$ may be determined. The second order rational approximations obtained for different values of $\alpha$ is as shown in Table 1.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$H_{RE}(s)$</th>
<th>$H_{CFE}(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>$1.994s^2 + 5.082s + 1.594$</td>
<td>$2.31s^2 + 7.98s + 1.71$</td>
</tr>
<tr>
<td>0.2</td>
<td>$1.594s^2 + 5.082s + 1.994$</td>
<td>$1.71s^2 + 7.98s + 2.31$</td>
</tr>
<tr>
<td>0.3</td>
<td>$1.225s^2 + 5.051s + 1.325$</td>
<td>$1.225s^2 + 7.92s + 1.44$</td>
</tr>
<tr>
<td>0.4</td>
<td>$1.325s^2 + 5.051s + 2.125$</td>
<td>$1.44s^2 + 7.92s + 2.64$</td>
</tr>
<tr>
<td>0.5</td>
<td>$1.257s^2 + 4.998s + 1.497$</td>
<td>$2.99s^2 + 7.82s + 1.19$</td>
</tr>
<tr>
<td>0.6</td>
<td>$1.099s^2 + 4.998s + 2.297$</td>
<td>$1.99s^2 + 7.82s + 2.99$</td>
</tr>
<tr>
<td>0.7</td>
<td>$0.930s^2 + 4.924s + 0.8941$</td>
<td>$3.36s^2 + 7.68s + 0.96$</td>
</tr>
<tr>
<td>0.8</td>
<td>$0.893s^2 + 4.924s + 2.499$</td>
<td>$0.96s^2 + 7.68s + 3.96$</td>
</tr>
<tr>
<td>0.9</td>
<td>$0.707s^2 + 4.828s + 0.7071$</td>
<td>$3.75s^2 + 7.54s + 0.72$</td>
</tr>
<tr>
<td>1.0</td>
<td>$0.707s^2 + 4.828s + 2.707$</td>
<td>$0.75s^2 + 7.54s + 3.72$</td>
</tr>
<tr>
<td>1.1</td>
<td>$0.930s^2 + 4.71s + 0.536$</td>
<td>$4.16s^2 + 7.28s + 0.56$</td>
</tr>
<tr>
<td>1.2</td>
<td>$0.536s^2 + 4.71s + 2.936$</td>
<td>$0.56s^2 + 7.28s + 4.16$</td>
</tr>
<tr>
<td>1.3</td>
<td>$0.317s^2 + 4.569s + 0.3791$</td>
<td>$4.59s^2 + 7.02s + 0.39$</td>
</tr>
<tr>
<td>1.4</td>
<td>$0.317s^2 + 4.569s + 3.149$</td>
<td>$0.39s^2 + 7.02s + 4.99$</td>
</tr>
<tr>
<td>1.5</td>
<td>$0.236s^2 + 4.404s + 0.2365$</td>
<td>$5.04s^2 + 6.72s + 0.24$</td>
</tr>
<tr>
<td>1.6</td>
<td>$0.236s^2 + 4.404s + 3.341$</td>
<td>$0.24s^2 + 6.72s + 0.04$</td>
</tr>
<tr>
<td>1.7</td>
<td>$0.109s^2 + 4.215s + 0.1095$</td>
<td>$2.51s^2 + 6.38s + 0.11$</td>
</tr>
<tr>
<td>1.8</td>
<td>$0.109s^2 + 4.215s + 3.711$</td>
<td>$0.11s^2 + 6.38s + 0.56$</td>
</tr>
</tbody>
</table>

A comparison of the two rational approximations (for $\alpha = 0.4$) is as shown in Figures 1 and 2. The phase response offered by the RE approximation is good as compared to CFE based method. From the magnitude response it is observed that the CFE based method produces good results for larger range of frequencies as compared to RE method. Keeping in view of the better linearity in phase angle, RE method is considered for the implementation.

3. FRACTIONAL ORDER LOW-PASS FILTER

Fractional order filters can meet the exact design criteria, but traditional integer-order filters were unable to do so. Comparing the design parameters to the integer-order filters, the fractional notion increases.
the level of safety a little amount. Fractional order filters make it simple to meet the requirements for the circuit
design and tuning parameters. Only fractional order filters make it feasible to vary and modify parameters like
the roll-off frequency to any desired slope. Thus, compared to integer-order filters, fractional order filters offer
greater design freedom.

A standard filter and a fractional-order filter vary in that additional parameters are included in the term
that determines the slope of attenuation of a given transfer function. Therefore, the equation for a fractional-
order filter’s attenuation of transfer function is \(20 \log_{10} \left( \frac{k_1}{s^{n+\alpha}} \right)\) (dB/decade). Where \(n\) is an unsigned integer number,
often between 1 and 10, and \(\alpha\) is defined as a real number in the range [0 1]. The transfer function of FOLPF
is given by (10) [24]:

\[
H_{LP}^{n+\alpha} = \frac{k_1}{s^n(s^{n+\alpha})}
\]  

(10)

where \(n\) is an integer and \(0 < \alpha < 1\). The following are the values of \(k\)’s as (11):

\[
k_1 = 1 \quad k_2 = 1.1796n^2 + 0.167\alpha + 0.21735 \quad k_3 = 0.19295\alpha + 0.81369
\]  

(11)

Substituting the second order approximation of the fractional order operator in the transfer function of the
low-pass filter, the equation becomes as (12):

\[
H_{LP}^{n+\alpha} = k_1 \left( a_2 s^2 + a_1 s + a_0 \right) \left( a_0 s^{n+1} + a_1 s^n + s^2 (k_3 a_2 + k_2 a_0) \right)
\]  

(13)

After rearranging the terms, the equation becomes as (13):

\[
H_{LP}^{n+\alpha} = k_1 \left( a_2 s^2 + a_1 s + a_0 \right)
\]  

(12)

(13)

For \(n = 1\), in (13) simplifies to (14):

\[
H_{LP}^{1+\alpha} = \frac{k_1 \left( a_2 s^2 + a_1 s + a_0 \right)}{a_0 (s^{1+\alpha} + c_1 s + c_2 + c_3)}
\]  

(14)

where,

\[
c_1 = (k_3 a_2 + k_2 a_0 + a_1) / a_0 \\
c_2 = (k_3 a_1 + k_2 a_1 + a_2) / a_0 \\
c_3 = (k_3 a_0 + k_2 a_2) / a_0
\]  

(15)

The equations indicate that the values of the filter coefficients are dependent upon the value of fra-
tional order \(\alpha\). Table 2 depict the values of filter coefficients for different value of fractional order. The transfer
functions of the FOLPF using different rational approximation techniques (for different values of \(\alpha\) as tabu-
lated in Table 3.

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(a_0)</th>
<th>(a_1)</th>
<th>(a_2)</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>2.31</td>
<td>7.98</td>
<td>1.71</td>
<td>3.497325</td>
<td>0.9053559</td>
<td>0.04038944</td>
</tr>
<tr>
<td>0.2</td>
<td>2.64</td>
<td>7.92</td>
<td>1.44</td>
<td>3.101633</td>
<td>0.9029765</td>
<td>0.08254491</td>
</tr>
<tr>
<td>0.3</td>
<td>2.99</td>
<td>7.82</td>
<td>1.19</td>
<td>2.794686</td>
<td>0.9580753</td>
<td>0.1200770</td>
</tr>
<tr>
<td>0.4</td>
<td>3.36</td>
<td>7.68</td>
<td>0.96</td>
<td>2.563302</td>
<td>1.046208</td>
<td>0.1501903</td>
</tr>
<tr>
<td>0.5</td>
<td>3.75</td>
<td>7.5</td>
<td>0.75</td>
<td>2.397695</td>
<td>1.149750</td>
<td>0.1721550</td>
</tr>
<tr>
<td>0.6</td>
<td>4.16</td>
<td>7.28</td>
<td>0.56</td>
<td>2.290444</td>
<td>1.255711</td>
<td>0.1942337</td>
</tr>
<tr>
<td>0.7</td>
<td>4.59</td>
<td>7.02</td>
<td>0.39</td>
<td>2.235792</td>
<td>1.354332</td>
<td>0.1941091</td>
</tr>
<tr>
<td>0.8</td>
<td>5.04</td>
<td>6.72</td>
<td>0.24</td>
<td>2.229228</td>
<td>1.438158</td>
<td>0.1966716</td>
</tr>
<tr>
<td>0.9</td>
<td>5.51</td>
<td>6.38</td>
<td>0.11</td>
<td>2.267138</td>
<td>1.501410</td>
<td>0.1957304</td>
</tr>
</tbody>
</table>

Table 2. Values of filter co-efficients for different values of \(\alpha\)
The above transfer function can be realized using only lossless integrators and multipliers.

\[ H(s) = \frac{1}{s^{n+1} + k_2 s^{n-2} + \ldots + k_{n-1} s + k_n} \]

(16)

\[ H_2(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s + a_n \]

(17)

\[ H(s) = H_1(s)H_2(s) \]

(18)

The above transfer function can be realized using only lossless integrators and multipliers.

\[ H_1 = \frac{V_{out}}{V_1} = A s^2 + B s + C \]

(19)

Now splitting the transfer function.

\[ H_1 = \frac{V_{out}}{V_1} = A s^2 + B s + C \]

\[ \Rightarrow V_{out} + (-A s^2 V_1 - B s V_1 - CV_1) = 0 \]

\[ \Rightarrow V_{out} + (-s^2 V_1 \frac{1}{17A} - s V_1 \frac{1}{17B} - V_1 \frac{1}{17C}) = 0 \]

(20)

\[ H_2 = \frac{V_1}{V_{in}} = \frac{1}{s^n + D s^{n-2} + E s + F} \]

(21)

The above equation can be rearranged as (22):

\[ V_{in} - (s^3 + Es) V_1 - (D s^2 + F) V_1 = 0 \]

\[ \Rightarrow V_{in} - (s^3 + s \frac{1}{17F}) V_1 - (s \frac{1}{17B} + \frac{1}{17C}) V_1 = 0 \]

(22)

The operational amplifier based realization valid for any fractional order \( \alpha \) is as shown in Figure 4.
5. RESULTS

To illustrate the methodology, a value of \( \alpha = 0.7 \) is considered. In (23) is the transfer function of RE approximation is taken into consideration.

\[
H(s) = \frac{3.791s^2 + 6.829s + 3.179}{s^3 + 6.829s^2 + 4.1712s + 0.6928}
\]  

(23)

The circuit realized is as shown in Figure 5. If frequency denormalization by a factor of 20k rad/sec and impedance scaling by a value of 10000, then all resistances and capacitance values will be practicable. The same circuit is used for the simulation purpose using TINA software. For the simulation purpose Texas instruments TINA software is used. The corresponding results of transient response and bode plot are as shown in Figures 6 and 7 respectively.
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6. CONCLUSION

This paper presents a novel realization of FOLPF using different approximation techniques. Initially, a comparative study of the existing approximation techniques for fractional order device is carried out. Then synthesis of the FOLPF is carried out using FLF topology. The proposed method is straightforward and makes use of an operational amplifier for the realization. This method makes use of only lossless integrators and amplifiers. A 100 mHz and 1 V_p-p sinusoidal signal is chosen as input. LM301A operational amplifier is chosen for simulation purpose. It is observed that there is a phase shift of 180 between input and output. The linear range of operation can be up to 100 mHz to 2 KHz. So, the procedure suggested can be used for the realization of fractional order filters.

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